CRANBROOK SCHOOL

Year 10 MATHEMATICS : Advanced

Term 4 1999   Time : 2 hrs/GJB, LD, CGH

Submit your work in three bundles :   A (answer sheet + question paper)
                                    B (qq. 31 - 33)
                                    B (qq. 34 – 36)

Section A :

In questions 1 - 20 answer the questions in the spaces provided on the separate answer sheet.
         In 1 - 10 circle the letter corresponding to your chosen answer.
         In 11 - 20 write the answers in the spaces provided.

In questions 21 - 30, **show all working** in the spaces provided on this question paper.

Enter your name in the spaces provided on the answer sheet and on this question paper.

NAME : _____ . _____

Answer questions 1 - 20 by circling the letter on the answer sheet corresponding to your choice of answer.

1. As a decimal $\dfrac{8}{1000} + \dfrac{7}{10} + \dfrac{6}{10000} =$

     A. 0.876   B. 0.7806   C. 0.786   D. 0.7086

2. After a pay rise of 20%, a man's new wage is $210. What was his original wage?

     A. $168   B. $175   C. $190   D. $170

3. If $a : b = 1 : 2$ and $b : c = 3 : 4$ then the value of $a : c$ is

     A. 1 : 4   B. 2 : 3   C. 3 : 8   D. 1 : 3

4. In a regular pentagon, the size of each interior angle is

     A. 108°   B. 72°   C. 90°   D. 120°

5. $2^{-\frac{1}{2}} =$

    A. -1 B. $\dfrac{1}{4}$ C. $-\dfrac{1}{\sqrt{2}}$ D. $\dfrac{1}{\sqrt{2}}$

6. $\left(2+\sqrt{3}\right)^{2} =$

    A. $4+3$ B. $5+2\sqrt{3}$ C. $7+4\sqrt{3}$ D. $7+\sqrt{12}$

. . .

7. $b^{2}-b-6 =$

    A. $(b+2)(b-3)$ B. $(b-2)(b-3)$ C. $(b-2)(b+3)$ D. $(b-6)(b-1)$

8. If $a>0$ and $b-a>0$, which of the following is FALSE?

    A. $a+b>0$ B. $b>0$ C. $ab>0$ D. $a-b>0$

9. If $7x=2x-45$, the value of x is

    A. -9 B. -5 C. 5 D. 9

10. In solving the equation $\dfrac{x+3}{3}+x=9$ we could first remove the fraction. In which of the following has this step been correctly done?

   A. $x+1+x=9$ B. $x+3+3x=9$ C. $x+3+3x=27$ D. $3x+9+3x=27$

In questions 11-20 you may simply write answers in the spaces provided on the answer sheet. Any necessary working may be done elsewhere. There is one mark for each question.

11. Write $4q^{2}-25t^{2}$ in factored form.

12. Solve for y: $7y + 5 = y + 12$

13. The probability of pulling a black marble out of a bag (containing black, red and white marbles) is $\frac{1}{3}$ and the probability of pulling a red marble from the same bag is $\frac{3}{8}$. If there are 48 marbles in the bag altogether, how many of them are white?

14. Solve for n: $3^{2n-1} = 27$

15. What fraction (in simplest form) is 30 metres of 60 kilometres?

16. Write as a fraction in simplest form: $3d^2 \div (3d)^2$.

17. Find all integral values of x for which $3 \leq 2x + 5 < 7$

18. In a certain election, there were only two candidates. The winner won by 120 votes, and received 60% of the votes cast. How many people voted?

Questions 19 and 20 refer to this diagram :

[DE is parallel to CB, AE = 8 cm, EB = 3 cm, AD = 6 cm]
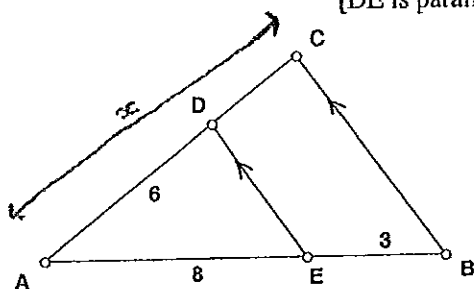

19. What is the length of CD in simplest fractional form?

20. If triangle ADE has an area of 64 cm², what is the area of trapezium EBCD?

In questions **21-30** there is space provided here to give setting out or reasons for your answer. There are two marks for each question.

NAME : _____

21.   If $\tan\theta = \dfrac{5}{12}$ give the exact value of $\cos\theta$.

22.   If the left hand sketch below is of the line $2x - 3y = 6$, draw on the right hand set of axes the graph of $2x - 3y > 6$.
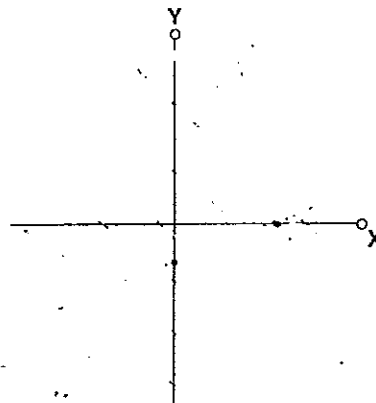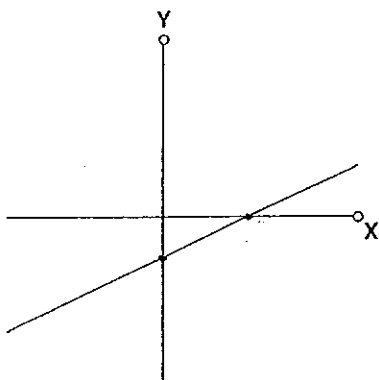
23.   Solve the equation $\dfrac{4x + 3}{5} - 2 = 1$

24.   For the scores   5, 7, 6, 10, 12, 18, 21, 26   use your calculator in statistics mode to write down the values of the

          Mean

          Standard Deviation

25. Find (in simplest form) the gradient of the line which passes through the points $(3,5)$ and $(-1,7)$.

. Find the equation of the line in question 25.

. Find the x and y intercepts of the line whose equation was found in question 26.

. Express as a single algebraic fraction: $\dfrac{x}{2(x-3)} + \dfrac{5}{x(x-3)}$

9. Find the volume of metal required to make 100,000 spherical all bearings, each of diameter 5cm (expressing your answer in m^3 correct to two decimal places].

1. Construct in the space below, and as accurately and neatly as possible, triangle ABC with sides AB = 15cm, AC = 11cm and BC = 9cm.

[Leave all construction lines in the diagram.

Write down the size of angle B in your triangle to the nearest degree.

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Section B :

In this section, show all working and pay particular attention to setting out. Marks may not be awarded for careless or untidy work, or where working is not shown.

Each question is worth 10 marks. Begin each question on a new sheet of examination paper.

Write only on one side of the paper. Place your name on each sheet of paper used.

. . .

31.

(a)   A is the point of intersection of the lines $x + 2y = 3$ and $2x - y = -5$

    (i)   Find the co-ordinates of A.

    (ii)   Find the equation of the line passing through A perpendicular to $x - 3y = 5$.

(b)   The midpoint of interval PQ, where P = $(-2,3)$, is $(4,-2)$. Find the co-ordinates of Q.

(c)   Draw a neat sketch of the line $2x - y + 3 = 0$, showing clearly its important features.

**32. (new page please)**

(a)   Given the following distribution

x	f
2	3
3	1
4	3
5	3
6	5

find (correct to three decimal places where necessary) the

(i) mode   (ii) median   (iii) mean   (iv) range   (v) standard deviation

(b)   Jeremy scores 75 on each of his tests in French and English. The statistics were as follows :

French   mean = 60, standard deviation = 10
English   mean = 65, standard deviation = 12

By using mathematical reasoning to support your answer, determine which would be considered better, his performance in French or his performance in English.

(c)   The probability that Anna wins any particular set of tennis is $\frac{3}{5}$.

    (i)   What is the probability she loses any particular set?

    (ii)   Determine the probability she wins a best-of-three-sets match in straight sets.

33. (new page please)

(a) Find θ if $\sin\theta = \dfrac{15\sin 68°12'}{32}$, giving your answer correct to the nearest minute.

(b) Find x, giving your answer correct to two significant figures.
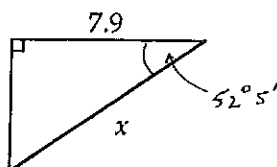
(c) A boat sails due north from a port P for 65 nautical miles, then changes course, sailing on a bearing of 323° for 80 nautical miles.

(i) Find the boat's final distance from P, correct to the nearest nautical mile.

(ii) the boat's final bearing from P, to the nearest degree.

34. (new page please)

(a) Simplify fully

(i) $\left(4 - 2\sqrt{3}\right)\left(4 + 2\sqrt{3}\right)$

(ii) $\sqrt{40} + \sqrt{160}$

(iii) $\dfrac{1}{\left(3 - \sqrt{3}\right)} - \dfrac{1}{\left(3 + \sqrt{3}\right)}$

(b) Our galaxy may be considered to be a sphere of radius 6×10^{19} m.

(i) What is this radius in km?

(ii) Find the surface area of a sphere of this size in km², correct to three significant figures.

(c) In every second our sun converts approximately 508×10^6 tonnes of hydrogen into 504×10^6 tonnes of helium. There is thus a loss associated with this conversion. What is the amount of this loss (expressing the answer in tonnes, and in scientific notation) in a period of 24 hours?

35. (new page please)

(a) The parabola $y = x^2 + x - 5$ is cut by the line $y = 3x + 10$. Find algebraically the points of intersection.

(b) Solve by completing the square $\quad x^2 - 6x - 1 = 0$
[Express your answers in simplest surd form.]

(c) Solve using the quadratic formula $\quad 2x^2 - 5x + 1 = 0$
[Give your answers correct to 2 decimal places.]

36. (new page please)

(a) Factor fully $\quad 25 - 9(x - 3)^2$

(b) Make p the subject of the equation $\quad 3r = \dfrac{.2}{p - 4}$

(c) Find the values of a and b.
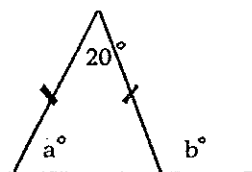
(d) Sketch a parallelogram ABCD with diagonal BD drawn.

Copy and complete the following proof that AB = DC.

Give a reason for each step of your proof.

Proof :   In triangles ABD and CDB

            Angle ABD = ...

            .... = Angle CBD

            BD ....

     $\therefore$ $\triangle ABD$..........$\triangle CDB$

     $\therefore$ ....

1. D	4. A	7. A
2. B	5. D	8. D
3. C	6. C	9. A
		10. C

11. $(2a+5t)(2a-5t)$

12. $y=\frac{7}{6}$

13. 14 white marbles

14. $n=2$

15. $\frac{1}{2000}$

16. $\frac{3}{4}$

17. $-1, 0$

18. 600 people

19. $2\frac{1}{4}$

20. 57 cm²

21.

$\cos\theta = \frac{12}{13}$

22.

23. $x=3$

24. Mean: 13.125
Standard Dev.: 7.32 (2dp)
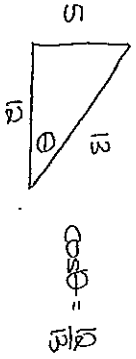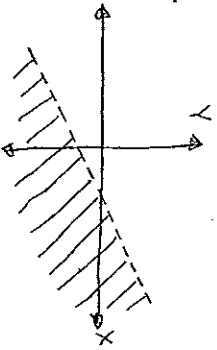
25. gradient = $-\frac{1}{4}$

26. $2y+x=13$

27. x-intercept: (13,0)
y-intercept: (0,6½)

28. $\dfrac{x^2+10}{2x(x-3)}$

29. 104719.76 m³ (2dp)

30. $\theta = 46°$

31.(a)(i) $x+2y=3$ $2x-y=-5$

Solving simultaneously:

$2(3-2y)-y=-5$ $x=3-2y$
$6-4y-y=-5$ $=-1\frac{2}{5}$
$5y=11$
$y=2\frac{1}{5}$ ∴ $A(-1\frac{2}{5}, 2\frac{1}{5})$.

(ii) $x-3y=5$
∴ gradient, $m=\frac{1}{3}$
∴ gradient of perpendicular line, $m_2=-3$.

$y=mx+b \Rightarrow 2\frac{1}{5}=(-3\times-1\frac{2}{5})+b \Rightarrow b=-2$

Equation: $3x+y+2=0$

(b) Midpoint, M: $\left(\dfrac{x_p+x_a}{2}, \dfrac{y_p+y_a}{2}\right) = (4,-2)$

ie $x_p+x_a=8$, $y_p+y_a=-4$
Given: $P(-2,3)$. That is $x_p=-2, x_a=3$.
hence, Q(10,-7).

(c) $2x-y+3=0$ x-intercept: $(-1\frac{1}{2},0)$
y-intercept: $(0,3)$.

$2x-y+3=0$
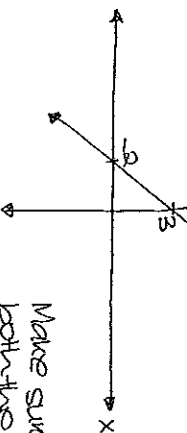
Make sure you label both the axes and the line!

34.(a).(i). $(4-2\sqrt{3})(4+2\sqrt{3}) = 16-12=4$

(ii). $2\sqrt{10}+4\sqrt{10} = 6\sqrt{10}$

(iii). $\frac{2+\sqrt{3}}{\sqrt{3}} - \frac{3\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

(b).(i). $r = 6\times10^{19}$ m $= 6\times10^{16}$ km.

(ii). Surface area, $S = 4\pi r^2$
$= 4.52\times10^{34}$ (3sf).

(c). Loss/second $= 508\times10^6 - 504\times10^6$
$= 4\times10^6$ tonnes
Loss/day $= 4\times10^6\times60\times60\times24$
$= 3.456\times10^{11}$ tonnes

35(a). $y = x^2+x-5$ $y = 3x+10$
Solving simultaneously:
$x^2+x-5 = 3x+10$
$x^2-2x-15 = 0$
$(x-5)(x+3) = 0$
$x=5, \ x=-3$
$y=25, \ y=1$
∴ points of intersection are $(-3,1)$ and $(5,25)$.

(b). $x^2-6x-1 = 0$
$x^2-6x+9 = 10$
$(x-3)^2 = 10$
$x-3 = \pm\sqrt{10}$
$x = 3\pm\sqrt{10}$

(c). $2x^2-5x+1 = 0$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{5\pm\sqrt{25-8}}{4} = 2.28, \ 0.22$ (2dp).

32.(a).(i). mode = highest frequency = 6.

(ii). There are 15 numbers.
The middle one is the 8th.
median = 5

(iii). mean = 4.4
(iv). range = 6-2 = 4.
(v). std.dev. = 1.497 (3dp).

(b). In French, Jeremy's mark was 1.5 standard deviations above the mean. In English, his mark was 0.83 standard deviations above the mean. French was the better performance.

(c).(i). $1-\frac{3}{5} = \frac{2}{5}$

(ii). $(\frac{3}{5})\times(\frac{3}{5}) = \frac{9}{25}$

33.(a). $\theta = 25°48'$

(b). $\cos(53°5') = \frac{7.9}{x}$
$x = 13$ (2sf).

(c). $323° - 180° = 143°$
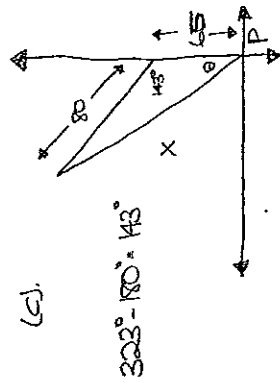
(i). Cosine rule:
$x^2 = 65^2+80^2-2(65)(80)\cos(143°)$
$x = 138$ nautical miles,
to the nearest naut. mile.

(ii). Sine rule:
$\frac{80}{\sin\theta} = \frac{138}{\sin143°}$ ∴ $\theta = 20°29'$

Bearing $= 340°$ T, to the nearest.

36.(a). $25 - 9(x-3)^2$

$= (5)^2 - [3(x-3)]^2$

$= [5 + [3(x-3)]][5 - [3(x-3)]]$

$= [5 + 3x - 9][5 - 3x + 9]$

$= (3x - 4)(14 - 3x)$.

(b). $3r = \dfrac{2}{p-4}$

$\dfrac{2}{3r} = p - 4$

$p = \dfrac{2}{3r} + 4$

(c).
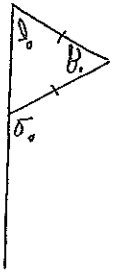
Isosceles Triangle.

$\therefore 2a° + 20° = 180°$

$a = 80°$

b is the exterior angle.

$b = 20° + a$

$= 100°$

(d).
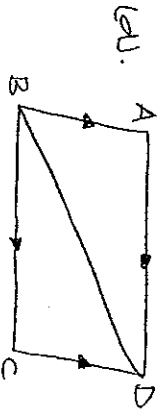
Proof: In triangles ABD and CDB

Angle ABD = Angle CDB (alternate angles).

Angle ADB = Angle CBD (alternate angles).

BD is similar to both triangles.

$\therefore \triangle ABD \cong \triangle CDB$ [ASA].

$\therefore AB = DC$ (corresponding sides in congruent triangles).