



## 14. Variation - Part 1

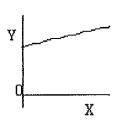
1. Which graph shows direct variation existing between the variables?

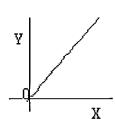
A.

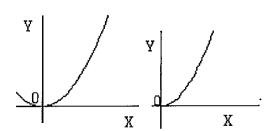
В.

C.

D.







Ans: B.

2. Which of the tables illustrates a  $\alpha$  b?

A.

b	6	4	1
a	3	2	1/2

В.

C.

b	1	2	3	
a	3	4	5	

D.

b	9	4	1	
a	3	2	1	

Ans: A

3. Given

х	5	3	2	
у	12	20	30	

then

A. y varies inversely	y as x			
B. y varies directly	as x			
C. y varies as x <sup>2</sup>				
D. no relationship be	etween x and y			
Ans: B				
4. If a varies as b	and $a = 12$ wh	nen $b = 3$ then	n, if $b = 7$ , a	has the value:
A. $5\frac{1}{7}$ B. 4	C. 28		D. 29	
Ans: C				
5 70 2 1 4				
5. If $y \propto x^2$ and $y = 4$	5 when $x = 3$ , t	hen when $x = 4$	, y equais:	
A. 80	B. 180	C. 20	D 45	
Ans: A				
6. Given tα√m. If t	t = 2 when m =	1, then if $m = 1$	then if $m = 9$ ,	t equals:
A. 9	В. 18	C. 3	D. 6	
Ans: D				
7. Given that $c \propto n^2$ ,	if $c = 50$ when	n = 10, then if	n = 5, c equals	:
A. 25	В. 100	C. 12½	<u>.</u>	D. 50
Ans: C				
8. If a α b then the ef	fect on a of tre	ebling b is:		
A. treble	B. divide by 3	C. add	3	D. cube
Ans: A				

9. If A  $\alpha$  m<sup>2</sup> then if m is halved, the value of A is:

A. multiplied by 4

- B. divided by 4
- C. doubled
- D. halved

Ans: B

10. If A  $\alpha \sqrt{m}$  then if m is multiplied by 4, the value of A is:

A. multiplied by 2

B. square root C. divided by 4

D. multiplied by 4

Ans: A

11.

m	2	4	5	
n	3	6	$7\frac{1}{2}$	

Show that direct variation exists between m and n.

Ans: 
$$\frac{n}{m} = \frac{3}{2}$$

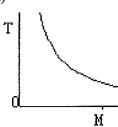
12.

Does y vary directly as x? Give reasons for your answer.

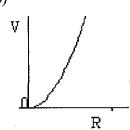
Ans: No,  $\frac{y}{x}$  not constant

13. State whether or not direct variation exists in each of the following:

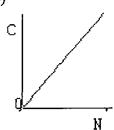
a)



b)

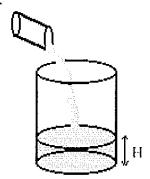


c)



Ans: (a) no (b) no (c) yes

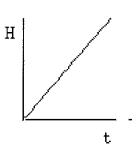
14.



Water flows into a cylindrical water tank at a constant rate.

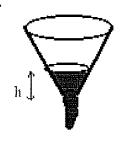
- a) Sketch a graph of the height of the water (H) against time (t);
- b) Does the height of the water vary directly with time?

Ans: (a)



(b) yes

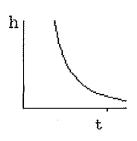
15.



Oil flows out of the filter at a constant rate.

- a) Sketch a graph of the height of the oil
- (h) against time (t);
- b) Does the height vary directly with time?

Ans: (a)



(b) no

16. If  $a \alpha b$  and b = 30 when a = 5, find the value of a when b = 7.

Ans: 42

17. If  $g \alpha h$  and g = 24 when h = 8, find the value of g when h = 11.

Ans: 33

18. If  $t \alpha s$  and t = 9 when s = 3, find the value of s if t = 24.

Ans: 8

19. If  $y \propto w$  and y = 35 when w = 10, find the value of w if y = 25.

Ans: 7.14

20.

Х	1	2	4
у	4	16	100

Show that  $y \propto x^2$ 

Ans: 
$$\frac{y}{x^2} = 4$$

21.

a	1	4	9	16
b	3	6	9	12
√a	•••		•••	

- a) Complete the table.
- b) Show that  $b \propto \sqrt{a}$

Ans: 1, 2, 3, 4 (b) 
$$\frac{b}{\sqrt{a}} = 3$$

- 22. Write down the relationship between the variables using k as the constant variation.
- a) The profit (\$P) varies directly as the number of participants (n);
- b) The volume (V) of a spherical ball varies directly as the cube of the radius (r).

Ans: (a) 
$$P = kn$$
 (b)  $V = kr^3$ 

- 23. Given x varies as the square of y, and x = 18 when y = 3:
- a) write an equation giving the relationship between x and y using k for constant of variations;
- b) find the value of the constant of variation;
- c) find the value of x when y = 11.

Ans: (a) 
$$x = ky^2$$
 (b) 2 (c) 242

24. 
$$t \alpha m^2$$
. If  $t = 24$  when  $m = 4$ 

- a) find the constant of variation;
- b) find the value of t if m = 5.

Ans: (a) 1.5 (b) 37.5

25.  $c \propto \sqrt{d}$  If c = 90 when d = 9

- a) find the constant of variation;
- b) find the value of c when d = 4.

Ans: (a) 30 (b) 60

26.  $e \propto \sqrt{f}$  If e = 12 when f = 4

- a) find the constant of variation;
- b) find the value of e when f = 9.

Ans: (a) 6 (b) 18

- 27. A transistor radio used 0.2 Amp of current when operated by a 9-volt battery. Given that voltage varies directly as the current for a given resistance:
- a) What is the resistance of the radio circuit?
- b) What voltage would use 0.6 Amp of current in the radio circuit?

Ans: (a)  $45\Omega$  (b) 27 Volt

- 28. A car travelling at constant speed travels 232 km in 4 hours. Given that the distance travelled is directly proportional to the time taken, find:
- a) the distance travelled in 7 hours;
- b) the time taken to travel 638 km.

Ans: (a) 406 km (b) 11 hours

29. The surface area of a ball varies directly as the square of its radius. If the surface area of a ball of radius 5 cm is 314.16 cm<sup>2</sup>, find the surface area of a ball of radius of 7 cm.

Ans: 502.65 cm

30. The distance of the horizon is proportional to the square root of the height above sea level. If I stand on the top of a cliff 100 m above sea level, I can see 35777 m to the horizon. How far could I see if I climbed half way down the cliff?

Ans: 25298 m

31. The distance a ball can be thrown varies directly as the square of the speed with which it leaves the hand. When the speed is  $16 \text{ ms}^{-1}$ , the distance is 24 m. Find the distance when the speed is  $12 \text{ ms}^{-1}$ .

Ans: 13.5 m

32. The profit made by a manufacturer varies directly with the number of hours the plant fully operates. If the profit is \$3000 when the plant fully operates for 8 hours, for how long must the plant operate for a profit of \$100000 to be made?

Ans:  $266\frac{2}{3}$  hours

33. For an investment of \$5000 I receive \$700. Given that the interest varies directly with time, how much interest wouldd I receive on an investment of \$6500?

Ans: \$910

34. If  $y \alpha x$  what is the resultant change in y when x is doubled?

Ans: Doubled

35. If  $a \alpha b$  what is the resultant change in a when b is halved?

Ans: Halved

36. If  $p \alpha q$  what change in q results in p doubling?

Ans: Doubled

37. If  $t \alpha y^2$  what change in t results from y doubling?

Ans: Multiplied by 4

38. If  $m \alpha n^2$  what change in m results from n being halved?

Ans: Divided by 4

39. If  $a \alpha \sqrt{b}$  what change in a results from b being doubled?

Ans: Multiplied by  $\sqrt{3}$ 

40. If  $g \alpha \sqrt{h}$  what change in g results from h being halved?

Ans: Multiplied by  $\sqrt{\frac{1}{2}}$ 

41. If a  $\alpha$  b<sup>2</sup> what change in b results in a doubling?

Ans: Multiplied by  $\sqrt{2}$ 

42. If  $t \alpha s^2$  what change in s results in t being halved?

Ans: Divided by  $\sqrt{2}$ 

43. If  $s \alpha \sqrt{m}$  what change in m results in s being doubled?

Ans: Multiplied by 4

44. If  $x \alpha \sqrt{y}$  what change in y results in x being trebled?

Ans: Multiplied by 9

45. The area of a square varies directly as the square of the side length. What is the effect on the area if the side length is increased by 20%?

Ans: Increased by 44%

46. The cost of a repair varies directly with the time taken by the technician. What is the effect on the cost if the technician takes 10% less time to complete the job?

Ans: Decreased by 10%

- 47. The wages bill for a firm employing 10 people varies directly with the wage paid to the employees.
- a) Assuming all employees are paid the same amount, what is the effect on the total wage bill of a wage rise of 5%?
- b) Would it make any difference if the employees were not all paid the same amount?

Ans: (a) 5% (b) yes - higher wage, bigger actual increase

48. Investigate the effect on the volume of a sphere of changing its surface area.

Ans: Change of volume with surface area is as r<sup>2</sup>

49. Which of the graphs shows an inverse variation existing between the variables?

A. Y X

 C. Y

Y 0 X

D.

Ans: B

50. For which of the following does b vary inversely as a?

A.

a	1	2	3
b	5	4	2

В.

a	2	3	4
b	4	6	8

C.

a	1	2	3
b	12	6	4

D.

a	1	2	3
b	-1	-2	-3

Ans: C

## 51. Given the following information:

a	2	4	6	8
b	12	6	4	3

which of the statements is true?

- A. b varies inversely as a
- B. b varies directly as a
- C. b varies as a
- D. there is no relationship between a and b.

Ans: A

52. Given that m varies inversely as n and that m = 5 when n = 4, the constant of variation is:

- A.  $\frac{5}{4}$  B.  $\frac{4}{5}$  C.  $\frac{1}{20}$

- D. 20

Ans: D

53. Given that y varies inversely as the square of x and y = 2 when x = 5, the constant of variation is:

- A. 10
- B. 50

Ans: B

C.

a	1	2	3
b	12	6	4

D.

a	1	2	3
b	-1	-2	-3

Ans: C

51. Given the following information:

a	2	4	6	8	
b	12	6	4	3	

which of the statements is true?

- A. b varies inversely as a
- B. b varies directly as a
- C. b varies as a
- D. there is no relationship between a and b.

Ans: A

52. Given that m varies inversely as n and that m = 5 when n = 4, the constant of variation is:

- A.  $\frac{5}{4}$  B.  $\frac{4}{5}$  C.  $\frac{1}{20}$

- D. 20

Ans: D

53. Given that y varies inversely as the square of x and y = 2 when x = 5, the constant of variation is:

- A. 10 B. 50 C.  $\frac{2}{25}$
- D.  $\frac{2}{5}$

Ans: B

54. If p varie variation is:	es inversely as the so	quare of q ar	and $p = 4$ when $x = 2$ then the constant of		
· A. 8	B. $\frac{1}{2}$	C. 16	D. 2		
Ans: C					
55. If $t \propto \frac{1}{h}$	and $t = 6$ when $h = 3$	s, the value of	ft when h = 9 is:		
A. 2	B.1	C. ½	D. 3		
Ans: A					
56. If $g \propto \frac{1}{h^2}$	and $g = 3$ when	h=4, the val	lue of $g$ when $h=2$ is:		
A. 24	B. 12	C. 6	D. 8		
Ans: B					
57. m varies	inversely as √n and	m = 6 when	n = 4. The value of m when $n = 9$ is:		
A. 2	B. 4/3	C. 1	D. 4		
Ans: D					
58. If b varie	es inversely as c, m	ultiplying c b	by 3 means that:		
A. b is	divided by 3	В.	b is multiplied by 3		
C. b is	left unchanged	D.	3 is subtracted from b		
Ans: A					
59. h varies inversely as k. If k is halved, then:					
A. h is	s halved	B. h is d	loubled C. h is increased by 2		

D. there is no change in h

Ans: B

60. a varies inversely as the square of b. If b is doubled, then:

A. a is halved

B. a is divided by 4

C. a is squared

D. there is no change in a

Ans: B

61. t varies inversely as the square of y. If t is multiplied by 4, then:

A. y is divided by 2 B. y is divided by 4

C. y is divided by 16

D. there is no change in y

Ans: A

62. c varies inversely as the square root of d:

A. d is multiplied by 9

B. d is divided by 3 C. d is multiplied by 3

D. d is divided by 9

Ans: C

63.

b	12	6	8	10
С	2	4	3	2.4

Show that inverse variation exists between b and c.

Ans: bc = 24

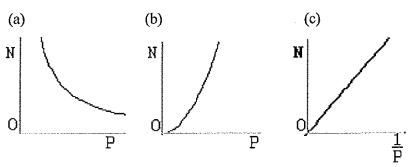
64.

а	30	20	5	
t	6	9	36	

Show that t varies inversely as a.

Ans: at = 180

65. State whether or not inverse variation exists between P and N in each of the following:



Ans: (a) yes (b) no (c) yes

66. Given that t varies inversely as y:

- a) find the constant of variation;
- b) complete the table:

S	2	4		40	
t	20		8	1	

Ans: (a) 40 (b) 10;5

67.

x	5	8		1 60
у	$\frac{1}{10}$		12	30

Given that y  $\alpha \frac{1}{x}$ 

- a) find the constant of variation;
- b) complete the table.

Ans: 
$$(a)^{\frac{1}{2}}$$
  $(b)^{\frac{1}{16}}$ ;  $\frac{1}{24}$ 

68.

m	2		4	4.5
n	6	3	•••	4

Given that m  $\alpha \frac{1}{n^2}$ 

- a) find the constant of variation;
- b) complete the table.

Ans: (a) 72 (b) 8; 4.24

69.

h	10	8		25.6
r	8	•••	10	5

Given that h  $\alpha \frac{1}{r^2}$ 

- a) find the constant of variation;
- b) complete the table

Ans: (a) 640 (b) 8.94; 2.53

70. m varies inversely as w. If m = 8 when w = 3, find:

- a) m when w = 6
- b) w when m = 10

Ans: (a) 4 (b) 2.4

71. y varies inversely as x. If y = 12 when x = 5, find:

a) y when x = 15

b) x when 
$$y = 7$$

72. y varies inversely as  $x^2$ . If y = 12 when x = 2, find:

a) y when 
$$x = 3$$

b) x when 
$$y = 8$$

73. g varies inversely as  $\sqrt{h}$ . If g = 12 when h = 4, find:

a) g when 
$$h = 9$$

b) h when 
$$g = 8$$

74. Write an equation using k for the contant of variation, to express the relationship:

- a) length (1) varies inversely as the width (w);
- b) height (h) varies inversely as the square of the radius(r).

Ans: (a) 
$$l = \frac{k}{w}$$
 (b)  $h = \frac{k}{r^2}$ 

75. The force, F, of gravitational attraction between two bodies varies inversely as the square of the distance, d, between them. The force is 20 when the distance is 6. Find:

- a) the constant of variation;
- b) the force when the distance is 9.

76. The time taken for a trip is inversely proportional to the average speed. If I can complete my trip in 8 hours if my average speed is 60 km/hr, how long will the trip take if I travel at 50 km/h?

77. The current I in an electrical circuit varies inversely as its resistance R. A current of 8 Amps passes through a part which has a resistance of 10 ohm. What would the current be if a part of resistance 6 ohm were put into the same circuit?

Ans: 13.3 Amp

78. The volume, V cm³, of a gas varies inversely as the pressure P N/cm². What is the pressure when the volume is 20 cm³?

Ans: 60 N/cm<sup>2</sup>

79. For a cylinder of a given volume, the height is inversely proportional to the square of the radius. If, for a given cylinder, the height is 20 cm when the radius is 5 cm, what would be the height of the cylinder if the radius were 3 cm?

Ans: 55,5 cm

80. For a cone of a given volume, the height is inversely proportional to the square of the base radius. If the height is 15 cm when the radius is 3 cm, what would be the radius if the height were 20 cm?

Ans: 2.6 cm

81. The time taken to do a job varies inversely as the number of workers employed. If 24 workers take 30 hours to do the job, how long would it have taken if 20 workers had been employed?

Ans: 36 hours

82. If a varies inversely as b, wht is the effect of doubling b?

Ans: Half

83. If m varies inversely as n, what is the effect on m if n is divided by 3?

Ans:  $m \times 3$ 

84. If 
$$x \propto \frac{1}{y^2}$$
, what is the effect on x of doubling y?

Ans:  $x \div 4$ 

85. If  $a \propto \frac{1}{\sqrt{b}}$ , what is the effect on a of multiplying b by 9?

Ans:  $a \div 3$ 

86. If  $m \propto \frac{1}{n^2}$ , what is the effect on *n* of multiplying *m* by 25?

Ans:  $n \div 5$ 

87. If  $t \propto \frac{1}{\sqrt{y}}$ , what is the effect on y of dividing t by 2:

Ans:  $y \times 4$ 

88. If the force F varies inversely as the square of the distance d (that is  $F \propto \frac{1}{d^2}$ ). If d is increased by 10%

- a) express the new distance as a percentage of the original distance;
- b) express your answer to Part (a) as a decimal;
- c) express the new force as a multiple of the original force;
- d) what percentage is the new force of the original force?
- e) what effect did the 10% increase in d have on F?

Ans: (a) 110% (b) 1.1 (c) 0.826F (d) 82.6% (e) decrease by 17.4%

89. If  $a \propto \frac{1}{\sqrt{h}}$ , what is the effect on a of decreasing b by 10%?

Ans: a multiplied by 1.05, that is a 5% increase in a

A. αα t	B.	$a \propto (bc)^2$	C. $a \alpha b + c^2$	D. $a \propto (b+c)^2$
Ans: A				
91. If <i>m</i> varies as:	s jointly as $p$ and	the square root	of $q$ , the relationsh	ip can be expressed
A. <i>m</i> ∝	$p + \sqrt{q}$ B.	$m \propto \sqrt{p+q}$	C. $m \propto p\sqrt{q}$	D. $m \propto \sqrt{pq}$
Ans: C				
92. m α nq If	m = 24 when $n = 2$	3 and $q = 2$ , the	constant of variation	n is:
A. 6	B. 4	C. 4	$1\frac{4}{5}$ D. 144	
Ans: B				
93. aαbc Ifa	= 48  when  b = 4 a	and $c = 2$ , the $cc$	onstant of variation i	is:
A. 6	В. 384	C. 8	B D. 24	
Ans: A				
94. gαht² If g	g = 36 when $h = 2$	and $t = 3$ , the co	onstant of variation i	is:
A. 6	B. 2	C. $7\frac{1}{5}$	D. 12	
Ans: B				
95. Given that i	$m \propto \frac{b}{c}$ and $m = 6$	when $b = 2$ and	c = 2, the constant of	of variation is
A. 1	B. $\frac{3}{2}$	C. 6	D. 3	
Ans: C				

90. If a varies jointly as b and the square of c the relationship can be expressed as:

96.  $g \alpha mn$  If g = 40 when m = 2 and n = 2, the value of g when m = 2 and n = 5 is:

A. 100

B. 20

C. 10

D. 8

Ans: A

97.  $y \alpha xz$  If y = 50 when x = 5 and z = 2, the value of y when x = 10 and z = 1 is:

A. 5

B. 50

C. 100

D. 25

Ans: A

98.  $a \propto \frac{b}{c}$ . If a = 6 when b = 6 and c = 2, the value of a when b = 12 and c = 3 is:

A.  $\frac{1}{6}$ 

B. 72

C. 8

D. 4

Ans: C

99.  $g \propto pq^2$  If g = 36 when p = 1 and q = 3, the value of g when p = 2 and q = 4 is:

A. 64

B. 32

C. 8

D. 16

Ans: A

100.

а	3	5	6	
b	8	7	3	
c	48	70	36	

Show that c varies jointly as a and b.

Ans: c = 2ab

101.

m	5	7	11
n	12	8	7
р	30	28	$38\frac{1}{2}$

Show that  $p \propto mn$ .

Ans: p===mn.

## VARIATION Part 3

102.

а	2	5	6	
b	2	3	2	
C	24	135	72	_

Show that  $c \propto ab^2$ .

Ans:  $c = 3ab^2$ 

103.

a	10	8	6
b	2	3	4
c	10	5 <sup>1</sup> / <sub>3</sub>	3

Show that  $c \propto \frac{a}{h}$ .

Ans:  $c = \frac{2a}{b}$ 

104. Express the following relationships as equations using k as the constant of variation:

- a) t varies jointly as m and n
- b) h varies directly as t and inversely as the square of d

Ans: (a) t = kmn (b)  $h = k \frac{t}{d^2}$ 

105. If  $z \propto xy$  and z = 40 when x = 5 and y = 4, find z when x = 6 and y = 8.

Ans: 96

106. If  $t \alpha uv$  and t = 120 when u = 5 and v = 6, find t when u = 8 and v = 3.

Ans: 96

107. If 
$$g \propto \frac{h}{k}$$
 and  $g = 8$  when  $h = 12$  and  $k = 3$ , find g when  $h = 18$  and  $k = 9$ .

Ans: 4

108. If 
$$a \propto \frac{m}{n^2}$$
 and  $a = 8$  when  $m = 16$  and  $n = 5$ , find  $a$  if  $m = 3$  and  $n = 2$ .

Ans:  $9\frac{3}{8}$ 

109. d 
$$\alpha$$
 m<sup>2</sup>n If d = 96 when m = 4 and n = 3, find n when d = 50 and m = 5.

Ans: 1

110. The interest paid on my investment varies jointly as the rate of interest and the length of time. If the interest is \$700 when the rate is 7% p.a. and the time 5 years, what would be the interest if the rate were 3% and the time 4 years?

Ans: \$240

111. The area of a figure varies jointly as the length and the width. If the area is 80 when the length is 20 and the width 8, find the area when the length is 16 and the width 10.

Ans: 80

112. The resistance R ohms of a wire varies directly as the length, I, and inversely as the square of its diameter, d. The resistance is 70 ohms when the wire has length 20 cm and a diameter of 1 mm. What is the resistance of a 30 cm length of wire which has a diameter of  $\frac{1}{2}$  mm?

Ans: 210 ohms

113. The energy of a moving body varies as its mass and the square of its velocity. If a mass of 10 kg has a velocity of 30m/sec its kinetic energy is 4500 joule. Find the kinetic energy of a mass of 2 kg moving at 50m/sec.

Ans: 2250 J

114. The volume of a cylinder varies jointly as the square of the radius and the height. If a cylinder of radius 7 cm and height 12 cm has a volume of 1848 cm<sup>3</sup>, find the volume when the radius is 6 cm and the height is 20 cm.

Ans: 2262.857 cm<sup>3</sup>

115. m \alpha pg What is the effect on m of:

- a) doubling p
- b) multiplying q by 3
- c) doubling p and multiplying q by 3

Ans: (a) double (b)  $\times$  3 (c)  $\times$  6

116.  $t \propto ab$  What is the effect on t of:

- a) halving a
- b) doubling b
- c) halving a and doubling b

Ans: (a) half (b) double (c) none

117.  $c \propto ab^2$ 

- a) doubling a
- b) doubling b
- c) doubling a and b

Ans: (a) double (b)  $\times$  4 (c)  $\times$  8

118. h  $\alpha$  kg<sup>3</sup> What is the effect on h of:

- a) doubling k
- b) multiplying g by 3
- c) doubling k and multiplying g by 3

Ans: (a) double (b)  $\times$  27 (c)  $\times$  54

119.  $x \propto \frac{a}{b}$  What is the effect on x of:

- a) doubling a
- b) halving b
- c) doubling a and halving b

Ans: (a) double (b) double (c)  $\times$  4

120.  $F \propto \frac{m_1 m_2}{d^2}$ . What is the effect on F of doubling  $m_1$  and  $m_2$ ?

Ans:  $\times$  4

121.  $R \propto \frac{m}{d^2}$ . What is the effect on R of doubling m and halving d?

Ans:  $\times$  8

122. E  $\alpha$  mV<sup>2</sup> If m is doubled, what change in V must occur for E to be unchanged?

Ans:  $\div \sqrt{2}$ 

123.  $F \propto \frac{m_1 m_2}{d^2}$ . If  $m_1$  and  $m_2$  are both doubled, what change in d must occur for F to be unchanged?

Ans: Ans:  $\times$  2

