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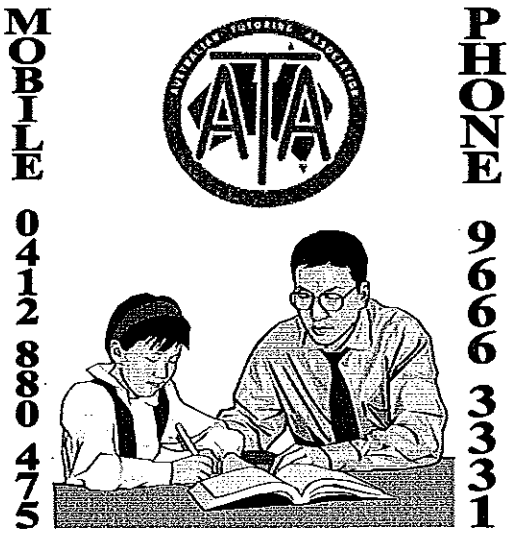

YEAR 10 – ADVANCED MATHS

REVIEW TOPIC A5: FURTHER REASONING IN ALGEBRA

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A5.1 Order of operation; Inequalities; Integers:

State whether **TRUE** or **FALSE** for each and give a reason or an example for each false statement.

1. (a) $5\times3-15\div7=0$	(b) $3+9\div2-1=5$
(c) $4+7\times2\geq(4+7)\times2$	(d) $\frac{8}{2+6}<\frac{6}{8+2}$
2. If p,q,r,s are consecutive numbers, then	
(a) $p+q+r$ is divisible by 3	(b) $p+q+r+s$ is a multiple of 4
(c) $qs+pr$ is even	(d) pqr is odd
(3) If a,b,c are positive integers and if $\frac{c}{a}$ and $\frac{c}{b}$ are integers, then the following are also integers: True or False? Give a reason or an example for your answer.	
(a) $\frac{b}{c}$	(b) $\frac{c}{a+b}$
(c) $\frac{c^2}{ab}$	(d) $\frac{ab}{c^2}$

A5.2 Odd, Even and Prime Numbers:

(1) State whether **TRUE** or **FALSE** for each and give a reason or an example for each false statement. If p is odd and q is even, then

(a) $p+q$ is odd	(b) $2q-3$ is even
(c) p^2+q is even	(d) p^3 is odd

2. If J is a positive integer, then

(a) $2J-1$ is always odd	(b) $J(J+1)$ is always even
(c) $J(J+1)(J+2)$ is divisible by 3	(d) J^2-J+11 is always prime

A5.3 Basic Algebra:

Write each as a TRUE statement.

(1) $a+a+a=a^3$	(2) $a(bc)=ab+ac$
(3) $a^4+a^4=a^8$	(4) $(-a)^3=a^3$
(5) $\frac{a+b}{a}=1+b$	(6) $\frac{2}{a}+\frac{3}{a}=\frac{5}{2a}$
(7) $\frac{a}{2}+\frac{a}{3}=\frac{2a}{5}$	(8) $(a+b)^2=a^2+b^2$

(9) $\sqrt{a^2+b^2}=a+b$	(10) $\frac{a-b}{b-a}=1$
(11) $\frac{a}{a+1}-\frac{a-1}{a}=\frac{1}{a+1}$	(12) $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$
(13) $\frac{\frac{a}{a+b}}{\frac{b}{a+b}}=\frac{ab}{(a+b)^2}$	(14) $\frac{\frac{ab}{a+b}}{\frac{b}{a^2-b^2}}=\frac{ab^2}{a-b}$

A5.4 Subject of the formula:

Make ***r*** the subject of the formula in each case:

(1) $1-p=r+2$	(2) $rq=p-1$
(3) $\frac{1}{p}+\frac{1}{q}=\frac{1}{r}$	(4) $\sqrt{p^2-r^2}=q$

(5) $pq+r=q+rp$	(6) $S=\dfrac{ar}{1-r}$
(7) $T=ar^{n-1}$	(8) $A=P\left(1+\dfrac{r}{100}\right)^n$

A5.5 Defining an operation such as *.

Example:

$a*b=\dfrac{a}{b}+\dfrac{b}{a}$ $\quad\therefore 1*2=\dfrac{1}{2}+\dfrac{2}{1}=2\dfrac{1}{2}$

(1) If $p*q=\dfrac{p+q}{p-q}$, evaluate the following:

(a) $5*2$	(b) $\dfrac{1}{2}*\dfrac{1}{3}$

(2) If $x*y=x^y+y^x$, evaluate the following:

(a) $2*4$	(b) $2*-4$
(c) $-2*4$	(d) $-2*-4$

A5.6 Number Patterns:

Example: **Q.** For the number pattern 2, 4, 6, __, __, __. Write down the next three terms and explain your reasoning.
A. Next 3 terms are 8, 10, 12 by adding 2 to the previous term.

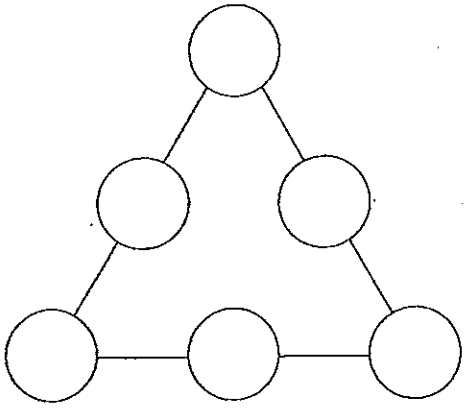
Write down the next terms in each of the following number patterns and explain your reasoning for doing so.

(1) 2, 4, 8, ___, ___, ___.	(2) 1, 4, 9, ___, ___, ___.
(3) 1, 2, 4, 7, ___, ___, ___.	(4) 2, 4, 12, 48, ___, ___, ___.
(5) 64, 32, 16, 8, ___, ___, ___.	(6) 1, 1, 2, 3, ___, ___, ___.

A5.7 Problem Solving: Some useful strategies to apply in problem solving are:

1. Simplify the problem in your own words.
2. Draw a clear and concise diagram.
3. Look for a pattern.
4. Construct a table if necessary.
5. Write an equation.
6. Guess and check the answer, if possible.
7. Use a related problem.
8. Break down the problem into smaller portions.

(1) Place the digits 1, 2, 3, 4, 5 and 6 in the circle so that the sum of the three numbers on each side of the triangle is 12.



(2)

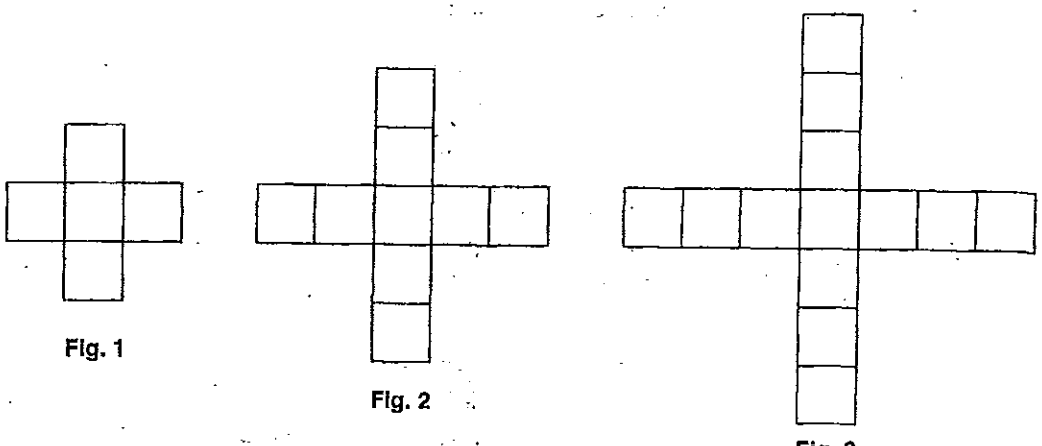


Fig. 1

Fig. 2

Fig. 3

(a) Complete the table of values below.

Figure number	**1**	**2**	**3**	**4**	**5**
Number of squares	**5**	**9**			

(b) By considering the number patterns, without drawing further diagrams, write down the number of squares in
(i) figure 10 (ii) figure 500

(Hint: Use the formula $T_n = a + (n-1)d$)

(c) Write down the number of the figure that has 165 squares.

(d) The number of squares in Figure f is denoted by s.
Write an equation that relates f and s.

(3) The diagram shows a sequence of sheep enclosures built using fences and posts.

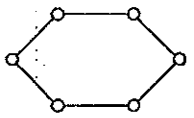


Fig. 1

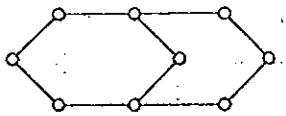


Fig. 2

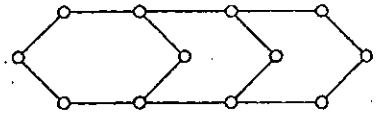


Fig. 3

(a) Sketch the next figure in the sequence.

(b) Complete the table below.

Figure number	**1**	**2**	**3**	**4**	**5**	**6**
Number of enclosures (E)	**1**	**2**				
Number of posts (P)	**6**	**9**				

(c) Write down the number of posts needed to build a figure having 20 enclosures.

(d) Write down, an expression to find the number of posts needed to build E enclosures.

(e) How many posts would be needed to build 560 enclosures?

(4) The diagram shows a sequence of arrangements of tables. Each table is in the shape of a trapezium and could seat 5 people. When the tables are joined together as shown below, each table cannot seat as many people.

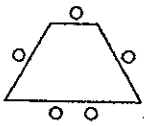


Fig. 1

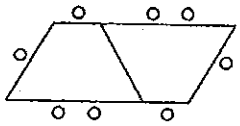


Fig. 2

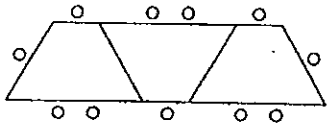


Fig. 3

(a) Complete the table below.

Figure number	**1**	**2**	**3**	**4**	**5**
Number of people	**5**				

(b) Using this arrangement, how many people can be seated if they used

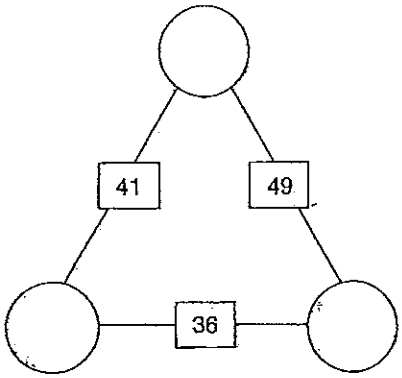
(i) 8 tables

(ii) n tables

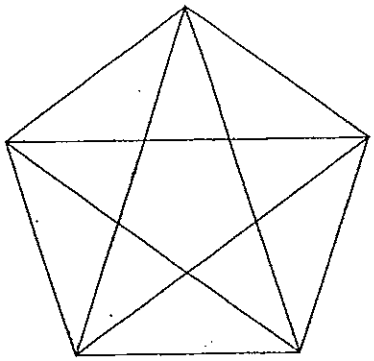
(iii) 50 tables

(c) For a school dinner, 260 people had to be seated.
How many tables like this did they need?

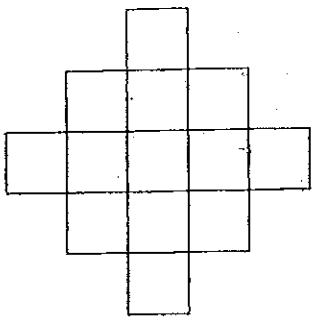
(5) In the following figure, called an arithmogon, the number that appears in a square is the sum of the numbers in the circles on each side of it. Fill in the missing numbers in the circles.



(6) How many triangles can you find in the diagram below?



(7) How many squares are there in the diagram below?



(8) In the number pattern shown below, find the values of p and q.

$$\begin{aligned}
1 + 3 &= 2^2 \\
1 + 3 + 5 &= 3^2 \\
1 + 3 + 5 + 7 &= 4^2 \\
&\vdots \\
1 + 3 + 5 + 7 + 9 + 11 &= p \\
&\vdots \\
1 + 3 + 5 + 7 + \ldots + 35 &= q
\end{aligned}$$

(9) Use the following number pattern to answer the questions below:

$$\begin{aligned}
1 &= 2^1 - 1 \\
1 + 2^1 &= 2^2 - 1 \\
1 + 2^1 + 2^2 &= 2^3 - 1 \\
1 + 2^1 + 2^2 + 2^3 &= 2^4 - 1 \\
&\vdots \qquad\quad \vdots
\end{aligned}$$

(a) Write down the next two lines of the pattern.

(b) Write down a simpler expression for the sum in the nth line.

(c) Find the value of the 15^{th} line in the sequence.

(10) Consider the number patterns in the table below and complete lines (a), (b) and (c)

	$6^3 - 125$	$1 \times [6^2 + (5 \times 6) + 25]$
	$7^3 - 125$	$2 \times [7^2 + (5 \times 7) + 25]$
	$8^3 - 125$	$3 \times [8^2 + (5 \times 8) + 25]$
(a)		$5 \times [10^2 + (5 \times 10) + 25]$
(b)	$12^3 - 125$	
(c)	$n^3 - 125$	

(d) By using your answers to part (c), find the two prime factors of 5707.
(Given that $18^2 = 324$ and $18^3 = 5832$)

Answers:

Page 1:

A5.1

(1)(a) False; $15-\frac{15}{7}=12\frac{6}{7}$ (b) False; $3+4\frac{1}{2}-1=6\frac{1}{2}$

(c) False; $18\leq 22$ (d) False; $1<\frac{3}{5}$

(2) Let $p=5,q=6,r=7,s=8$

(a) True; e.g. $5+6+7=18;6+7+8=21$

(b) False; $5+6+7+8=26$ is not a multiple of 4.

(c) False; $6\times 8+5\times 7=48+35=83$; Even + Odd = Odd

(d) False; $5\times 6\times 7=210$; Odd X Even X Odd = Even

(3) Let $a=5,b=3,c=15$;i.e. $\frac{15}{5},\frac{15}{3}$

(a) False; Reciprocal of an integer is not an integer

(b) False; $\frac{15}{5+3}=\frac{15}{8}$ is not an integer

(c) True; Integer X Integer = Integer

(d) False; Reciprocal of an integer is not an integer

Page 2:

A5.2

(1) (a) True; Odd + Even = Odd

(b) False; Even – Odd = Odd

(c) False; Odd + Even = Odd

(d) True; Odd x Odd x Odd = Odd

(2) (a) True; Even – Odd = Odd

(b) True; Even x Odd = Even

(c) True; e.g. $\frac{10.11.12}{3}=440$

(d) False; e.g. $20^2-20+1=391$ which is divisible by 17 = 23

A5.3

(1) $a+a+a=3a$ or $a.a.a=a^3$

(2) $a(bc)=abc$ or $a(b+c)=ab+ac$

(3) $a^4+a^4=2a^4$ or $a^4.a^4=a^8$

(4) $(-a)^3=(-a)(-a)(-a)=-a^3$

(5) $\frac{a}{a}+\frac{b}{a}=1+\frac{b}{a}$

(6) $\frac{2}{a}+\frac{3}{a}=\frac{5}{a}$

(7) $\frac{3a+2a}{6}=\frac{5a}{6}$

(8) $(a+b)^2=a^2+2ab+b^2$

Page 3:

(9) $\sqrt{a^2+b^2}=\sqrt{a^2+b^2}$

(10) $\frac{a-b}{-(a-b)}=-1$

(11) $\frac{a^2-a^2+1}{a(a+1)}=\frac{1}{a(a+1)}$

(12) $\frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}$

(13) $\frac{a}{a+b}\times\frac{a+b}{b}=\frac{a}{b}$

(14) $\frac{ab}{(a+b)}\times\frac{(a+b)(a-b)}{b}=a(a-b)$

A5.4

(1) $r=-p-1$

(2) $r=\frac{p-1}{q}$

(3) $r=\frac{pq}{p+q}$

(4) $r=\pm\sqrt{p^2-q^2}$

Page 4:

(5) $r=q$

(6) $r=\frac{S}{S+a}$

(7) $r=\sqrt[n-1]{\frac{T}{a}}$

(8) $r=100\left(1-\sqrt[n]{\frac{A}{P}}\right)$

A5.4

(1) (a) $2\frac{1}{3}$

(b) $\frac{1}{5}$

(2) (a) 32

(b) $16\frac{1}{16}$

(c) $16\frac{1}{16}$

(d) $\frac{1}{8}$

Page 5:

A5.6

(1) 16, 32, 64. Multiply the previous number by 2.

(2) 16, 25, 36. Square each number.

(3) 11, 16, 22. Add the difference between the previous two numbers to the last number.

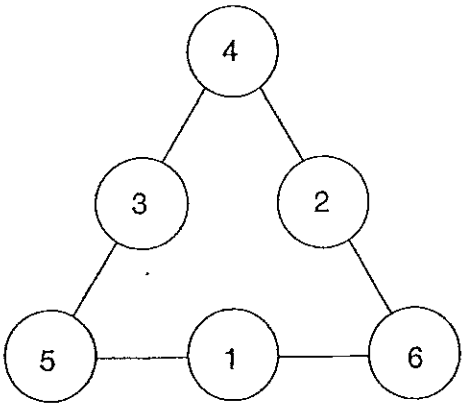
(4) 240, 1 440, 10 080. Each term is multiplied by ratio plus 1 of the previous two numbers.

(5) 4, 2, 1. Half the previous number.

(6) 5, 8, 13. Add the sum of the previous two terms to obtain the succeeding term.

A5.7

(1)



Page 6:

(2) (a)

Figure number	**1**	**2**	**3**	**4**	**5**
Number of squares	**5**	**9**	***13***	***17***	***21***

(b) (i) 41 (ii) 2 001 (c) Figure 41 (d) $s = 4f + 1$

Page 7:

(3) (a) Tutor to check diagram.

(b)

Figure number	**1**	**2**	**3**	**4**	**5**	**6**
Number of enclosures (E)	**1**	**2**	***3***	***4***	***5***	***6***
Number of posts (P)	**6**	**9**	***12***	***15***	***18***	***21***

(c) 63 posts (d) $P = (3E + 3)$ (e) 1683 posts

Page 8:

(4) (a)

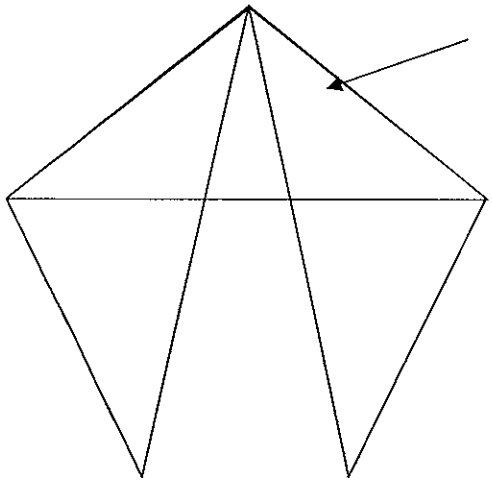
Figure number (n)	**1**	**2**	**3**	**4**	**5**
Number of people	**5**	*8*	*11*	*14*	*17*

(b) (i) 26 people (ii) $(3n+2)$ people (iii) 152 people (c) 86 tables

Page 9:

(5) 27, 14, 22 *(6) 35 (7) 18

* (6)



Top figure: 6 triangles x 5 – 5 = 25
Plus 5 triangles in the Star Shape Plus 5 triangles in the middle: Total = 35

Page 10:

(8) $p=6^2, q=18^2$

(9) (a) $1+2^1+2^2+2^3+2^4=2^5-1$ (b) 2^n-1 (c) 32 767
$1+2^1+2^2+2^3+2^4+2^5=2^6-1$

Page 11:

(10) (a) 10^3-125

(b) $7\times\left[12^2+(5\times12)+25\right]$

(c) $(n-5)\times\left[n^2+(5\times n)+25\right]$

(d) 13 and 439