ENRICHMENT EXERCISES

7 True or False

"FURTHER ... (All)"

Number

Write true or false for each of the following, giving reasons for each.

1 (a) $3 \times 4 - 12 \div 6 = 0$
(b) $3 \times (4 + 6) \div 2 = 15$
(c) $3 \times 2 + 4 \div 9 = 2$
(d) $8 + 3 \times 2 > (8 + 3) \times 2$

2 (a) The sum of 3 consecutive numbers is always divisible by 3.
(b) The sum of 5 consecutive numbers is always divisible by 5.
(c) The sum of 7 consecutive numbers is always divisible by 7.

3 If x, y, z, w are consecutive numbers then:
(a) $x + y + z + w$ is a multiple of 4
(b) $yz - xw = 2$
(c) $yw - xz$ is even

4 (a) If n is odd:
(i) $n + 8$ is odd
(ii) $n^2 + 5$ is odd
(iii) $2n^2 + 5$ is odd
(iv) n^3 is odd
(b) If n is even:
(i) n^2 is even
(ii) $n^2 + 5$ is even
(iii) $3n + 2$ is even

5 If n is a positive integer:
(a) $n^2 + 1$ is even, if n is odd
(b) $2n + 1$ is always odd
(c) n^2 is even, if n is even
(d) $n^2 - n + 11$ is always prime
(e) $n^2 - n + 41$ is always prime
(f) $n^2 + n + 41$ is always prime
(g) $(n - 1)! + 1$ is divisible by n

6 A, B and N are whole numbers.
(a) If $\frac{N}{A}$ and $\frac{N}{B}$ are whole numbers then the following are always whole numbers:
(i) $\frac{N}{A + B}$
(ii) $\frac{A}{N}$
(iii) $\frac{N}{AB}$
(iv) $\frac{N^2}{A}$
(v) $\frac{N^2}{AB}$
(b) If $\frac{N}{AB}$ is a whole number then the following are always whole numbers:
(i) $\frac{N}{A}$
(ii) $\frac{N}{B}$
(iii) $\frac{AB}{N}$
(iv) $\frac{2N}{AB}$
(v) $\frac{N}{2AB}$

7 (a) $13! + 11$ is a prime number

(b) $13! + 2, 13! + 3, 13! + 4, 13! + 5 \ldots 13! + 13$ is a set of 12 consecutive composites

(c) $1 + 2 \times 2! + 3 \times 3! + 4 \times 4! = 5! - 1$

8 If p and q are prime numbers, then the HCF of:

(a) p and q is 1

(b) p^2 and q is 1

(c) p^2 and q^2 is 1

(d) p^2 and pq is p^2

9 If p and q are prime numbers, then the LCM of:

(a) p and q is pq

(b) p^2 and q is pq

(c) p^2 and q^2 is p^2q^2

(d) p^2 and pq is pq

10 (a) If the LCM of p and 22 is 154, then the only value that p can take is 7.

(b) If the LCM of p and 130 is 2210, then p can be either 17 or 85.

11 If 4897^{583} is multiplied out, the last digit in the final product is 1.

12 (a) $\frac{1}{7}$ has a cycle of six repeating digits.

(b) $0.\dot{2}$ is a rational number, i.e. can be written as $\frac{p}{q}$.

(c) $0.\dot{2}\dot{3}$ is a rational number.

13 (a) Annabell buys a toaster for her shop for \$50 and marks it up by 45%. Later during a sale, the toaster is advertised at '45% off marked price'. The price of the toaster is the original wholesale price of \$50.

(b) An employer gets 20% discount on all goods sold in the shop. However, he has to pay tax at 5%. It does not matter which is calculated first, tax or discount, on any item that he buys.

14 A local Council has set a rate of 1.34 cents for each dollar in the unimproved capital value of land. If Mrs Nelson's land has a value of \$85 000, then her rate due is \$1139.

15 (a) If you want to increase something by 20%, you just multiply it by 1.2.

(b) If you want to decrease something by 20%, you just multiply it by 0.8.

16 The initial cost of a hire car is $\$P$ plus a charge of x cents per kilometre. The total cost, C, in dollars for travelling y kilometres is:

(a) $C = (P + 100x)y$

(b) $C = (P + \frac{x}{100})y$

(c) $C = P + \frac{xy}{100}$

(d) $C = P + 100xy$

Algebra

Write true or false for each of the following. If a statement is false, correct it.

1 (a) $3a + a = 3a^2$

(b) $a - 3a = -2a$

(c) $3a - b + 2a = a - b$

(d) $2(a + b) = 2a + b$

(e) $2(1 + a) + 3a = 2 + 5a$

(f) $a^2 + a^2 = a^4$

(g) $(2a)^3 = 2a^3$

(h) $a(a + b) = a^2 + ab$

(i) $p(qr) = p^2qr$

(j) $(p - q)^2 = p^2 - q^2$

(k) $2p(pq - 1) = 2p^2q - 2p$

(l) $a(b + c) = (b + c)a$

(m) $a + a^2 = a^3$

(n) $7p - 8p = -p$

(o) $5pq - 3qp + 2p^2 = 2pq + 2p^2$

2 Watch those signs!

(a) $(-a)^2 = -a^2$

(b) $-a^2 = -(a)^2$

(c) $-(bc)^2 = b^2c^2$

(d) $-2(b - c) = -2b + 2c$

(e) $(-3c)^3 = -3c^3$

(f) $-(3c)^3 = -27c^3$

(g) $\frac{-4}{-6x} = \frac{2}{3}x$

(h) $(-4p)(-6q) = -24pq$

(i) $-5^2 = -25$

(j) $(-5)^2 = -25$

(k) $(-2)^4 = -2^4$

(l) $(-1)^{21}\, a = -a$

3 Watch those fractions!

(a) $\frac{4 + a}{4 + b} = \frac{a}{b}$

(b) $\frac{1}{2}p + \frac{1}{2}p = 1$

(c) $\frac{4 + 2}{6p} = \frac{1}{p}$

(d) $\frac{p^2}{q^2} = \frac{p}{q}$

(e) $\frac{a}{3} = \frac{1}{3}a$

(f) $\frac{1}{a} + \frac{1}{b} = \frac{2}{a + b}$

(g) $\frac{5}{a} + \frac{5}{b} = \frac{5}{ab}$

(h) $\frac{3p + q}{3q + r} = \frac{p}{r}$

(i) $\frac{p + q}{q} = \frac{p}{q} + 1$

(j) $\frac{a + b}{5} = \frac{a}{5} + b$

(k) $\frac{3a + 4}{2} = 3a + 2$

(l) $\frac{m + 4}{4} = m$

(m) $\frac{1}{a} + \frac{1}{a} = \frac{2}{a}$

(n) $\frac{3p + 6}{3} = 3p + 2$

(o) $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

4 Watch those powers!

(a) $(a + b)^2 = a^2 + b^2$

(b) $\sqrt{a^2 + b^2} = a + b$

(c) $\frac{1}{2}(-4)^2 = (-2)^2$

(d) $(\sqrt{r})^2 = r$

(e) $(3a)^{-1} = \frac{1}{3a}$

(f) $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$

(g) $\frac{1}{(5p)^2} = 25p^{-2}$

(h) $(\frac{2}{3})^a = \frac{2^a}{3^a}$

(i) $\frac{1}{5a}(-25a)^2 = -5a$

(j) $\sqrt{p}(\sqrt{p} + \frac{1}{\sqrt{p}}) = p + 1$

(k) $(\sqrt{p})^4 = p^2$

(l) $(\sqrt{p} + \frac{1}{\sqrt{p}})^2 = p + \frac{1}{p}$

5 (a) $\frac{4t}{5} - \frac{3t}{4} = \frac{t}{20}$

(b) $\frac{x}{x+1} - \frac{x-1}{x} = \frac{1}{x(x+1)}$

(c) $\frac{x}{x+y} - \frac{x-y}{x} = \frac{y}{x+y}$

6 If x, y and z are different numbers then:

(a) $(x + y) + z = x + (y + z)$

(b) $(x - y) + z = x - (y + z)$

(c) $(x - y) - z = x - (y - z)$

(d) $(xy)z = x(yz)$

(e) $(x \div y) \div z = x \div (y \div z)$

(f) $xz \div y = x \div (y \div z)$

7 If x, y and z are integers, and $xyz = 0$:

(a) All of x, y and z are zero.

(b) One and only one of x, y and z is zero.

(c) At least one of x, y and z is zero.

(d) Some, but not all, of x, y and z are zero.

8 (a) If $E = \frac{1}{2}mv^2$ then $v = \sqrt{\frac{m}{2E}}$

(b) If $v = \sqrt{u^2 + 2as}$ then $a = \frac{v^2 - u^2}{2s}$

(c) If $d = \sqrt{\frac{3h}{2}}$ then $h = \frac{2}{3}d^2$

(d) If $r = \frac{3hp}{n+1}$ then $n = \frac{3hp - r}{r}$

(e) If $A = P(1 + \frac{r}{100})^n$ then $r = 100(\frac{A^{\frac{1}{n}}}{P} - 1)$

(f) If $V = \frac{1}{3}\pi r^2 h$ then $h = \frac{3V}{\pi r^2}$

9 If the operation $*$ is defined by $a * b = (-1)^b a + (-1)^a b$ so that

$2 * 3 = (-1)^3 2 + (-1)^2 3$

$= -2 + 3 = 1$ then:

(a) $3 * 4 = 4 * 3$

(b) $(3 * 4) * 5 = 3 * (4 * 5)$

(c) $a * 0 = a$

10 If $x * y = 2x^2 + y^2$ then:

(a) $3 * 5 = 43$

(b) $3 * 5 = 5 * 3$

(c) $1 * (2 * 3) = (1 * 2) * 3$

11 If $m \otimes n = \frac{mn}{m+n}$

(a) $8 \otimes 5 = 5 \otimes 8$

(b) $a \otimes b = b \otimes a$

(c) $2 \otimes (3 \otimes 4) = (2 \otimes 3) \otimes 4$

(d) $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

12 (a) The sequence formed by the difference between pairs of consecutive square numbers is the sequence of even numbers.

Investigate differences like $8^2 - 7^2$ or $7^2 - 6^2$. If the statement is true, try to prove your result algebraically.

(b) Since $5^2 + 1^2 = 26 = 2(9 + 4) = 2(3^2 + 2^2)$

$6^2 + 2^2 = 40 = 2(16 + 4) = 2(4^2 + 2^2)$

$7^2 + 1^2 = 50 = 2(16 + 9) = 2(4^2 + 3^2)$

$6^2 + 4^2 = 52 = 2(25 + 1) = 2(5^2 + 1^2)$

It is always true that $2(a^2 + b^2)$ is the sum of two squares.

13 (a) If 1, 3, 6, 10 . . . are triangular numbers, then the nth triangular number is:

$T(n) = \frac{1}{2}n(n + 1)$

(b) Since $1 + 3 = 4$

$3 + 6 = 9$

$6 + 10 = 16$

....

the above pattern can be generalised, so that the sequence formed by the sums of pairs of consecutive triangular numbers, is the sequence of square numbers.

14 Each of the following start with a true statement. Then at some stage it becomes false. Find the flaw in the following mathematical fallacies.

(a) If

$-20 = -20$

$4 - 24 = 100 - 120$

$4 - 24 + 36 = 100 - 120 + 36$ adding 36 to both sides

$(2 - 6)^2 = (10 - 6)^2$

$2 - 6 = 10 - 6$ finding the square root of both sides

$2 = 10$ adding 6 to both sides

(b) $11 > 8$ is a true statement

$-4 > -7$ subtract 15 from both sides

$16 > 49$ square both sides

(c) If $f(n) = n^2 - n + 41$

$f(1) = 1 - 1 + 41 = 41$ a prime number

$f(2) = 4 - 2 + 41 = 43$ a prime number

$f(3) = 9 - 3 + 41 = 47$ a prime number

$f(4) = 16 - 4 + 41 = 53$ a prime number

$f(5) = 25 - 5 + 41 = 61$ a prime number

$f(6) = 36 - 6 + 41 = 71$ a prime number

$f(7) = 49 - 7 + 41 = 83$ a prime number, and so on . . .

therefore $f(n) = n^2 - n + 41$ is always prime

(d) If

$a = b$

$a^2 = ab$ multiplying by a

$a^2 - b^2 = ab - b^2$ subtracting b^2

$(a - b)(a + b) = b(a - b)$

$a + b = b$ dividing by $a - b$

$b + b = b$ substituting $a = b$

$2b = b$

$2 = 1$ dividing by b

(e) If $x = 4$

$x^2 = 16$ squaring both sides

$x^2 - 4x = 16 - 4x$ subtracting $4x$ from both sides

$x(x-4) = 4(4-x)$

$x(x-4) = -4(x-4)$

$x = -4$ dividing by $x - 4$

(f) If $x < y$

$x(x-y) < y(x-y)$ multiplying by $x - y$

$x^2 - xy < yx - y^2$

$x^2 - 2xy + y^2 < 0$ bringing all terms to the LHS

$(x-y)^2 < 0$

Measurement

Write true or false for each of the following, giving reasons for each.

1 If each side of a triangle is doubled, the area is multiplied by 4.

2 If the radius of a circle is increased by 50%, the area is increased by 125%.

3 Of all rectangles with a given perimeter, the square has the smallest area.

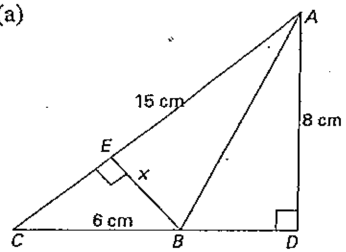
4 A rectangle of length x and width y has the same perimeter as the square of side length $\frac{x+y}{2}$.

5 If a 20 cm diameter pizza serves 2 people, a 30 cm diameter pizza serves 3 people.

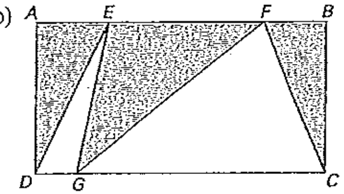
6 If a small $2 \times 2 \times 2$ Rubik's cube cost \$4.00, then it is fair to charge \$13.50 for a $3 \times 3 \times 3$ Rubik's cube, if the cost is only dependent on the number of small cubes.

7 If the cost of a mini 7 cm diameter and 2 cm high pizza is \$2 then for a 14 cm diameter and 3 cm high pizza the charge should be \$6.

8 (a) If in this diagram, $AC = 15$ cm, $CB = 6$ cm and $AD = 8$ cm, then $EB = 3\frac{1}{5}$ cm.



(b) The unshaded part of the rectangle $ABCD$, where $AB = 24$ cm, $BC = 8$ cm and $EF = 10$ cm is 96 cm^2



9 (a) If a person walks at 6 km/h for t hours, then he walks $6t$ km.

(b) The length of a side of a square whose area is 4 m^2 is 20 cm.

(c) m minutes and s seconds is equal to $60m + s$ seconds.

(d) c centimetres and m millimetres is the same as $100c + m$ millimetres.

(e) If the length of a rectangle is 2 cm and the breadth is 7 mm, then the area is 140 mm^2.

(f) If the area of a square is h hectares, then each side is 100 h metres.

(g) If the volume of a cube is 27 m^3, then each side is 30 cm.

(h) A swimming pool is filled with k kilolitres of water. This is the same as $1000k$ litres.

(i) If a cup has a capacity of 250 mL, then from p litres of milk, $40p$ cups can be filled.

(j) If k metres^3 of top soil are dumped in a heap on a garden and spread on a lawn so that it is 5 cm thick everywhere, then the area it covers is $200000k$ cm^3.

(k) A train travelling at 60 km/h takes k seconds to pass a man standing on a station. The length of the train is $1000k$ metres.

(l) If the surface area of a cube is k cm^2, then each side of the cube is $10\sqrt{\frac{k}{6}}$ cm.

(m) The volume of a cube of steel with a density of 7.9 g/cm^3 and mass of m grams is $7.9m$ cm^3.

(n) A box measuring $0.5 \text{ m} \times 20 \text{ cm} \times 12$ cm holds 12 L.

(o) If 1 pound equals 0.45359 kg and there are 14 pounds in a stone, then a person who weighs 8 stone 12 pounds weighs approximately 56 kg.

(p) In w weeks and h hours there are $1080w + 60h$ hours.

(q) A runner's heart beat is 120 beats per minute. If he runs 500 metres in 80 seconds then his heart beat a total of 90 times during his run.

(r) If \$A 1.00 = \$US 0.6 then \$US k = \$A $\frac{5k}{3}$

(s) If a phone call costs \$5 for the first minute, and \$1.20 a minute after the first minute, then if a conversation cost \$$d$ (where $d > 5$) then it lasted $\frac{d-5}{1.2}$ minutes.

(t) If the area of right angled trapezium with parallel sides of lengths 8 cm and 12 cm is k metres^2, then the height is $100k$ metres.

10 For a certain recipe a cube of butter with sides 6 cm is used. If a cube of butter with sides 9 cm is used, when this same recipe is made for more people, then all ingredients in the original recipe will have to be increased by $1\frac{1}{2}$.

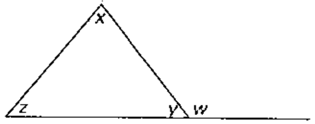
11 A family drives from A to B at an average speed of 60 km/h. On the return trip from B to A, late at night with little traffic, their average speed is 80 km/h. The average speed for the whole trip from A to B and back is 70 km/h.

12 It takes 80 seconds for Amy to walk up a stationary escalator, and 60 seconds to reach the top when she is stationary on a moving escalator. If she walks up the moving escalator at the same speed as she walked up when it was stationary, it will take her $34\frac{2}{7}$ seconds to reach the top.

Space

Write true or false for each of the following, giving reasons for each.

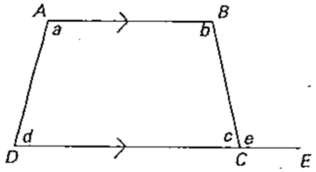
1



Using the diagram on the left.

(a) $y + w = 180°$
(b) $w = z + y$
(c) $w = x + z$
(d) $w > x$
(e) $x + z < w + y$
(f) $x = w$

2

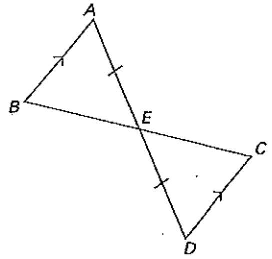


Using the diagram on the left.

(a) $d = b$
(b) $e = b$
(c) $a + b = 180°$
(d) $a + d = 180°$
(e) $c + e = 180°$
(f) $a + d + c + e = 360°$

3 (a) A parallelogram with a pair of adjacent sides equal is called rectangle.
(b) A rectangle has all its angles equal.
(c) A square belongs to the family of rectangles.
(d) A rectangle belongs to the family of squares.
(e) A trapezium has its opposite sides parallel.
(f) The diagonals of a kite bisect each other at right angles.
(g) The diagonals of a rhombus bisect each other at right angles.
(h) The diagonals of a parallelogram are equal.
(i) A rhombus is a special parallelogram with adjacent sides equal.
(j) A square is a special rhombus with all angles equal to 90°.

4

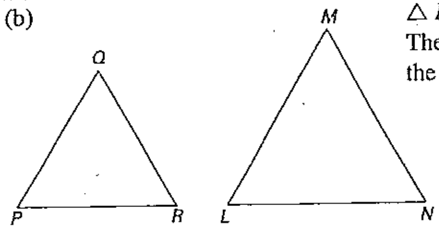


In this diagram, BEC and AED are straight lines, $AB // DC$ and $AE = ED$. Then $\triangle ABE \equiv \triangle DCE$.

5 (a) If a line does not lie in a plane, it meets it in just one point.
(b) Two distinct planes can have three non-collinear points in common.

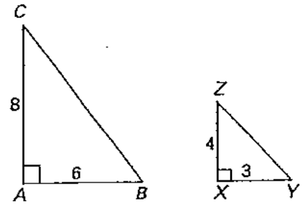
6 (a) 45° is really the same as 315°

(b)



$\triangle PQR /// \triangle LMN$.
The angles in $\triangle PQR$ are smaller than the angles in $\triangle LMN$.

(c)

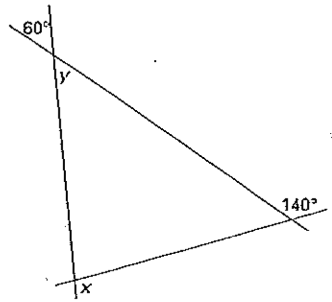


$\triangle ABC /// \triangle XYZ$.
The angles in $\triangle ABC$ are bigger than the angles in $\triangle XYZ$.

(d) Max is swinging on a swing. If the seat sweeps through 50°, then his feet will sweep through a greater angle than 50°.

(e) It is possible to draw a triangle which has:
  (i) two right angles.
  (ii) all its angles the same.
  (iii) one angle equal to the sum of the other two.

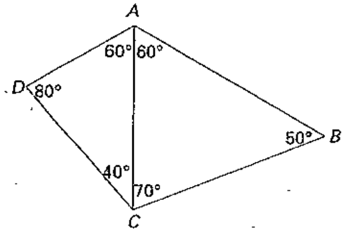
7



Three straight lines intersect as shown in the diagram. It follows that:

(a) $y = 60°$ and $x = 140°$
(b) $y = 40°$ and $x = 100°$
(c) $y = 60°$ and $x = 100°$
(d) $y = 60°$ and $x = 40°$

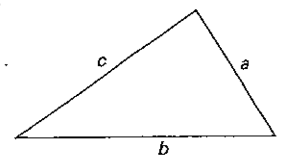
8 In this diagram the longest side is AC.



9 If a 3-metre pole casts a shadow 4 metres long, then the height of a tree which casts a shadow 12 metres long is 11 metres.

10 Pythagoras' theorem can be generalised, so that if similar figures are drawn on the sides of a right triangle, the largest figure has an area equal to the sum of the areas of the two smaller figures.

11



In any triangle with sides a, b and c:

$a + b > c$

$a + c > b$

$b + c > a$

(a) only one of the above statements must be true.

(b) all of the above statements must be true.

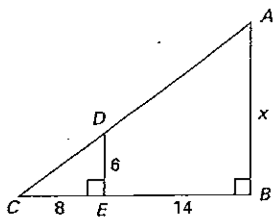
12 (a) A rotation of 30° clockwise followed by a rotation of 60° clockwise is equivalent to a rotation of 90° clockwise.

(b) The shape and size of a figure are unchanged under a rotation.

(c) Under reflection, a figure and its image are not necessarily congruent (identically the same in shape and size).

(d) If a figure is rotated about any point through 360°, the image coincides with the original figure.

13



If $\triangle ACB$ is similar to $\triangle DCE$, then:

(a) $\frac{x}{6} = \frac{14}{8}$

(b) $\frac{22}{x} = \frac{8}{6}$

(c) $\frac{14}{6} = \frac{8}{x}$

(d) $\frac{x}{14} = \frac{6}{8}$

(e) $\frac{x}{6} = \frac{22}{8}$

Chance and data

Write true of false for each of the following, giving reasons for each.

1 When 1 coin is flipped, there are two outcomes (H, T) so the probability of each is $\frac{1}{2}$. When two coins are flipped, there are three outcomes (HH, TT and a H and a T), so the probability of each is $\frac{1}{3}$.

2 When one die is rolled, there are 6 outcomes, so the probability of each is $\frac{1}{6}$. When rolling two dice, there are 11 possible outcomes for the sum on the dice (2, 3, 4 . . . 12), so the probability of each is $\frac{1}{11}$.

3 A set of 12 cards is numbered with the positive integers from 1 to 12. If the cards are shuffled and one card is selected at random, the probability that the number on the card is divisible by 2 or 5 is:

(a) $\frac{1}{6}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{7}{12}$

4 (a) The probability of correctly answering exactly two out of three true or false questions if the person is guessing is $\frac{2}{3}$.

(b) The probability of correctly answering at least three out of four true or false questions if the person is guessing is $\frac{1}{2}$.

(c) The probability of drawing two kings on two consecutive draws from a pack of cards if the card is not replaced after the first draw is $\frac{1}{221}$.

(d) The probability that the sum of two dice equals 7 is $\frac{3}{8}$.

5 If the probability of a pregnancy resulting in twins is $\frac{1}{92}$ and in triplets is $\frac{1}{9000}$ and a large hospital delivered 20000 pregnancies in 10 years, we would expect to have:

(a) 200 twins

(b) 2 triplets

6 Kevin had three test scores of 88, 86 and 84, while Jonathan had scores of 72, 95 and 94. When comparing Kevin's average with Jonathan's:

(a) Kevin's was 1 point higher.

(b) Kevin's was 1 point lower.

(c) Both averages were the same.

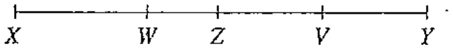
(d) Jonathan's was 2 points higher.

(e) Jonathan's was 2 points lower.

7 (a) It is posible to find 3 different whole numbers whose mean is 1?
(b) 3 different whole numbers have a mean of 2. There are four sets of such numbers.

8 If there are n scores that have been placed in order from the smallest to the largest, then the median is the $\frac{n+1}{2}$th value.

9 If $XY = 2XZ = 3XW = 4VY$ and if P is a point selected randomly on the segment XY, then the probability that P is between:
(a) X and Z is $\frac{1}{2}$
(b) V and Y is $\frac{1}{3}$
(c) Z and V is $\frac{1}{4}$
(d) W and Z is $\frac{1}{8}$



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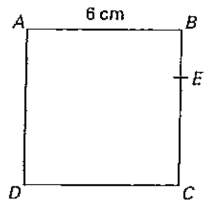


$ABCD$ is a square with each side 6 cm. E is a point on BC so that $BC = 3BE$.
If P is a point chosen at random on AD then the probability that:
(a) area $\triangle BPE$ is 12 cm² is 1.
(b) area $\triangle ABP$ is greater than 9 cm² is $\frac{1}{2}$.

Chapter 7 True or False

Number

1 (a) False LHS = 12 − 2 = 10
(b) True LHS = 15
(c) False LHS = $6\frac{4}{9}$
(d) False LHS = 14 RHS = 22

2 (a) True 3 consecutive numbers will always have the form $3n$, $3n + 1$, $3n + 2$ (though not necessarily in that order) and the sum of these numbers is $9n + 3 = 3(3n + 1)$ divisible by 3.
(b) True 5 consecutive numbers will always have the form $5n$, $5n + 1$, $5n + 2$, $5n + 3$, $5n + 4$ (though not necessarily in that order), and the sum of these numbers is $25n + 10 = 5(5n + 2)$ divisible by 5.
(c) True similar argument to the above

3 Let $x = k - 1$, $y = k$, $z = k + 1$, $w = k + 2$ is one way of writing 4 consecutive numbers
(a) False as $x + y + z + w = 4k + 2$
(b) True as $yz - xw = k(k + 1) - (k - 1)(k + 2)$
$= 2$
(c) False as $yw - xz = k(k + 2) - (k - 1)(k + 1)$
$= 2k + 1$ this is always odd

4 (a) every odd number is of the form $2k + 1$ so let $n = 2k + 1$
(i) True $n + 8 = 2k + 9$
(ii) False $n^2 + 5 = 4k^2 + 4k + 6$ is an even number
(iii) True $2n^2 + 5 = 8k^2 + 8k + 7$ an odd number
(iv) True $n^3 = 8k^3 + 12k^2 + 6k + 1$ or odd × odd × odd = odd
(b) every even number is of the form $2k$ so let $n = 2k$
(i) True $n^2 = 4k^2$
(ii) False $n^2 + 5 = 4k^2 + 5$
(iii) True $3n + 2 = 6k + 2$

5 (a) True if n is odd n^2 is odd as (odd)(odd) = odd and $n^2 + 1$ is even as odd + 1 is even
(b) True $2n$ is always even irrespective of whether n is odd or even and so $2n + 1$ is always odd
(c) True if n is even n^2 is even
let $n = 2m$ then $n^2 = 2m \times 2m = 4m^2$, an even number
(d) False this expression will produce primes for $n = 1$ to 10
however, when $n = 11$, $n^2 - n + 11 = 11^2$, a composite number
(e) False this expression will produce primes for $n = 1$ to 40
however, when $n = 41$, $n^2 - n + 41 = 41^2$, a composite number

(f) False this expression will produce primes for $n = 1$ to 39
however, when $n = 40$, $n^2 + n + 41 = 40(40 + 1) + 41$
$= 41 \times 41$, a composite number

(g) False $(n-1)! + 1$ is divisible by n if and only if n is a prime number.
This is known as Wilson's theorem, e.g. when $n = 6$.
$5! + 1 = 121$ is not divisible by 6.

6 (a) If the result is false, then one counter example is sufficient to disprove it.

(i) False if $\frac{N}{A} = \frac{6}{3}$ and $\frac{N}{B} = \frac{6}{2}$ then $\frac{N}{A+B} = \frac{6}{3+2}$ is not a whole number.

(ii) False if $\frac{N}{A} = \frac{6}{3}$ then $\frac{A}{N} = \frac{3}{6}$ which is not a whole number.

(iii) False if $\frac{N}{A} = \frac{24}{8}$ and $\frac{N}{B} = \frac{24}{12}$ then $\frac{N}{AB} = \frac{24}{8 \times 12}$ is not a whole number.

(iv) True since if $\frac{N}{A}$ is a whole number then A is a factor of N, and hence A is a factor of N^2.

(v) True since if $\frac{N}{A}$ is a whole number and since $\frac{N}{B}$ is a whole number

then $\frac{N^2}{AB} = \frac{N}{A} \times \frac{N}{B}$ is (a whole number)(a whole number)
$=$ a whole number

(b) (i) True since if $\frac{N}{AB}$ is a whole number, then $A \times B$ is a factor of N so A alone is a factor of N.

(ii) True similar reason to the above.

(iii) False since $\frac{N}{AB} = \frac{24}{3 \times 4}$ is a whole number but $\frac{AB}{N} = \frac{3 \times 4}{24}$ is not a whole number.

(iv) True since if $A \times B$ is a factor of N then $A \times B$ is a factor of $2N$.

(v) False since $\frac{N}{AB} = \frac{30}{2 \times 5}$ is a whole number but $\frac{N}{2AB} = \frac{30}{2 \times 2 \times 5}$ is not a whole number.

7 (a) False as $13! + 11 = 13 \times 12 \times 11 \times 10 \times \ldots \times 3 \times 2 \times 1 + 11$
$= 11(13 \times 12 \times 10 \times \ldots \times 3 \times 2 \times 1 + 1)$ hence composite

(b) True as the numbers are obviously consecutive and by a similar proof to the above are all composite.

8 (a) True the only factors of p are p and 1, while the only factors of q are q and 1, so 1 is the HCF

(b) True the only factors of p^2 are p^2, p and 1, while the only factors of q are q and 1, so 1 is the HCF

(c) True the only factors of p^2 are p^2, p and 1, while the only factors of q^2 are q^2, q and 1, so 1 is the HCF

(d) False the only factors of p^2 are p^2, p and 1, while the factors of pq are pq, p, q and 1, so p is the HCF

9 See question 15 for the factors of each
(a) True
(b) False LCM is p^2q
(c) True
(d) False LCM is p^2q

10 (a) False as $22 = 2 \times 11$ and $154 = 2 \times 11 \times 7$,
so p can be either 7, or $7 \times 2 = 14$, or 7×11, or $7 \times 2 \times 11 = 154$

(b) False as $130 = 2 \times 5 \times 13$ and $2210 = 2 \times 5 \times 13 \times 17$.
so p can be either 17 or $17 \times 2 = 34$ or $17 \times 5 = 85$ or $17 \times 13 = 221$
or $17 \times 2 \times 5 = 170$ or $17 \times 2 \times 13 = 442$ or $17 \times 5 \times 13 = 1105$
or $17 \times 2 \times 5 \times 13 = 2210$

11 False since we are only interested in the units digit, we need to know 7^{583}
Looking at the last digits only for various powers of 7

7^1 7
7^2 9
7^3 3
7^4 1
7^5 7
7^6 9
7^7 3
7^8 1 so it can be seen that the last digits are repeating in a cyclical order, with every fourth power ending in a 1. Therefore 7^{580} ends in a 1, and 7^{583} ends in a 3.

12 It is not possible to be convinced about the validity of these statements from the calculator as it does not not give you sufficient number of decimal places.

(a) True doing a long division by 7 into 1.0000 . . . will convince you

(b) True it is equal to $\frac{2}{9}$ the proof is as follows

$$\begin{aligned} \text{let } x &= 0.222222\ldots \\ 10x &= 2.222222\ldots \\ 9x &= 2 \quad \text{so } x = \frac{2}{9} \end{aligned}$$

(c) True

$$\begin{aligned} \text{let } x &= 0.232323\ldots \\ 100x &= 23.232323\ldots \\ 99x &= 23 \quad \text{so } x = \frac{23}{99} \end{aligned}$$

13 (a) False After the mark up by 45% the price of toaster is \$72.50.
After the '45% off marked price' the new price is
$\$72.50 \times 0.55 = \$39.88$.

(b) True Let an item originally cost \$100
After 20% discount, price = \$80
After 5% tax, price = \$84
If tax is added first, price = \$105
After discount price = \$84

14 True as rate = $\$\dfrac{85\,000 \times 1.34}{100}$

15 (a) True since 20% = 0.2 so $X + 0.2X = 1.2X$
(b) True since $X = X - 0.2X = 0.8X$

16 (a) False
(b) False
(c) True
(d) False

Algebra

1 (a) False as $3a + a = 4a$
(b) True
(c) False as $3a - b + 2a = 5a - b$
(d) False as $2(a + b) = 2a + 2b$
(e) True
(f) False as $a^2 + a^2 = 2a^2$
(g) False as $(2a)^3 = 8a^3$
(h) True
(i) False as $p(qr) = pqr$
(j) False as $(p - q)^2 = p^2 - 2pq + q^2$
(k) True
(l) True
(m) False as $a + a^2$ are unlike terms and we can't add them
(n) True
(o) True

2 (a) False as $(-a)^2 = a^2$
(b) True
(c) False as $-(bc)^2 = -b^2c^2$
(d) True
(e) False as $(-3c)^3 = -27c^3$
(f) True
(g) False as $\dfrac{-4}{-6x} = \dfrac{2}{3x}$
(h) False as $(-4p)(-6q) = 24pq$
(i) True
(j) False as $(-5)^2 = 25$
(k) False as $(-2)^4 = 2^4$
(l) True

3 (a) False as $\dfrac{4+a}{4+b}$ cannot be simplified
(b) False as $\dfrac{1}{2}p + \dfrac{1}{2}p = p$
(c) True
(d) False as $\dfrac{p^2}{q^2}$ cannot be simplified
(e) True
(f) False as $\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{b+a}{ab}$
(g) False as $\dfrac{5}{a} + \dfrac{5}{b} = \dfrac{5(b+a)}{ab}$
(h) False as $\dfrac{3p+q}{3q+r}$ cannot be simplified
(i) True
(j) False as $\dfrac{a+b}{5} = \dfrac{a}{5} + \dfrac{b}{5}$
(k) False as $\dfrac{3a+4}{2} = \dfrac{3a}{2} + 2$
(l) False as $\dfrac{m+4}{4} = \dfrac{m}{4} + 1$
(m) True
(n) False as $\dfrac{3p+6}{3} = p + 2$
(o) True

4 (a) False as $(a + b)^2 = a^2 + 2ab + b^2$
(b) False as $\sqrt{a^2 + b^2}$ cannot be simplified
(c) False as $\dfrac{1}{2}(-4)^2 = 8$, while $(-2)^2 = 4$
(d) True
(e) True
(f) False as $\sqrt{a} + \sqrt{b}$ cannot be simplified
(g) False as $\dfrac{1}{(5p)^2} = \dfrac{1}{25p^2}$, while $25p^{-2} = \dfrac{25}{p^2}$
(h) True
(i) False as $\dfrac{1}{5a}(-25a)^2 = \dfrac{1}{5a}(625a^2) = 125a$
(j) True
(k) True
(l) False as $(\sqrt{p} + \dfrac{1}{\sqrt{p}})^2 = p + 2 + \dfrac{1}{p}$

5 (a) True
(b) True
(c) False LHS = $\dfrac{y^2}{x(x+y)}$

6 (a) True
(b) False as LHS = $x - y + z$ and RHS = $x - y - z$
(c) False as LHS = $x - y - z$ and RHS = $x - y + z$
(d) True
(e) False as LHS = $\dfrac{x}{y} \div z = \dfrac{x}{yz}$ and RHS = $x \div \dfrac{y}{z} = x\dfrac{z}{y} = \dfrac{xz}{y}$
(f) True as LHS = $\dfrac{xz}{y}$ and RHS = $\dfrac{xz}{y}$

7 If $xyz = 0$ then we can only say with certainty that (c) is true. (a), (b) and (d) may be true in some cases but not always, hence we have to say they are false.

8 (a) False $\quad v = \pm\sqrt{\dfrac{2E}{m}}$
(b) True
(c) True
(d) True
(e) False $\quad r = 100[(\dfrac{A}{P})^{\frac{1}{n}} - 1]$
(f) True

9 (a) True as LHS $= 3 * 4 = (-1)^4 3 + (-1)^3 4 = 3 - 4 = -1$ and RHS $= 4 * 3 = (-1)^3 4 + (-1)^4 3 = -4 + 3 = -1$
(b) True similar to above
(c) True as LHS $= a * 0 = (-1)^0 a + (-1)^a 0 = a =$ RHS

10 (a) True as LHS $= 3 * 5 = 2 \times 9 + 25 = 43$ and RHS $= 43$
(b) False as LHS $= 3 * 5 = 43$ $\quad$ RHS $= 5 * 3 = 2 \times 25 + 9 = 59$
(c) False as LHS $= 1 * (2 * 3) = 1 * 17 = 291$ $\quad$ RHS $= (1 * 2) * 3 = 6 * 3 = 81$

11 (a) True as LHS $= 8 \otimes 5 = \dfrac{40}{8+5} = \dfrac{40}{13}$ $\quad$ RHS $= 5 \otimes 8 = \dfrac{40}{5+8} =$ LHS
(b) True as LHS $= a \otimes b = \dfrac{ab}{a+b}$ $\quad$ RHS $= b \otimes a = \dfrac{ba}{b+a} =$ LHS
(c) True as LHS $= 2 \otimes (3 \otimes 4) = 2 \otimes \dfrac{12}{7} = \dfrac{24}{7} \div (2 + \dfrac{12}{7}) = \dfrac{24}{7} \times \dfrac{7}{26} = \dfrac{12}{13}$

RHS $= (2 \otimes 3) \otimes 4 = \dfrac{6}{5} \otimes 4 = \dfrac{24}{5} \div (\dfrac{6}{5} + 4) = \dfrac{24}{5} \times \dfrac{5}{26} = \dfrac{12}{13} =$ LHS

(d) True as LHS $= a \otimes (b \otimes c) = a \otimes \dfrac{bc}{b+c} = \dfrac{abc}{b+c} \div (a + \dfrac{bc}{b+c}) = \dfrac{abc}{b+c} \times \dfrac{b+c}{ab+ac+bc} = \dfrac{abc}{ab+ac+bc}$

RHS $= (a \otimes b) \otimes c = \dfrac{ab}{a+b} \otimes c = \dfrac{abc}{a+b} \div (\dfrac{ab}{a+b} + c) = \dfrac{abc}{a+b} \times \dfrac{a+b}{ab+ac+bc} = \dfrac{abc}{ab+ac+bc} =$ LHS

12 (a) False as $n^2 - (n-1)^2 = 2n - 1$ which is always odd
(b) True as $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

13 (a) True as all triangular numbers can be generated by $\frac{1}{2}n(n+1)$
(b) True as

$$\frac{1}{2}n(n+1) + \frac{1}{2}(n+1)(n+2) = \frac{1}{2}(n+1)(n+n+2) = \frac{1}{2}(n+1)(2n+2) = \frac{1}{2}(n+1)2(n+1) = (n+1)^2$$

14 (a) The fifth step, finding square root of both sides is not valid, as what we are actually saying is $\sqrt{(-a)^2} = \sqrt{(a)^2}$ is equivalent to $-a = a$.
(b) The third step, as we are multiplying both sides of an inequality by a negative number, hence the sign of the inequality has to change.
(c) The last step is invalid, since $f(41) = 41^2 - 41 + 41 = 41^2$ which is composite.
(d) The fifth step, dividing by $a - b$ is invalid, since this is equivalent to dividing by zero as $a = b$.
(e) The last step, dividing by $x - 4$ is invalid, since this is equivalent to dividing by zero as $x = 4$.
(f) The second step, multiplying by $x - y$ which is negative, should change the inequality.

Measurement

1 True as area $= \frac{1}{2}$ base $\times$ height $= \frac{1}{2}bh$

enlarged area $= \frac{1}{2}(2b) \times (2h) = 4(\frac{1}{2}bh) = 4$ original area

Note that the enlarged triangle is similar to the original one, so if the sides are in the ratio 2 : 1 then it can be proved that the heights are in the same ratio.

or

Using the formula

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$ and a, b and c are the sides of the triangle,

New area $= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)} = 4\sqrt{s(s-a)(s-b)(s-c)}$

2 True If the original radius is $2r$ then the increased radius is $3r$.
Original area is $4\pi r^2$ and increased area is $9\pi r^2$.
The increase is $5\pi r^2$, which is 125% of the original area.

3 False You should experiment by drawing rectangles with a given perimeter on square paper and show that the square has the largest area not the smallest.

4 True as perimeter of rectangle $= 2(x+y)$ and
perimeter of square $= 4(\dfrac{x+y}{2}) = 2(x+y)$

5 False If diameter = 20 cm then radius = 10 cm and area = 100π cm^2
one person eats 50π cm^2
now if diameter = 30 cm then radius = 15 cm and area = 225π cm^2
and since one person's serve is 50π cm^2
$4\frac{1}{2}$ serves can be made from 225π cm^2

6 True In a $2\times2\times2$ Rubik's cube there are 8 cubes and so each cube costs 50 cents.
A $3\times3\times3$ Rubik's cube consits of 27 cubes and so the cost is $50c\times27$
= \$13.50.

7 False Volume of small pizza $=\pi\frac{7}{2}\times\frac{7}{2}\times2=\frac{49\pi}{2}$ cm^3
Volume of large pizza $=\pi\times7\times7\times3=49\pi\times3=6$(volume of small pizza)
Price of large pizza = $6\times$ price of small pizza
= \$12

8 (a) True If we consider area of $\triangle ABC$ in two different ways,
$\frac{1}{2}(6)(8)=\frac{1}{2}(15)(x)$ so $x=3\frac{1}{5}$ cm

(b) True though it seems as though there is not enough information given, we can show that half the rectangle is shaded and half unshaded.
$EF = 10$ cm is not useful at all.

$$\text{area } \triangle DEG + \text{area } \triangle GFC = \frac{1}{2}DG\times h+\frac{1}{2}GC\times h$$
$$=\frac{1}{2}(DG+GC)\times h$$
$$=\frac{1}{2}DC\times h=\frac{1}{2}\times24\times8=96 \text{ cm}^2$$

9 (a) True
(b) False as length of side = 2 m = 200 cm or
$4 \text{ m}^2 = 4\times10\,000$ (since $1 \text{ m}^2 = 10\,000 \text{ cm}^2$)
(c) True
(d) False $10c+m$ millimetres
(e) True as 2 cm = 20 mm
(f) False as h hecrares = $10\,000h$ metres^2 so each side is $100\sqrt{h}$ metres
(g) False as each side is 3 m = 300 cm
(h) True as 1 kilolitre = 1000 litres
(i) False as 1 litre = 1000 mL so p litres = $1000p$ and
this gives $\frac{1000p}{250}=4p$ cups
(j) True as k metres^3 = 100^3k cm^3 so area is $\frac{1\,000\,000k}{5}=200\,000k$ cm^3

(k) False as 60 km per hour $=\frac{60\times1000}{60\times60}$ metres per second
$=\frac{50}{3}$ metres per second
so train $=\frac{50k}{3}$ metres long
(l) True as k cm^2 = $100k$ mm^2 and since surface area = $6s^2=100k$
each side = $\sqrt{\frac{100k}{6}}$ cm
(m) False since if the density is 7.9 g/cm^3 and mass is m grams then the volume of cube of steel is $\frac{m}{7.9}$ cm^3
(n) True as 1000 mL = 1L and $50\times20\times12=12\,000$ mL
(o) True as 8 stone 12 pounds $=8\times14+12$
= 124 pounds
$=124\times0.45359$
= 56.25 kg (correct to 2 dec. pl.)
(p) False since in w weeks and h hours $=(7w\times24+h)60$ minutes
$=10080w+60h$ minutes
(q) False as if he runs for 80 seconds, then
his heart beat a total of $\frac{120\times80}{60}=160$ times during his run.
(r) True as \$A 1.00 = \$US 0.6 then \$US 1.00 = \$A $\frac{10}{6}$ so \$US k = \$A $\frac{5k}{3}$
(s) False since $\frac{d-5}{1.2}$ minutes does not take into account the first minute of conversation. Answer should be $\frac{d-5}{1.2}+1$ minutes
(t) False as area of trapezium $=\frac{1}{2}(8+12)h=10\,000k$ cm^2
so $h=1000k$ cm $=10k$ metres

10 False volume of butter in original recipe $=6\times6\times6$ cm^3
volume of butter in larger recipe $=9\times9\times9$ cm^3
$=3.375\times$ (original recipe)

11 False Average speed $=\frac{\text{total distance}}{\text{total time}}$

$$S=\frac{2d}{(t+T)} \quad \text{and} \quad 60=\frac{d}{t}\ \ 80=\frac{d}{T}$$
$$=\frac{2d}{\frac{d}{60}+\frac{d}{80}}=2d\div(\frac{4d+3d}{240})$$
$$=\frac{480}{7} \text{ km/h}$$
$$=68.6 \text{ km/h}$$

12 True speed on moving escalator = speed of escalator + Amy's walking speed

$$S = \frac{d}{t} = \frac{d}{60} + \frac{d}{80}$$

$$\frac{1}{t} = \frac{1}{60} + \frac{1}{80}$$

$$= \frac{4+3}{240} = \frac{7}{240}$$

$$t = \frac{240}{7}$$

$$= 34\tfrac{2}{7} \text{ secs}$$

Space

1 (a) True supplementary angles on a straight line
(b) False
(c) True exterior angle equals sum of two interior opposites
(d) True since the above is true
(e) True since $x + z = w$
(f) False

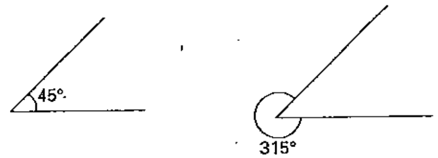
2 (a) False
(b) True these are equal alternate angles since $AB//DE$
(c) False
(d) True these are cointerior supplementary angles as $AB//DE$
(e) True supplementary angles on a straight line
(f) True since $a + d + c + b = 360$ sum of angles in quadrilateral and $b = e$

3 (a) False
(b) True as all angles are 90°
(c) True as a square satisfies the definition of rectangles
(d) False as a rectangle does not satisfy the definition of a square, since adjacent sides are not equal for a rectangle
(e) False as only one pair of sides are parallel
(f) False as diagonals intersect each other at right angles and one diagonal is bisected, but not both
(g) True
(h) False
(i) True
(j) True

4 True as $\angle ABE = \angle DCE$ (alt. angles since $AB//CD$)
$\angle BAE = \angle EDC$ (alt. angles since $AB//CD$)
$AE = ED$
$\triangle ABE \equiv \triangle DCE$ (2 angles and one side of one triangle is equal to 2 angles and the corresponding side of the other triangle)

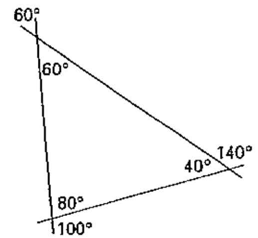
5 (a) False as the line can be parallel to the plane
(b) False Two distinct planes cannot have three non-collinear points in common since if planes intersect, they will intersect in a straight line, which will give collinear points.

6 (a) False



(b) False If triangles are similar then corresponding angles are equal.
(c) False If triangles are similar then corresponding angles are equal.
(d) False If the seat and the rope sweep 50° then his feet will also sweep 50°.
(e) (i) False angle sum of triangle is 180° and the triangle has 3 angles
(ii) True an equilateral triangle
(iii) True isosceles right angled (45°, 45°, 90°)

7



Three straight lines intersect as shown in the diagram. It follows that:
(a) False
(b) False
(c) True
(d) False

8 False as in $\triangle ADC$, AC is the longest side, since the longest side is always opposite the largest angle; however, in $\triangle ABC$, AB is the longest side opposite $\angle ACB$.

9 False as for every 3 metres of height there is a 4-metre long shadow, so a 6-metre tree would have an 8-metre shadow and a 9-metre tree would have an 12-metre shadow.

10 True This is an amazing result and can be proved for any similar figure (see section on Pythagoras).

11 (a) False In any triangle the sum of two sides must always be greater than the third side.
(b) True

12 (a) True
(b) True
(c) False, a figure and its image are always congruent under reflection.
(d) True

13 Since $\triangle$s are similar, corresponding sides are in proportion

$\frac{DE}{EC} = \frac{AB}{CB}$

(a) False, as it should be $\frac{x}{6} = \frac{22}{8}$

(b) True, as $\frac{CB}{BX} = \frac{CE}{ED}$

(c) False, 14 is a side of triangle

(d) False, 14 is a side of triangle

(e) True, correct proportion

Chance and data

1 False When two coins are flipped, the outcomes are HH, TT and HT and TH and each is equally likely with a probability of $\frac{1}{4}$ for each.

2 False again each outcome is not equally likely

3 The cards that can be selected are 2, 4, 5, 6, 8, 10 or 12, i.e. 7 possibilities out of 12.

(a) False
(b) False
(c) False
(d) True

4 (a) False as, P(correctly answering exactly two out of three)

$= P(CCW \text{ or } CWC \text{ or } WCC) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

(b) False as, P(correctly answering at least three out of four)

$= P(CCCC \text{ or } CCCW \text{ or } CCWC \text{ or } CWCC \text{ or } WCCC) = \frac{5}{16}$

(c) True as, $P(\text{two kings}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

(d) False as, $P(\text{sum } 7) = P$ (6 and 1 or 5 and 2 or 4 and 3 or 3 and 4 or 2 and 5 or 1 and 6) $= \frac{6}{36} = \frac{1}{6}$

5 (a) False $\frac{20\,000}{92} = 217.39 = 217$ twins approx. (note that as an approximation 200 is acceptable)

(b) True

6 We can consider the deviations from 80, so average $= 80 + \frac{8+6+4}{3} = 86$

and $= 80 + \frac{-8+15+14}{3} = 87$

(a) False
(b) True
(c) False
(d) False
(e) False

7 (a) True they are 0, 1, and 2

(b) False there are only 3 such sets, they are 1, 2, 3 or 0, 1, 5 or 0, 2, 4

8 True convince yourself with numbers

9 (a) True as $\frac{XZ}{XY} = \frac{1}{2}$

(b) False as $\frac{VY}{XY} = \frac{1}{4}$ so $P(P$ between V and $Y)$ should be $\frac{1}{4}$

(c) True as $\frac{ZV}{XY} = \frac{1}{4}$

(d) False as $XY = 2XZ = 3XW$ so $WZ = \frac{1}{6}XY$, so

$P(P$ between W and $Z)$ should be $\frac{1}{6}$

10 (a) False as $BE = 2$ cm and area $\triangle BPE = \frac{1}{2} \times 2 \times 6 = 6\text{ cm}^2$ so probability that area is 12 cm^2 is 0.

(b) True as area $\triangle ABP > 9\text{ cm}^2$ if $\frac{1}{2} \times AP \times 6 > 9$ that is if $AP > 3$. Now if a point P is chosen on AD at random, then $P(AP > 3) = \frac{1}{2}$.