

TEST 9**Algebraic Manipulation
and Formulae****Marks: /80****Time: 1 hour 30 minutes**

Name:

Date:

INSTRUCTIONS TO CANDIDATES**Section A (40 marks)****Time: 45 minutes**

1. Answer all the questions in this section.
2. Calculators may not be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [] at the end of each question.

-
- 1 Make x the subject of the formula.

$$\frac{x}{a} + \frac{x}{3b} = c$$

Answer [2]

C.E.M. Tutor
Tel: 9555 3331
Suite 207/474 Gardeners Rd
ROSEBERY NSW 2018

- 2 (a) Given that $\frac{3u - 2v}{u - 4v} = \frac{4}{5}$, find the value of $\frac{u}{v}$.
- (b) (i) Find the HCF of $20a^2b^3$ and $5a^5b^2$.
(ii) Find the LCM of $12pq^2r^3$ and $21p^2qr^4$.

3 Si
(a
(b

Answer (a) [2]

(b) (i) HCF = [1]

(ii) LCM = [1]

3 Simplify each of the following.

(a) $\frac{2x^2y \times 6xy^2}{18xy}$

(b) $\frac{8x^3y}{5(x+2)} + \frac{10xy^3}{(x+2)^2}$

C.E.M. TUNON
Tel: 9666 3331
Suite 201/414 Gardens Rd
ROSEBERY NSW 2018

Answer (a) [1]

(b) [2]

- 4 (a) Express $\frac{5}{3x-2} - \frac{3}{x+3}$ as a single fraction in its simplest form.
- (b) Solve the equation $x + 4 = \frac{x+4}{x-2}$.

Answer (a) [2]

(b) [3]

5 (a) Find the (i) HCF and the (ii) LCM of $2x^2y^2z^3$, $6x^2yz$ and $8x^3y$.

(b) Given that $k = \frac{g^2h - h^2}{g^2 - k}$, express g in terms of k and h .

CLEM TUISON
Tel: 9986 3894
Suite 201/414 Gardens Rd
ROSEBERY NSW 2018

Answer (a) (i) HCF = [1]

(ii) LCM = [1]

(b) [2]

6 (a) Simplify $\frac{x^2 - 4xy + 4y^2}{x^2 - 2xy}$.

(b) Express $\frac{3}{x-y} - \frac{2}{x+y}$ as a single fraction in its lowest term.

Answer (a) [2]

(b) [2]

7 (a)

(b)

7 (a) Simplify $\frac{a^2 + ab + ac + bc}{a^2 + ab - ac - bc}$.

(b) Solve the equation $\frac{x}{x-1} + x = \frac{x-2}{1-x} - 2$.

Answer (a) [2]

(b) [3]

8 Solve the following equations.

(a) $5 + \frac{7}{x-2} = 4x$

(b) $\frac{4}{3x-3} + \frac{3}{1-x} = 2$

Answer (a) [2]

(b) [2]

9 Solve the following equations.

(a) $\frac{1}{3x-12} - \frac{x^2-1}{4+3x-x^2} = 0$

(b) $\frac{5}{2-x} - \frac{3}{4-2x} + \frac{1}{x-2} = 2$

Answer (a) [3]

(b) [2]

10 The area of a rectangle is $\frac{45x^2}{14y^2}$ cm² and its breadth is $\frac{5x}{7y}$ cm.

Find an expression in terms of x and y for

- (a) the length of the rectangle,
- (b) the perimeter of the rectangle.

Answer (a) cm [2]

(b) cm [2]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks)

Time: 45 minutes

1. Answer all the questions in this section.
 2. Calculators may be used in this section.
 3. All working must be clearly shown. Omission of essential working will result in loss of marks.
 4. The marks for each question is shown in brackets [] at the end of each question.
-

11 (a) Given that $\frac{a}{3} = \sqrt{\frac{b+1}{b-1}}$, express b in terms of a .

(b) Simplify $\frac{6a^2b - 15a^3b^3 + 21a^3b}{3ab}$.

(c) When x^2 is divided by $(x - 5)$, the quotient is 20 and the remainder is 1. Find the possible values of x .

Answer (a) [2]

(b) [2]

(c) [3]

12 (a) Given that $a = \frac{1}{b} - \frac{1}{c}$, $b = d^2 - 1$ and $c = 1 + d$, express a in terms of d in its simplest form.

(b) Find the number which, increased by its reciprocal is equal to $\frac{37}{6}$.

13 (a)

(b)

Answer (a) [3]

(b) [5]

13 (a) Express the following as a single fraction in its lowest terms.

(i) $\frac{2}{x+2y} - \frac{x-6y}{x^2-4y^2}$

(ii) $x+y - \frac{2xy}{x+y}$

(b) Solve the equation $\frac{x}{x-2} + \frac{x-2}{x} = \frac{5}{2}$.

Answer (a) (i) [3]

(ii) [2]

(b) [3]

C.E.M. Tutor
Tel: 9888 3331
Suite 201/414 Gardeners Rd
ROSEBURY NSW 2818

14 Benjamin can buy x pens for \$30.

(a) Write down, an expression in terms of x , for the cost of each pen.

He could also buy 5 more rulers than pens using the same amount of money.

(b) Write down, an expression in terms of x , for the cost of each ruler.

10 rulers will cost \$3 less than 10 pens.

(c) Write down an equation in terms of x to represent the information given and show that it reduces to $x^2 + 5x - 500 = 0$. [3]

(d) Hence solve the equation $x^2 + 5x - 500 = 0$ to find the values of x .

(e) Find the cost of each ruler.

Answer (a) \$ [1]

(b) \$ [1]

(d) [2]

(e) \$ [1]

- 15 A fruitseller bought some durians for \$300. He paid \$ x for each kilogram of durians.
- Find, in terms of x , an expression for the number of kilograms of durians he bought.
 - The fruitseller had to throw away 3 kg of durians that were rotten and sold the remainder for \$2 per kg more than he paid for it. Write down, an expression in terms of x , for the sum of money he received.
 - He made a profit of \$132. Write down, an equation in terms of x , to represent this information and show that it reduces to $x^2 + 46x - 200 = 0$. [3]
 - Hence solve the equation $x^2 + 46x - 200 = 0$ for the values of x .
 - How many kilograms of durians did he sell altogether?

Answer (a) kg [1]

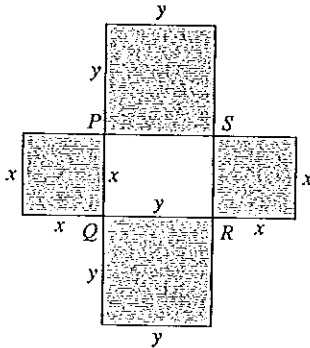
(b) \$ [2]

(d) [2]

(e) kg [1]

C.L.M. Judd
Tel: 9665 3331
Suite 201/414 Gardeners Rd
ROSEBERRY NSW 2018

(b)



Let x cm and y cm be the lengths of PQ and QR respectively.

Perimeter of $PQRS = 26$ cm (Given)

$$2(x + y) = 26 \quad \text{Divide both sides by 2.}$$

$$x + y = 13$$

Area of the 4 squares = 178 cm² (Given)

$$x^2 + x^2 + y^2 + y^2 = 178$$

$$2x^2 + 2y^2 = 178 \quad \text{Divide both sides by 2.}$$

$$x^2 + y^2 = 89$$

$$(x + y)^2 = x^2 + 2xy + y^2 \quad \text{Use } (a + b)^2 = a^2 + 2ab + b^2.$$

$$(x + y)^2 = (x^2 + y^2) + 2xy \quad \text{Substitute } x + y = 13$$

$$(13)^2 = 89 + 2xy \quad \text{and } x^2 + y^2 = 89.$$

$$2xy = 169 - 89 = 80$$

$$xy = \frac{80}{2} = 40$$

$$\text{Area of } PQRS = xy$$

$$= 40 \text{ cm}^2$$

15. (a) Time taken = $\frac{80 \text{ km}}{x \text{ km/h}}$ Use Total time taken = $\frac{\text{Total distance travelled}}{\text{Average speed}}$

$$= \left(\frac{80}{x}\right) \text{ h}$$

(b) Time taken = $\frac{30 \text{ km}}{(x - 15) \text{ km/h}}$ Average speed is 15 km/h slower. \therefore average speed = $(x - 15) \text{ km/h}$

$$= \left(\frac{30}{x - 15}\right) \text{ h}$$

(c) Total time taken = $10 \text{ 30} - 07 \text{ 30} = 3 \text{ h}$

$$\begin{array}{c} \text{Rest} \\ \downarrow \\ \frac{80}{x} + 1 + \frac{30}{x - 15} = 3 \end{array}$$

$$\frac{80}{x} + \frac{30}{x - 15} = 2$$

$$\frac{80(x - 15) + 30x}{x(x - 15)} = 2$$

$$80(x - 15) + 30x = 2x(x - 15) \quad \text{Multiply both sides by } x(x - 15).$$

$$80x - 1200 + 30x = 2x^2 - 30x \quad \text{by } x(x - 15).$$

$$2x^2 - 140x + 1200 = 0 \quad \text{Divide both sides by 2.}$$

$$x^2 - 70x + 600 = 0 \quad \text{Shown.}$$

(d) $x^2 - 70x + 600 = 0$

$$(x - 60)(x - 10) = 0$$

$$\therefore x - 60 = 0 \quad \text{or} \quad x - 10 = 0$$

$$x = 60 \quad \text{or} \quad x = 10$$

(e) $x = 60$ Reject $x = 10$.

$$\text{Time taken} = \frac{80 + 30}{60}$$

$$= 1 \frac{5}{6} \text{ h}$$

$$= 1 \text{ h } 50 \text{ min}$$

$$07 \text{ 30} + 1 \text{ h } 50 \text{ min} = 09 \text{ 20}$$

\therefore the motorist will reach Town B at 09 20.

Test 9: Algebraic Manipulation

Section A

1. $\frac{x}{a} + \frac{x}{3b} = c$

$$x\left(\frac{1}{a} + \frac{1}{3b}\right) = c \quad \text{Extract common factor, } x.$$

$$x\left(\frac{3b + a}{3ab}\right) = c \quad \text{The LCM of } a \text{ and } 3b \text{ is } 3ab.$$

$$x = \frac{3abc}{3b + a} \quad \text{Cross multiply and express } x \text{ in terms of the other variables.}$$

2. (a) $\frac{3u - 2v}{u - 4v} = \frac{4}{5}$ Cross-multiply.

$$5(3u - 2v) = 4(u - 4v)$$

$$15u - 10v = 4u - 16v$$

$$11u = -6v$$

Divide both sides by $11v$.

$$\frac{u}{v} = -\frac{6}{11}$$

(b) (i) $20a^2b^3 = 2^2 \times 5 \times a^2 \times b^3$

$$5a^3b^2 = 5 \times a^3 \times b^2$$

$$\therefore \text{HCF} = 5 \times a^2 \times b^2$$

$$= 5a^2b^2$$



Teacher's Tip

To obtain the Highest Common Factor (HCF), find the largest number of factors which are common to both expressions.

(ii) $12p^2q^3r^3 = 2^2 \times 3 \times p^2 \times q^3 \times r^3$

$$21p^2qr^4 = 3 \times 7 \times p^2 \times q \times r^4$$

$$\therefore \text{LCM} = 2^2 \times 3 \times 7 \times p^2 \times q^3 \times r^4$$

$$= 84p^2q^3r^4$$



Teacher's Tip

To obtain the Least Common Multiple (LCM), find the smallest group of factors which contain all the factors of the two expressions.

$$3. (a) \frac{2x^2y \times 6xy^2}{18xy}$$

$$= \frac{2x^2y^2}{3}$$

$$(b) \frac{8x^3y}{5(x+2)} \div \frac{10xy^3}{(x+2)^2}$$

Change \div to \times by inverting the divisor.

$$= \frac{8x^3y}{5(x+2)} \times \frac{(x+2)^2}{10xy^3}$$

$$= \frac{4x^2(x+2)}{25y^2}$$

$$4. (a) \frac{5}{3x-2} - \frac{3}{x+3}$$

$$= \frac{5(x+3) - 3(3x-2)}{(3x-2)(x+3)}$$

The LCM of $(3x-2)$ and $(x+3)$ is $(3x-2)(x+3)$.

$$= \frac{5x + 15 - 9x + 6}{(3x-2)(x+3)}$$

$$= \frac{21 - 4x}{(3x-2)(x+3)}$$

Teacher's Tip
Do not expand the denominator since the question requires the answer in its simplest form, i.e. the factorised form.

$$(b) x + 4 = \frac{x+4}{x-2}$$

$$(x-2)(x+4) = x+4$$

$$x^2 + 4x - 2x - 8 = x + 4$$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4) = 0$$

$$\therefore x-3 = 0 \quad \text{or} \quad x+4 = 0$$

$$x = 3 \quad \text{or} \quad x = -4$$

Multiply both sides by $(x-2)$.

$$5. (a) 2x^2y^2z^3 = 2 \times x^2 \times y^2 \times z^3$$

$$6x^2yz = 2 \times 3 \times x^2 \times y \times z$$

$$8x^3y = 2^3 \times x^3 \times y$$

(i) HCF = $2 \times x^2 \times y = 2x^2y$

(ii) LCM = $2^3 \times 3 \times x^3 \times y^2 \times z^3 = 24x^3y^2z^3$

$$(b) k = \frac{g^2h - h^2}{g^2 - k}$$

Cross-multiply and express g in terms of other variables.

$$k(g^2 - k) = g^2h - h^2$$

$$kg^2 - k^2 = g^2h - h^2$$

$$kg^2 - g^2h = k^2 - h^2$$

$$g^2(k-h) = k^2 - h^2$$

Collect all terms containing g to the left-hand side and then factorise.

$$g^2 = \frac{k^2 - h^2}{k-h}$$

Use $a^2 - b^2 = (a+b)(a-b)$.

$$= \frac{(k+h)(k-h)}{(k-h)}$$

$$= k+h$$

$$g = \pm\sqrt{k+h}$$

Take square root on both sides.

$$6. (a) \frac{x^2 - 4xy + 4y^2}{x^2 - 2xy}$$

$$= \frac{(x-2y)(x-2y)}{x(x-2y)}$$

$$= \frac{x-2y}{x}$$



Teacher's Tip

Factorise both the numerator and denominator completely before cancelling common factors.

$$(b) \frac{3}{x-y} - \frac{2}{x+y}$$

The LCM is $(x-y)(x+y)$.

$$= \frac{3(x+y) - 2(x-y)}{(x-y)(x+y)}$$

$$= \frac{3x + 3y - 2x + 2y}{(x-y)(x+y)}$$

$$= \frac{x+5y}{(x-y)(x+y)}$$

$$7. (a) \frac{a^2 + ab + ac + bc}{a^2 + ab - ac - bc}$$

$$= \frac{a(a+b) + c(a+b)}{a(a+b) - c(a+b)}$$

Factor out $(a+b)$, the common factor of the numerator and denominator.

$$= \frac{(a+b)(a+c)}{(a+b)(a-c)}$$

$$= \frac{a+c}{a-c}$$

$$(b) \frac{x}{x-1} + x = \frac{x-2}{1-x} - 2$$

Note: $1-x = -(x-1)$

$$\frac{x}{x-1} + x = -\left(\frac{x-2}{x-1}\right) - 2$$

$$\therefore \frac{x-2}{1-x} = -\left(\frac{x-2}{x-1}\right)$$

$$\frac{x}{x-1} + x + \frac{x-2}{x-1} + 2 = 0$$

$$x + x(x-1) + (x-2) + 2(x-1) = 0$$

$$x + x^2 - x + x - 2 + 2x - 2 = 0$$

$$x^2 + 3x - 4 = 0$$

$$(x-1)(x+4) = 0$$

$$\therefore x-1 = 0 \quad \text{or} \quad x+4 = 0$$

$$x = 1 \text{ (rejected)} \quad \text{or} \quad x = -4$$

$x-1 \neq 0$
 $\therefore x \neq 1$



Teacher's Tip

$x = 1$ is rejected because division by 0 is undefined.
 $x = 1$ is not a solution.

8. (a) $5 + \frac{7}{x-2} = 4x$
 $5(x-2) + 7 = 4x(x-2)$ Multiply throughout
 $5x - 10 + 7 = 4x^2 - 8x$ by the LCM, $(x-2)$.
 $4x^2 - 13x + 3 = 0$
 $(4x-1)(x-3) = 0$
 $\therefore 4x-1 = 0$ or $x-3 = 0$
 $x = \frac{1}{4}$ or $x = 3$

(b) $\frac{4}{3x-3} + \frac{3}{1-x} = 2$ Factorise the denominator
 first before finding the LCM.
 $\frac{4}{3(x-1)} + \left(-\frac{3}{x-1}\right) = 2$
 $\frac{4}{3(x-1)} - \frac{3}{x-1} - 2 = 0$
 $4 - 3(3) - 2[3(x-1)] = 0$ Multiply throughout by
 the LCM, $3(x-1)$.
 $4 - 9 - 6x + 6 = 0$
 $6x = 1$
 $x = \frac{1}{6}$

9. (a) $\frac{1}{3x-12} - \frac{x^2-1}{4+3x-x^2} = 0$
 $\frac{1}{3(x-4)} - \frac{x^2-1}{(4-x)(1+x)} = 0$ Factorise each of
 the denominators
 first.
 $\frac{1}{3(x-4)} + \frac{x^2-1}{(x-4)(x+1)} = 0$
 $(x+1) + 3(x^2-1) = 0$ Multiply
 $x+1+3x^2-3 = 0$ throughout by
 the LCM,
 $3x^2+x-2 = 0$
 $(3x-2)(x+1) = 0$ $3(x+1)(x-4)$.
 $\therefore 3x-2 = 0$ or $x+1 = 0$
 $x = \frac{2}{3}$ or $x = -1$ (rejected)

(b) $\frac{5}{2-x} - \frac{3}{4-2x} + \frac{1}{x-2} = 2$
 $\frac{5}{2-x} - \frac{3}{2(2-x)} - \frac{1}{2-x} = 2$
 $2(5) - 3 - 2(1) = 2[2(2-x)]$ Multiply
 $10 - 3 - 2 = 8 - 4x$ throughout by the
 LCM, $2(2-x)$.
 $4x = 3$
 $x = \frac{3}{4}$

10. (a) Area of rectangle = $\frac{45x^2}{14y^2}$ (Given) Area of
rectangle
= Length \times
Breadth
 (Length) $\times \left(\frac{5x}{7y}\right) = \frac{45x^2}{14y^2}$
 Length = $\frac{45x^2}{14y^2} \times \frac{7y}{5x} = \frac{9x}{2y}$ (Given)
 $= \frac{9x}{2y}$ cm

(b) Perimeter of rectangle
 $= 2(\text{Length} + \text{Breadth})$
 $= 2\left(\frac{9x}{2y} + \frac{5x}{7y}\right)$
 $= 2\left[\frac{7(9x) + 2(5x)}{7 \cdot 14y}\right]$ The LCM of $2y$ and
 $7y$ is $14y$.
 $= \frac{63x + 10x}{7y}$
 $= \frac{73x}{7y}$ cm

Section B

11. (a) $\frac{a}{3} = \sqrt{\frac{b+1}{b-1}}$
 $\left(\frac{a}{3}\right)^2 = \frac{b+1}{b-1}$ Square both sides.
 $\frac{a^2}{9} = \frac{b+1}{b-1}$ Cross-multiply.
 $a^2(b-1) = 9(b+1)$
 $a^2b - a^2 = 9b + 9$
 $a^2b - 9b = a^2 + 9$
 $b(a^2 - 9) = a^2 + 9$
 $b = \frac{a^2 + 9}{a^2 - 9}$

(b) $\frac{6a^2b - 15a^3b^3 + 21a^3b}{3ab}$
 $= \frac{3a^2b(2 - 5ab^2 + 7a)}{3ab}$ Extract common factor
 of numerator, $3a^2b$.
 $= a(2 - 5ab^2 + 7a)$

(c) $\frac{x^2}{x-5} = 20 + \frac{1}{x-5}$ Given that when x^2 is
divided by $(x-5)$, the
quotient is 20 and the
remainder is 1.
 $x^2 = 20(x-5) + 1$ Multiply
 $x^2 = 20x - 100 + 1$ throughout by
 the LCM,
 $x^2 - 20x + 99 = 0$ $(x-5)$.
 $(x-9)(x-11) = 0$
 $\therefore x-9 = 0$ or $x-11 = 0$
 $x = 9$ or $x = 11$

Check answer:

When $x = 9$: $\frac{9^2}{9-5} = \frac{81}{4} = 20$ remainder 1.

When $x = 11$: $\frac{11^2}{11-5} = \frac{121}{6} = 20$ remainder 1.

$$\begin{aligned}
 12. (a) \quad a &= \frac{1}{b} - \frac{1}{c} \\
 &= \frac{1}{d^2 - 1} - \frac{1}{1 + d} \quad \text{Given that } b = d^2 - 1 \\
 &\quad \text{and } c = 1 + d. \\
 &= \frac{1}{(d + 1)(d - 1)} - \frac{1}{d + 1} \\
 &= \frac{1 - (d - 1)}{(d + 1)(d - 1)} \quad \text{Use } a^2 - b^2 = (a + b)(a - b). \\
 &\quad d^2 - 1 = d^2 - 1^2 \\
 &\quad = (d + 1)(d - 1) \\
 &= \frac{1 - d + 1}{(d + 1)(d - 1)} \\
 a &= \frac{2 - d}{(d + 1)(d - 1)} \quad \text{The LCM is } (d + 1)(d - 1).
 \end{aligned}$$

(b) Let x be the number. The reciprocal of a number

Then $\frac{1}{x}$ is the reciprocal. A is the number $\frac{1}{A}$.

$$\begin{aligned}
 x + \frac{1}{x} &= \frac{37}{6} \quad \text{Given the number added to its} \\
 6x^2 + 6 &= 37x \quad \text{reciprocal is equal to } \frac{37}{6}. \\
 6x^2 - 37x + 6 &= 0 \quad \text{Multiply throughout by the} \\
 (6x - 1)(x - 6) &= 0 \quad \text{LCM, } 6x. \\
 \therefore 6x - 1 &= 0 \quad \text{or} \quad x - 6 = 0 \\
 x &= \frac{1}{6} \quad \text{or} \quad x = 6
 \end{aligned}$$

\therefore the number may be taken as $\frac{1}{6}$ and the reciprocal 6 or the number can be taken as 6 and the reciprocal as $\frac{1}{6}$.

Check answer: $6 + \frac{1}{6} = \frac{37}{6}$

$$\begin{aligned}
 13. (a) (i) \quad &\frac{2}{x + 2y} - \frac{x - 6y}{x^2 - 4y^2} \\
 &= \frac{2}{x + 2y} - \frac{x - 6y}{(x + 2y)(x - 2y)} \\
 &= \frac{2(x - 2y) - (x - 6y)}{(x + 2y)(x - 2y)} \quad \text{Use } a^2 - b^2 \\
 &\quad = (a + b)(a - b) \\
 &= \frac{2x - 4y - x + 6y}{(x + 2y)(x - 2y)} \quad \text{on } x^2 - 4y^2. \\
 &= \frac{x - 2y}{(x + 2y)(x - 2y)} \quad x^2 - 4y^2 \\
 &= \frac{-(-x + 2y)}{-(-x + 2y)(x - 2y)} \quad = (x + 2y)(x - 2y) \\
 &= \frac{1}{x - 2y} \quad \text{The LCM is} \\
 &\quad (x + 2y)(x - 2y).
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad x + y - \frac{2xy}{x + y} \\
 &= \frac{x(x + y) + y(x + y) - 2xy}{x + y} \quad \text{The LCM} \\
 &\quad \text{is } (x + y). \\
 &= \frac{x^2 + xy + xy + y^2 - 2xy}{x + y} \\
 &= \frac{x^2 + y^2}{x + y}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \frac{x}{x - 2} + \frac{x - 2}{x} &= \frac{5}{2} \\
 2x^2 + 2(x - 2)^2 &= 5x(x - 2) \quad \text{Multiply} \\
 2x^2 + 2(x^2 - 4x + 4) &= 5x^2 - 10x \quad \text{throughout by} \\
 2x^2 + 2x^2 - 8x + 8 &= 5x^2 - 10x \quad \text{the LCM,} \\
 x^2 - 2x - 8 &= 0 \quad 2x(x - 2). \\
 (x + 2)(x - 4) &= 0. \\
 \therefore x + 2 = 0 \quad \text{or} \quad x - 4 = 0 \quad \text{Use } (a - b)^2 \\
 x = -2 \quad \text{or} \quad x = 4 \quad = a^2 - 2ab + b^2 \text{ to} \\
 &\quad \text{expand } (x - 2)^2.
 \end{aligned}$$

14. (a) Cost of each pen = $\$ \left(\frac{30}{x} \right)$

(b) Cost of each ruler = $\$ \left(\frac{30}{x + 5} \right)$

(c) $10 \left(\frac{30}{x + 5} \right) = 10 \left(\frac{30}{x} \right) - 3$ 10 rulers cost \$3 less than 10 pens. (Given)

$$\begin{aligned}
 \frac{300}{x + 5} - \frac{300}{x} + 3 &= 0 \quad \text{Multiply} \\
 300x - 300(x + 5) + 3x(x + 5) &= 0 \quad \text{throughout} \\
 300x - 300x - 1500 + 3x^2 + 15x &= 0 \quad \text{by the LCM,} \\
 3x^2 + 15x - 1500 &= 0 \quad x(x + 5). \\
 x^2 + 5x - 500 &= 0 \quad \text{Divide throughout by 3.} \\
 &\quad \text{Shown}
 \end{aligned}$$

(d) $x^2 + 5x - 500 = 0$
 $(x - 20)(x + 25) = 0$
 $\therefore x - 20 = 0 \quad \text{or} \quad x + 25 = 0$
 $x = 20 \quad \text{or} \quad x = -25$

(e) $\therefore x = 20$ Reject $x = -25$.

Cost of each ruler = $\$ \left(\frac{30}{20 + 5} \right)$
 $= \$1.20$

15. (a) No. of kilograms of durians bought

$$= \frac{300}{x}$$

(b) Sum of money received

$$\begin{aligned}
 &= \$ \left[\left(\frac{300}{x} - 3 \right) \times (x + 2) \right] \quad \text{Sum of money} \\
 &= \$ \left[\left(\frac{300 - 3x}{x} \right) (x + 2) \right] \quad \text{received} \\
 &= \$ \left[\frac{3(100 - x)(x + 2)}{x} \right] \quad = (\text{No. of kg left}) \\
 &\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \times (\text{Selling price of} \\
 &\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{each kg})
 \end{aligned}$$

Sum of money - \$300 = Profit.

(c) $\frac{3(100 - x)(x + 2)}{x} - 300 = 132$

$$\frac{3(100 - x)(x + 2)}{x} = 432 \quad \text{Divide both sides} \\
 (100 - x)(x + 2) = 144x \quad \text{by 3.}$$

$$\begin{aligned}
 100x + 200 - x^2 - 2x &= 144x \quad \text{Multiply throughout} \\
 x^2 + 46x - 200 &= 0 \quad \text{Shown by } x.
 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad x^2 + 46x - 200 &= 0 \\ (x - 4)(x + 50) &= 0 \\ \therefore x - 4 = 0 \quad \text{or} \quad x + 50 = 0 \\ x = 4 \quad \text{or} \quad x = -50 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad \therefore x = 4 \quad \text{Reject } x = -50. \\ \text{No. of kilograms of durians sold} \\ = \frac{300}{4} - 3 \\ = 72 \end{aligned}$$

Test 10: Simultaneous Linear Equations

Section A

1. Method 1: Elimination Method

$$\begin{aligned} 2x + 3y &= 12 \quad \text{--- (1)} \\ x - 4y &= -5 \quad \text{--- (2)} \\ (2) \times 2: \quad 2x - 8y &= -10 \quad \text{--- (3)} \\ (1) - (3): \quad 3y - (-8y) &= 12 - (-10) \\ 3y + 8y &= 12 + 10 \\ 11y &= 22 \\ y &= \frac{22}{11} = 2 \end{aligned}$$

To eliminate x , make the coefficients of x to be equal by multiplying (2) by 2.

$$\begin{aligned} \text{Substitute } y = 2 \text{ into (2):} \\ x - 4(2) &= -5 \\ x - 8 &= -5 \\ x &= -5 + 8 \\ x &= 3 \end{aligned}$$

Substitute the value of y into any of the original equations to find the value of x .

\therefore the solution set is $x = 3, y = 2$.

Teacher's Tip

1) In this method, we first eliminate one of the two unknowns. Either of the unknowns may be eliminated first. In the above question, it is easier to eliminate x .

2) Make a habit of checking your answer either mentally or by writing down.

Check: Substitute $x = 3, y = 2$ into (1):

$$\text{LHS} = 2(3) + 3(2) = 6 + 6 = 12 = \text{RHS}$$

Method 2: Substitution Method

$$\begin{aligned} 2x + 3y &= 12 \quad \text{--- (1)} \\ x - 4y &= -5 \quad \text{--- (2)} \end{aligned}$$

$$\text{From (2):} \quad x = -5 + 4y \quad \text{--- (3)}$$

Make x the subject of the equation.

Substitute (3) into (1):

$$\begin{aligned} 2(-5 + 4y) + 3y &= 12 \\ -10 + 8y + 3y &= 12 \\ 11y &= 22 \\ y &= \frac{22}{11} = 2 \end{aligned}$$

Substitute (3) into (1) to obtain an equation with only the y variable.

Substitute $y = 2$ into (3):

$$\begin{aligned} x &= -5 + 4(2) \\ &= -5 + 8 \\ &= 3 \end{aligned}$$

\therefore the solution set is $x = 3, y = 2$.

Teacher's Tip

In this method, we solve one of the equations for one unknown and substitute this expression into the other equation to solve for the remaining unknown.

$$\begin{aligned} 2. \quad 4x - 3y &= 18 \quad \text{--- (1)} \\ 7x + 5y &= 11 \quad \text{--- (2)} \end{aligned}$$

$$\begin{aligned} (1) \times 5: \quad 20x - 15y &= 90 \quad \text{--- (3)} \\ (2) \times 3: \quad 21x + 15y &= 33 \quad \text{--- (4)} \\ (4) + (3): \quad 41x &= 123 \\ x &= \frac{123}{41} = 3 \end{aligned}$$

To eliminate y , make the coefficients of y to be equal by multiplying (1) by 5 and (2) by 3.

Substitute $x = 3$ into (1):

$$\begin{aligned} 4(3) - 3y &= 18 \\ 12 - 3y &= 18 \\ -3y &= 6 \end{aligned}$$

$$y = \frac{6}{-3} = -2$$

\therefore the solution set is $x = 3, y = -2$.

$$\begin{aligned} 3. \quad 2x - y &= 11 \quad \text{--- (1)} \\ -2x + 5y &= -7 \quad \text{--- (2)} \end{aligned}$$

Rewrite $-2x + 5y + 7 = 0$ as $-2x + 5y = -7$.

$$\begin{aligned} (1) + (2): \quad 4y &= 4 \\ y &= \frac{4}{4} = 1 \end{aligned}$$

Substitute $y = 1$ into (1):

$$\begin{aligned} 2x - 1 &= 11 \\ 2x &= 12 \\ x &= \frac{12}{2} = 6 \end{aligned}$$

\therefore the solution set is $x = 6, y = 1$.

Teacher's Tip

Note that the coefficients of x are equal but one is positive and the other is negative. To eliminate x , add the two equations.

$$\begin{aligned} 4. \quad h - 5k &= -3 \quad \text{--- (1)} \\ h + 3k &= 1 \quad \text{--- (2)} \end{aligned}$$

Since the coefficients of h are equal, subtract (1) from (2) to eliminate h .

$$\begin{aligned} (2) - (1): \quad 3k - (-5k) &= 1 - (-3) \\ 3k + 5k &= 1 + 3 \\ 8k &= 4 \end{aligned}$$

$$k = \frac{4}{8} = \frac{1}{2}$$