

Probability

Probability is a measure of how likely it is that an event will happen.

Definitions Probability Multiplication Expected
Principle Value

Definitions

- The rolling of a die, the drawing of a card, etc. are called **experiments**.
- A **trial** is a single part of an experiment which consists of many trials.

e.g. Rolling the die 10 times would be 10 trials.

- An **outcome** is the result of a trial of an experiment.
- The **sample space (S)** is the set of all the possible outcomes of an experiment.
- An **event (E)** is a part of the sample space.

e.g. An experiment could be the tossing of two coins.

The sample space for this experiment is:

$$S = \{(\text{head, head}), (\text{head, tail}), (\text{tail, head}), (\text{tail, tail})\}$$

An event could be:

$$E = \{\text{both the coins are the same}\} = \{(\text{head, head}), (\text{tail, tail})\}$$

Probability

Probabilities are expressed as *fractions, decimal fractions or percentages*.

For equally likely outcomes, the theoretical probability of the event E occurring is given by:

$$P(E) = \frac{\text{Number of ways that event E can occur}}{\text{Total number of possible outcomes}}$$

For experiments, the experimental probability of an event E happening is given by:

$$P(E) = \frac{\text{Number of times event E occurs}}{\text{Total number of trials}}$$

For experiments, the more trials that are carried out, the nearer the experimental probability will be to the theoretical probability given above.

Example 1	Answer
When two coins are tossed together, what is the probability that they will both be the same?	$S = \{(H, H), (H, T), (T, H), (T, T)\}$ $E = \{(H, H), (T, T)\}$ $P(E) = \frac{2}{4} = \frac{1}{2}$
Example 2	Answer
If a die is rolled, what is the probability of getting more than a 4?	$S = \{1, 2, 3, 4, 5, 6\}$ $E = \{5, 6\}$ $P(E) = \frac{2}{6} = \frac{1}{3}$

Probabilities must always be in the range from 0 to 1.

e.g. $0 \leq P(E) \leq 1$

If the probability of an event happening is 0, the event cannot occur.

If the probability of an event happening is 1, the event is certain to occur.

e.g. $P(\text{throwing a six-sided die and scoring a } 7) = 0$

$P(\text{throwing a six-sided die and scoring less than } 7) = 1$

Multiplication Principle

To find the probability of multiple events occurring, use the multiplication principle and multiply the individual probabilities together.

e.g. $P(\text{throwing a "6" on a standard dice followed by a "5"})$

$= P(\text{throwing a "6"}) \times P(\text{throwing a "5"})$

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Expected value

The expected value of an event occurring from a certain number of trials is the number of times the event is expected to occur.

Expected value = (Probability of event) x (number of trials)

$$= P(E) \times n$$

Example	Answer
If a die is thrown 60 times, how many times would you expect a 4 or a 5 to be thrown?	$P(4 \text{ or } 5) = \frac{2}{6} = \frac{1}{3}$ Expected value = $\frac{1}{3} \times 60 = 20$ times.



Probability

1. If we take a normal pack of 52 cards, what is the probability of drawing:

- (a) A king?
- (b) The king of diamonds?
- (c) A club?
- (d) A five, six or a seven?
- (e) A red or a black card?



2. A die is thrown. What is the probability of throwing:

- (a) A five?
- (b) A number less than a 3?
- (c) An even number?

3. Jane's birthday is on 22 November. What is the probability that her birthday will be on:

- (a) A Friday?
- (b) A day beginning with a 't'?

4. In a town, three out of eight children own a bicycle. If there are 2000 children in the town, how many would you expect to own a bicycle?

5. A bag contains 12 white balls and 6 red balls. A student takes a ball from the bag.

What is the probability that the ball is:

- (a) White?
- (b) Red?
- (c) Green?

6. If two one dollar coins are tossed together:

- (a) List the sample space of the outcomes, e.g. $\{(T, T), \dots\}$
- (b) What is the probability of getting:

- (i) Two heads?
- (ii) At least one tail?

7. A die is rolled 60 times. How many times would you expect to throw:

- (a) a 5 or a 6?
- (b) an even number?
- (c) numbers less than 4?

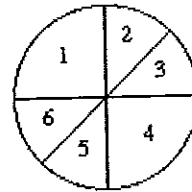
8. A die is rolled twice and the total is noted. Find the probability of getting:

- (a) Two fours.
- (b) A total of 7.
- (c) A double.

9. A blindfolded person throws a dart at a dart board split into 6 sections as shown below and marked with numbers from 1 to 6 as in the diagram. All darts hit the board.

The circle shown has angles of 90° and 45°

- (a) What is the probability of scoring a 5?
- (b) What is the probability of scoring an even number?
- (c) If the person throws 20 darts, how many times would you expect a 4 to be scored?



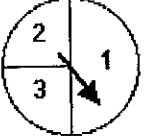
Probability - ANSWERS

1	(a) $\frac{1}{13}$	(b) $\frac{1}{52}$	(c) $\frac{1}{4}$
	(d) $\frac{3}{13}$	(e) 1	
2	(a) $\frac{1}{6}$	(b) $\frac{1}{3}$	(c) $\frac{1}{2}$
3	(a) $\frac{1}{7}$	(b) $\frac{2}{7}$	
4	750		
5	(a) $\frac{2}{3}$	(b) $\frac{1}{3}$	(c) 0
6	(a) {(T, T), (T, H), (H, H), (H, T)}	(b) (i) $\frac{1}{4}$	(ii) $\frac{3}{4}$
7	(a) 20	(b) 30	(c) 30
8	(a) $\frac{1}{36}$	(b) $\frac{1}{6}$	(c) $\frac{1}{6}$
9	(a) $\frac{1}{8}$	(b) $\frac{1}{2}$	(c) 5

Probability

Unit Test #46

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "Submit Test" button. Clicking the "Begin Test Again" button will clear all the answers.

Q1:	Two six-sided dice are rolled one after the other. The number of ways in which a total score of eight can be made is:	A. 3 B. 4 C. 5 D. 6	Answer 1:	<input type="checkbox"/>
Q2:	A ball is drawn from a bag containing 7 red, 3 white, and 6 blue balls. The probability that it is NOT a red ball is:	A. $\frac{3}{16}$ B. $\frac{7}{16}$ C. $\frac{9}{16}$ D. $\frac{10}{16}$	Answer 2:	<input type="checkbox"/>
Q3:	Malua spins the pointer.  The probability that it stops on an odd number is	A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$	Answer 3:	<input type="checkbox"/>
Q4:	A fair coin is tossed 4 times. Each time it lands heads up. When it is tossed a fifth time, the probability that it will land heads up is:	A. $\frac{1}{32}$ B. $\frac{1}{5}$ C. $\frac{1}{2}$ D. 1	Answer 4:	<input type="checkbox"/>
Q5:	40 balls are numbered 1 to 40. One ball is selected at random. The probability that the number on the chosen ball will contain the numeral "1" is	A. $\frac{4}{40}$ B. $\frac{10}{40}$ C. $\frac{12}{40}$ D. $\frac{13}{40}$	Answer 5:	<input type="checkbox"/>
Q6:	A six-sided die is rolled 60 times. How many times would you expect the die with the number "6" facing up?	A. 6 B. 10 C. 20 D. 5	Answer 6:	<input type="checkbox"/>
	A dart is thrown at the board shown			

Q7:	<table border="1" data-bbox="550 168 686 302"> <tr><td>30</td><td>27</td><td>22</td></tr> <tr><td>21</td><td>23</td><td>42</td></tr> <tr><td>19</td><td>52</td><td>35</td></tr> </table> <p>A prize is won if the number hit is a multiple of 5 or a multiple of 7. The probability that a prize will be won with one throw is:</p>	30	27	22	21	23	42	19	52	35	<p>A. $\frac{2}{9}$ B. $\frac{3}{9}$ C. $\frac{4}{9}$ D. $\frac{5}{9}$</p>	Answer 7:	<input type="checkbox"/>
30	27	22											
21	23	42											
19	52	35											
Q8:	An event with a probability of 1:	<p>A. is certain to occur B. has an even chance of occurring. C. is unlikely to occur D. will never occur</p>	Answer 8:	<input type="checkbox"/>									
Q9:	Two six-sided dice are rolled and the number on each die is recorded. What is the probability that a total of 12 is recorded?	<p>A. $\frac{6}{36}$ B. $\frac{12}{36}$ C. $\frac{1}{12}$ D. $\frac{1}{36}$</p>	Answer 9:	<input type="checkbox"/>									
Q10:	What is the probability that two heads occur when two coins are tossed?	<p>A. 0.5 B. 0.25 C. 0.4 D. 0.2</p>	Answer 10:	<input type="checkbox"/>									

Enter your initial and surname here:

[Submit Test](#) [Begin Test Again](#)

ANSWERS Test #46

1. ~~A~~ 2. C 3. D 4. C 5. D 6. B 7. C 8. A 9. D 10. B

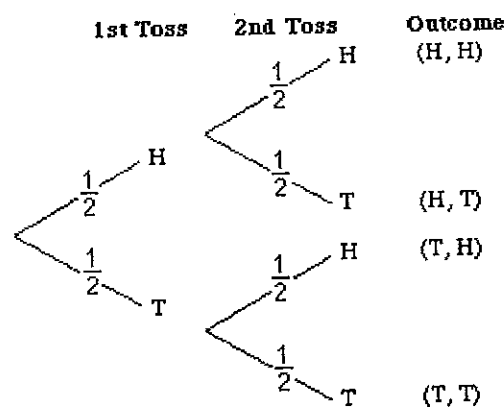
Probability Trees

A **tree diagram** can be used to show probabilities of events.

The diagram looks like the branches of a tree 🌳.

- The probability of each event should be marked on each branch.
- The probabilities on each set of branches must always add up to 1.

e.g. The tree diagram shows the probabilities when 2 coins are tossed. The coins are "fair" that is there is an equal probability that a "head" or a "tail" will be face up.



From the diagram there are 4 possible outcomes:

- Head followed by head
- Head followed by tail
- Tail followed by head
- Tail followed by tail

Therefore the probability of each of these events occurring is $\frac{1}{4}$ (1 out of 4 possibilities).

This can also be calculated using the multiplication principle. e.g. $P(H,H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Tree diagrams are useful for dealing with problems of games of chance.

One such game involves picking coloured balls from a bag. There are 8 balls in a bag, 5 of them are red and 3 of them green. If two balls are picked out at random and not replaced, what is the probability of getting two of the same colour?

Draw a tree diagram.

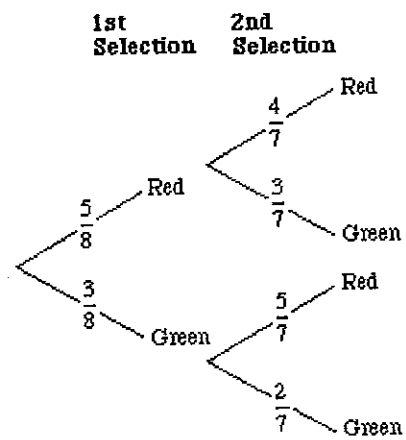
The second selection is out of 7 because one ball has been removed.

From the diagram:

$P(\text{Two balls are same colour})$

$= P(\text{red, red}) + P(\text{green, green})$

$$\begin{aligned} &= \frac{5}{8} \times \frac{4}{7} + \frac{3}{8} \times \frac{2}{7} \\ &= \frac{5}{14} + \frac{3}{28} \\ &= \frac{13}{28} \end{aligned}$$



Probability Trees

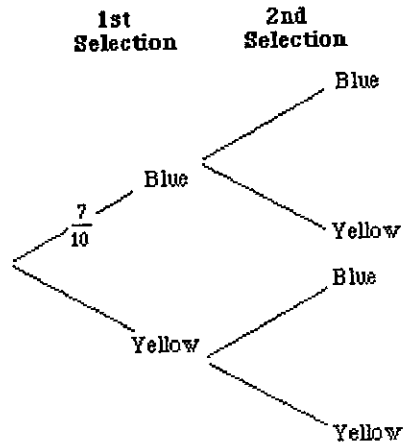
1. A bag contains 7 blue balls and 3 yellow balls. Two balls are drawn at random.

(a) Copy and complete the probability tree diagram.

(b) What is the probability of two blue balls being selected?

(c) What is the probability of balls of different colours being selected?

(d) What is the probability of either or both of the balls not being yellow?



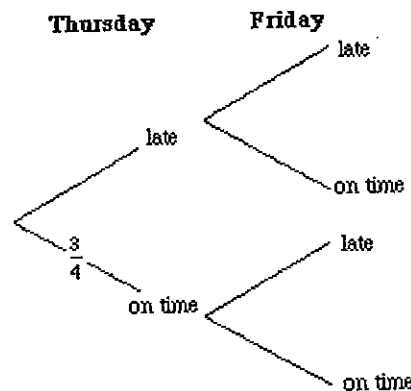
2. The probability that a train is late at a certain station on a Thursday morning is $\frac{1}{4}$. If the train is late on Thursday, the probability that it is also late on Friday is $\frac{1}{12}$, but if it is on time on Thursday the probability that it is late on Friday is $\frac{1}{4}$.

(a) Copy and complete the probability tree below to show this information.

(b) What is the probability that the train is late on both days?

(c) What is the probability that the train is late on one day and on time on the other day?

(d) What is the probability that the train is on time on both days?



3. David and Hayden play two games of tennis. The probability of David winning the first game is $\frac{1}{3}$.

If he wins the first game, the probability of him winning the second game is $\frac{2}{3}$, but if he loses the first game the probability of him losing the second game is $\frac{1}{2}$.

(a) Draw a probability tree for these two games.

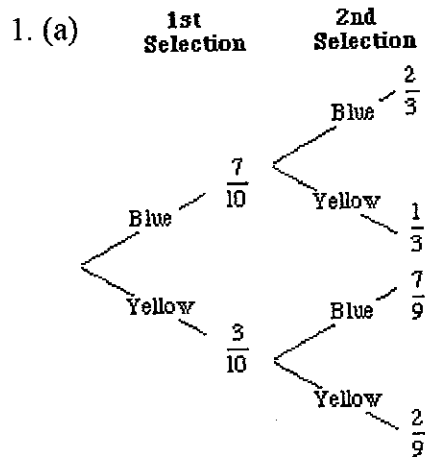
(b) What is the probability of David winning both games?

(c) What is the probability of David losing both games?

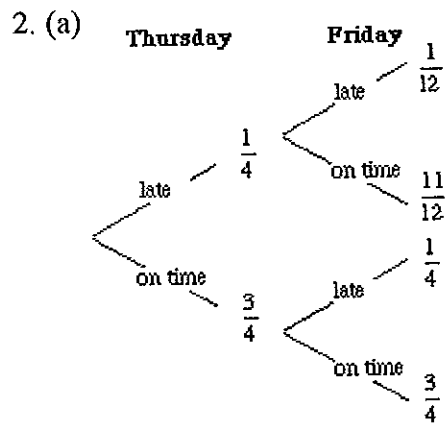
(d) What is the probability of Hayden winning one game and losing one game?



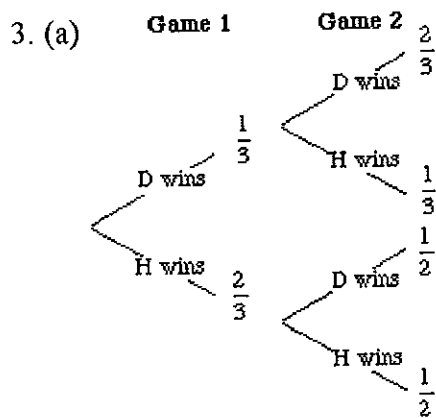
Probability Trees



(b) $\frac{7}{15}$ (c) $\frac{7}{15}$ (d) $\frac{1}{15}$



(b) $\frac{1}{48}$ (c) $\frac{5}{12}$ (d) $\frac{9}{16}$

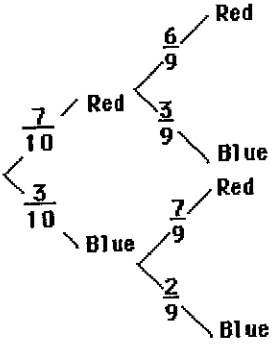


(b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

Probability Trees

Unit Test #47

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "Submit Test" button. Clicking the "Begin Test Again" button will clear all the answers.

<p>Q1:</p>	 <p>What name is given to the type of diagram shown above?</p>	<p>A. Bar graph B. Pie chart C. Tree diagram D. Box and whisker diagram</p>	<p>Answer 1:</p>	<input type="checkbox"/>
<p>Q2:</p>	<p>The diagram in Question 1 shows the probabilities when two coloured balls are drawn from a bag containing 7 red balls and 3 blue balls without replacing the first ball drawn.</p> <p>What is the probability the first ball is red?</p>	<p>A. $\frac{7}{10}$ B. $\frac{3}{10}$ C. $\frac{7}{9}$ D. $\frac{6}{9}$</p>	<p>Answer 2:</p>	<input type="checkbox"/>
<p>Q3:</p>	<p>Using the diagram in question 1.</p> <p>Which answer would give the probability that both balls drawn were red?</p>	<p>A. $\frac{3}{10} \times \frac{7}{9}$ B. $\frac{7}{10} \times \frac{3}{9}$ C. $\frac{7}{10} \times \frac{6}{9}$ D. $\frac{3}{10} \times \frac{2}{9}$</p>	<p>Answer 3:</p>	<input type="checkbox"/>
<p>Q4:</p>	<p>Using the diagram in question 1.</p> <p>Which answer would give the probability that both balls drawn were blue?</p>	<p>A. $\frac{3}{10}$ B. $\frac{47}{90}$ C. $\frac{2}{9}$ D. $\frac{1}{15}$</p>	<p>Answer 4:</p>	<input type="checkbox"/>
<p>Q5:</p>	<p>How many branches does the tree diagram have in question 1?</p>	<p>A. 1 B. 2 C. 3</p>	<p>Answer 5:</p>	<input type="checkbox"/>

		D. 6		
Q6:	<p style="text-align: center;">Tossing of two coins</p> <p>The above diagram indicates an experiment with how many trials?</p>	<p>A. 1 B. 2 C. 3 D. 6</p>	Answer 6:	<input type="checkbox"/>
Q7:	In the diagram in question 6, what is the probability that the two coins tossed are both heads?	<p>A. 0.5 B. 0.25 C. 0.2 D. 1</p>	Answer 7:	<input type="checkbox"/>
Q8:	In the diagram in question 6, what is the probability that the two coins tossed are different ?	<p>A. 0.5 B. 0.25 C. 0.2 D. 1</p>	Answer 8:	<input type="checkbox"/>
Q9:	Complete the set for the sample space for the experiment shown in question 6. $\{(H,H), (H,T), (.,.), (T,T)\}$	<p>A. (T, H) B. (H, T) C. (H, H) D. (T, T)</p>	Answer 9:	<input type="checkbox"/>
Q10:	When a six-sided die and a coin are thrown together, what is the probability that a "3" and a "head" result?	<p>A. $\frac{1}{2}$ B. $\frac{1}{12}$ C. $\frac{1}{6}$ D. $\frac{1}{8}$</p>	Answer 10:	<input type="checkbox"/>

Enter your initial and surname here:

ANSWERS TEST #47

1. C 2. A 3. C 4. D 5. D 6. B 7. B 8. A 9. A 10. B