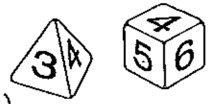
Probability

If you are dealing with an experiment that has more than one stage or event, a table or tree diagram is a useful way to record the outcomes.

When the events are independent

Example
A 1 to 4 dice and a 1 to 6 dice are thrown together.
(The result on one dice does not restrict the result on the other.)
Calculate the probability of
(a) getting a total of 2
(b) getting a total of 5 or more.



The probability of getting a total of 2 is $\frac{1}{24}$ (1 possibility out of 24 equally likely ones).

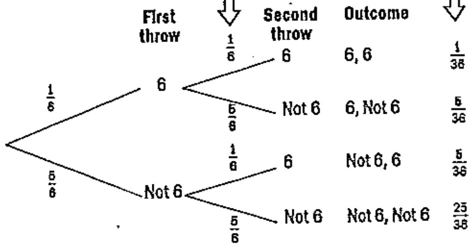
The probability of getting a total of 5 or more is $\frac{18}{24}$ (18 possibilities out of 24 equally likely ones).

	Number on second dice					
Number on first dice	1	2	3	4	5	6
1	②	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

Example
Calculate the probability of getting just one 6 when an ordinary fair dice is thrown twice.

The dice has no memory so the probabilities for the second throw are the same as for the first.

To get these probabilities multiply together the probabilities along the branches.



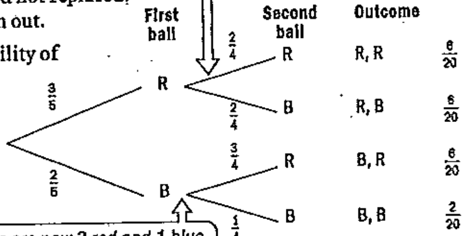
The probability of just one 6 is $\frac{5}{36} + \frac{5}{36} = \frac{10}{36}$.

When the result for one event affects the probabilities for another event

Example
A bag contains 3 red balls and 2 blue ones.
A ball is taken out and not replaced.
A second ball is taken out.

Calculate the probability of getting just one red.

There are now 2 red and 2 blue so the probabilities change.



The probability of just one red is $\frac{6}{20} + \frac{6}{20} = \frac{12}{20}$.

There are now 3 red and 1 blue so the probabilities change.

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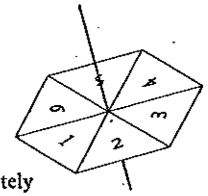
1 A bag contains 10 white balls, 8 blue balls and 7 yellow balls.
Pat picks one ball at random.
(a) What is the probability that she picks a blue ball?
(b) She wins a prize if she picks a blue ball or a yellow ball.
What is the probability that she wins a prize?

MEG (SMP)

2 Dinah has a biased spinner. In a trial she had these results.

Number shown	1	2	3	4	5	6
Frequency	5	16	19	20	14	26



(a) (i) Use Dinah's results to estimate the probability of obtaining a six with this spinner, if she spins it once.
(ii) What should Dinah do so that she can estimate more accurately the probability of obtaining a six with this spinner?
(b) Estimate the probability of obtaining a four followed by a one with two spins.

MEG/ULEAC (SMP)

3 Ambrose has to drive through two sets of traffic lights on his way to work.
These lights work independently.
At the first set the probability he has to stop is 0·6.
At the second set this probability is 0·7.
(a) Draw a tree diagram to show this information.
(b) Calculate the probability that Ambrose is stopped by at least one of these sets of lights.

MEG (SMP)

4 Clair estimates that the probability of a drawing pin landing 'pin up' is 0·85.
(a) What is the probability of a drawing pin landing 'pin down'?
(b) Claire now drops three drawing pins.
Calculate the probability that
(i) all three land 'pin up', (ii) at least one lands 'pin down'.

MEG (SMP)

5 There are three terms in a school year.
The teaching staff have one training day in each term.
They choose each training day at random.
It is equally likely for each training day to be on any of the five days of the week from Monday to Friday.
(a) Calculate the probability that the three training days will all be on a Friday.
(b) Calculate the probability that none of the three training days will be on a Friday.
(c) Calculate the probability that the three training days will all be on the same day of the week.
(d) Calculate the probability that the three training days will be on different days of the week.

ULEAC

6 From a standard pack of playing cards a mini-pack is made up containing the four kings, the four queens and the four jacks.

(a) John takes two cards from this mini-pack.
Calculate the probability that they will be of the same colour.

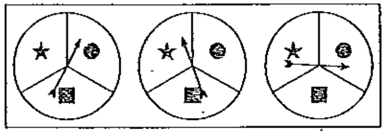
(b) John's cards are replaced and the mini-pack is shuffled.
Jill now takes two cards from the mini-pack.
Calculate the probability that one of these will be a queen and the other will not.

(c) Jill's cards are replaced and the mini-pack is again shuffled.
Jo now takes two cards from the mini-pack.
Show that the probability that they will be of the same colour, and one will be a queen but the other will not, is $\frac{8}{33}$. MEG/ULEAC (SMP)

7 The school canteen at St Frideswide's High School serves chips on one day a week and peas on two days a week.
These days are chosen at random in a five day week.

By drawing a tree diagram, or otherwise, calculate the probability that on Wednesday next week either chips or peas (but not both) will be served. MEG (SMP)

8 The spinners in this game have been tested and found to be fair.
All three spinners are spun at the same time.



(a) What is the probability that they all point to a star when they stop?

You pay 20p to play the game.
You win £1 if all three arrows point to the same shape when the spinners stop.

(b) When you play the game, what is the probability that you win £1?

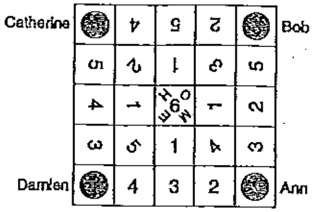
(c) The game was played 400 times at a fete.
How much money do you expect it made for the organisers, after the winnings had all been paid back?

9 There are 6 marbles in a bag, three red, two blue and one green.
Jane takes out a marble at random three times.

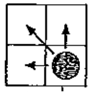
(a) If she replaces the marble each time, what is the probability that
  (i) all three are the same colour,
  (ii) all three are different colours?

(b) If she takes them out without replacement, what is the probability that
  (i) all three are the same colour,
  (ii) all three are different colours?

10 In the game 'Hurry home' players start with counters on the four blank corner squares.
An ordinary dice numbered 1 to 6 is thrown by each player in turn.
On each throw all players must move their counters to an adjoining square if the number in that square comes up on the dice.



These are examples of adjoining squares.

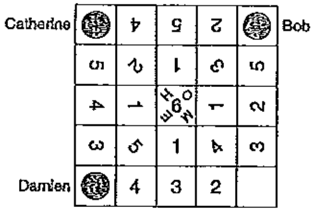


The centre square has the number 6 and the player who reaches that square first wins the game.
If two or more players reach the centre at the same time the game is declared a draw.

(a) On the first throw of the dice what is the probability that
  (i) Ann must move,
  (ii) Ann, Bob and Catherine must all move,
  (iii) nobody has to move?

(b) What is the probability that Ann wins the game on the second throw of the dice?

(c) Bob, Catherine and Damien start a new game with their counters on the corner squares.
The first throw of the dice is a 5 and the second throw is a 4.
What is the probability that Bob wins or draws on the fourth throw of the dice?



NISEAC

11 The game 'Dicey' is played with two coloured dice.
The red dice is a standard cubical dice with faces numbered 1, 2, 3, 4, 5, 6.
The blue dice is a tetrahedron with faces numbered 1, 2, 3, 4.
A turn consists of these dice being thrown and the scores added according to the following rules:

1 The red dice is always thrown first.

2 (i) If the score on the red dice is even, then the red dice is thrown again.
  (ii) If the score on the red dice is odd, then the blue dice is used for the second throw.

Using a tree diagram, or otherwise, calculate

(a) the probability that the total score in one turn is 12,

(b) the probability that the total score in one turn is 3. MEG (SMP)

Answers and hints ► page 146

ANSWERS

Probability (page 86)

*Probabilities can be given as fractions, decimals or percentages but **not** as ratios. For example in question 1, 8:25 would be penalised.*

1 (a) $\frac{8}{25}$ or 0·32 or 32%

(b) $\frac{8+7}{25} = \frac{15}{25} = \frac{3}{5}$ or 0·6 or 60%

2 (a) (i) $\frac{26}{100}$ or $\frac{13}{50}$ or 0·26 or 26%

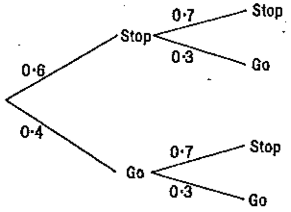
(ii) She would need to do some more 'spins'.

(b) The probability of a four is $\frac{20}{100} = \frac{1}{5}$.

The probability of a one is $\frac{5}{100} = \frac{1}{20}$.

So the probability of a four followed by a one is $\frac{1}{5} \times \frac{1}{20} = \frac{1}{100}$ or 0·01.

3 (a)



(b) *There are two ways of answering this question.*
***Either** add the three outcomes in which he stops.*

$(0·6\times0·7)+(0·6\times0·3)+(0·4\times0·7)$
$= 0·42+0·18+0·28$
$= 0·88$

***Or** use the fact that the probability that he stops at least once is*
1 − (the probability that he does not stop at all).

$1-(0·4\times0·3) = 1-0·12 = 0·88$

Remember to show all your working so that you can gain marks for method even if you make a mistake in the calculation.

4 (a) 0·15

(b) (i) $0·85\times0·85\times0·85 = 0·614\,125$
$= 0·61$ (to 2 s.f.)

A probability cannot be bigger than 1; if you got the answer 2·55 you made the common mistake of adding instead of multiplying.

(ii) $1-0·614125 = 0·39$ (to 2 s.f.)

5 (a) $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5} = \frac{1}{125}$

(b) $\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5} = \frac{64}{125}$

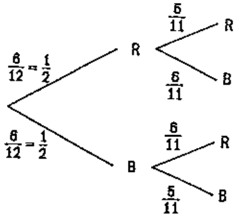
(c) $\frac{1}{125}\times5 = \frac{1}{25}$

The answer to (a) mulitplied by 5.

(d) $\frac{4}{5}\times\frac{3}{5} = \frac{12}{25}$

Whatever the first day is, there are 4 choices for the second day and 3 choices for the third.

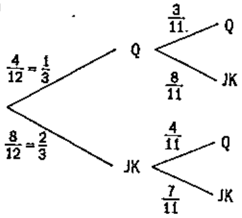
6 (a)



The probability of two reds or two blacks is $\frac{1}{2}\times\frac{5}{11}+\frac{1}{2}\times\frac{5}{11} = \frac{5}{22}+\frac{5}{22} = \frac{5}{11}$.

You might have got this answer by thinking 'whatever colour card he takes first there will be five of that colour out of the 11 that are left'.

(b)



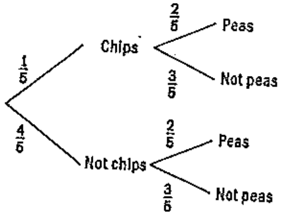
The probability of one queen only is $\frac{1}{3}\times\frac{8}{11}+\frac{2}{3}\times\frac{4}{11} = \frac{8}{33}+\frac{8}{33} = \frac{16}{33}$.

(c) The probability of a red queen and red king is $\frac{2}{12}\times\frac{2}{11} = \frac{4}{132} = \frac{1}{33}$.

There are eight such possible combinations, all equally likely.

So the probability of the required outcome is $\frac{1}{33}\times8 = \frac{8}{33}$.

7



$\frac{1}{5}\times\frac{3}{5}+\frac{4}{5}\times\frac{2}{5} = \frac{3}{25}+\frac{8}{25}$
$= \frac{11}{25}$ or 0·44

8 (a) $\frac{1}{27}$

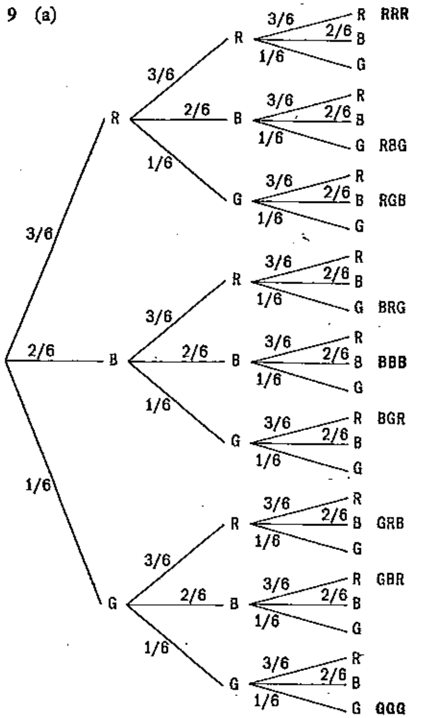
(b) The probability of three arrows pointing to the same shape is $\frac{1}{27}+\frac{1}{27}+\frac{1}{27} = \frac{1}{9}$.

So the probability of winning £1 is $\frac{1}{9}$ or $0·\dot{1}$.

(c) The money taken is £80.

The money kept is £80 − 400 × $\frac{1}{9}$ × £1 = £35·56.

9 (a)



The outcomes listed on the right are the ones you need for the question.

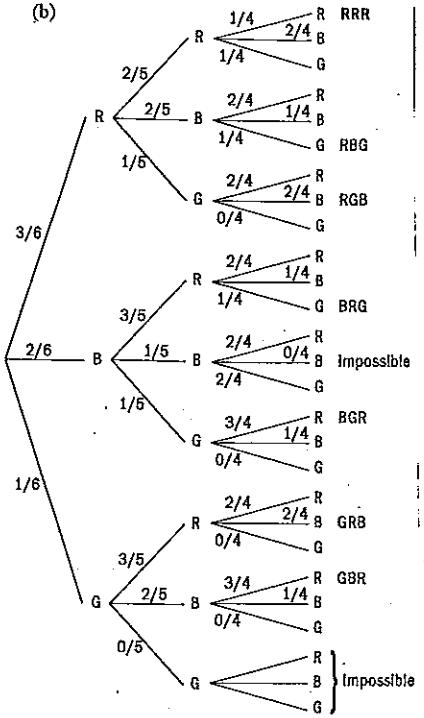
(i) The probability of all three marbles being the same colour is

$(\frac{3}{6}\times\frac{3}{6}\times\frac{3}{6})+(\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6})+(\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6})$
$= \frac{27}{216}+\frac{8}{216}+\frac{1}{216} = \frac{36}{216} = \frac{1}{6}$.

(ii) The probability of all three marbles being different colours is

$6(\frac{1}{2}\times\frac{1}{3}\times\frac{1}{6}) = 6\times\frac{1}{36} = \frac{1}{6}$.

(b)



All the probabilities are marked above, but it is only necessary to work out the fractions along the branches you are interested in.

(i) The probability of all three marbles being the same colour is

$\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4} = \frac{1}{20}$.

The only possible outcome is three reds.

(ii) The probability of all three marbles being different colours is

$(\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4})+(\frac{3}{6}\times\frac{1}{5}\times\frac{2}{4})+(\frac{2}{6}\times\frac{3}{5}\times\frac{1}{4})$
$+(\frac{2}{6}\times\frac{1}{5}\times\frac{3}{4})+(\frac{1}{6}\times\frac{3}{5}\times\frac{2}{4})+(\frac{1}{6}\times\frac{2}{5}\times\frac{3}{4})$
$= 6\times\frac{1}{20}$
$= \frac{3}{10}$.

10 (a) (i) $\frac{1}{2}$

(ii) $\frac{1}{6}$

If all three players are to move, the throw must be a '2'.

(iii) $\frac{1}{3}$

If nobody moves, the throw must be a '1' or '6'.

(b) $\frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

Ann needs a '4' and then a '6' to win in two throws.

(c) $\frac{2}{6}\times\frac{1}{6} = \frac{2}{36} = \frac{1}{18}$

Bob needs a '1' or '3' on his third throw and a '6' on his fourth throw.

11 (a) $\frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$

The only way to score 12 in two throws is to throw two sixes, both with red dice.

(b) *'3' can be scored by throwing a '1' on the red dice followed by '2' on the blue, or '2' on the red dice followed by '1' also on the red. The probability is found by adding the individual probabilities together.*

$(\frac{1}{6}\times\frac{1}{4})+(\frac{1}{6}\times\frac{1}{6}) = \frac{1}{24}+\frac{1}{36} = \frac{5}{72}$

More help or practice
Independent events ▸ Book Y4 pages 76 to 77
Tree diagrams ▸ Book Y4 pages 78 to 79
Adding probabilities ▸ Book Y4 pages 80 to 81
Dependent events ▸ Book Y4 pages 83 to 85
Conditional probability ▸ Book YX2 pages 8 to 12