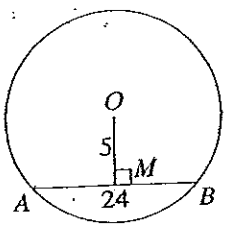
4B 2006 | **CIRCLE GEOMETRY** | 15th August

Use your own paper. Attempt all questions. Show all necessary working.

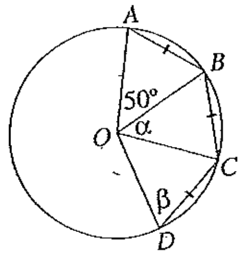
1. Find the value of the pronumerals in the diagrams below. Show all reasoning.

a)

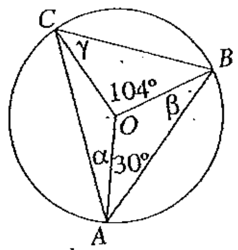


Find AO.

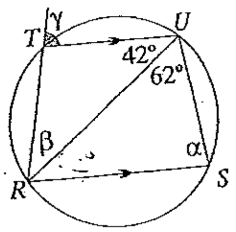
b)



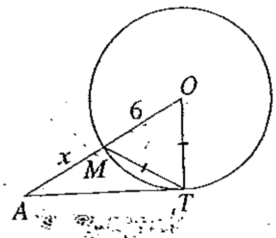
c)



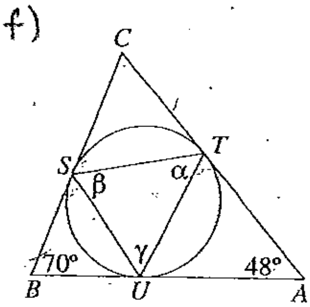
d)



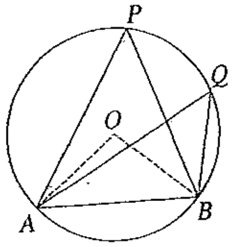
e)



f)

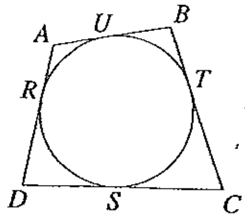


2.



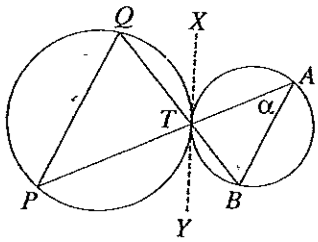
Copy and complete the following proof:
Given: Let AB be any arc of a circle with centre O. Let P and Q be any two points on the circle on the opposite arc.
Aim: To prove: $\angle P = \angle Q$
Construction:
Proof:

3.



Prove that $AB + DC = AD + BC$.

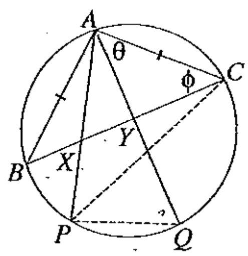
4.



The two circles touch externally a T, and XTY is the common tangent at T.
Prove that $AB \parallel QP$.

5. AB and CD are two parallel chords in a circle. AD and BC intersect at E. If $\angle ABC = 35°$, show that $AE = BE$.

6.



In the diagram above, $AB = AC$.

(a) Prove that $\angle CPQ = \theta$.

(b) Prove that $\angle CPA = \phi$.

(c) Hence prove that PQYX is cyclic.

CIRCLE GEOMETRY

a) AM = 12 (The interval OM through the centre bisects the chord AB) ✓

$AO^2 = AM^2 + OM^2$ (2)

$= 25 + 144$

$= 169$ ✓

AO = 13 (Pythagoras' thm)

) Since AB = BC = CD

$\alpha = 50°$ ✓

∠COD = 50° ✓ (3)

(Equal chords subtend the same angle at the centre)

Δ COD isosceles

$\beta = 65°$ (base angles of isosceles) ✓

) ∠BAC = 52° (Angle at the circumference is half the angle at the centre) ✓

$\alpha = 22°$ (adjacent angles) ✓ (4)

$\beta = 30°$ (base angles of isosceles) ✓

$\gamma = 38°$ (base angles of isosceles). ✓

d) ∠URS = 42° (alternate angles UT ∥ RS) ✓

$\alpha = 180° - 62 - 42$

$= 76°$ (angle sum Δ) ✓ (4)

$\gamma = 76$ (exterior angle of cyclic quad) ✓

$\beta = 34°$ (exterior angle of Δ) ✓

e) OT = MT = OM = 6 (radii)

∠MOT = ∠OTM

= 60° (equilateral Δ) ✓

AT tangent

∴ ∠OTA = 90° (tangent ⊥ radius) ✓

(5) ∠MTA = 30° (adjacent angles) ✓

∠MAT = 30° (angle sum Δ) ✓

Δ AMT isosceles

x = MT

= 6 (equal sides of isosceles) ✓

f) Δ ABC ✓

∠ACB = 62° (angle sum of Δ)

CS = CT (Two tangents from an external point have equal lengths) ✓

∠CTS = ∠CST

= 59° (base angles isosceles) ✓

Δ TCS and Δ AUT and Δ BUS isosceles

(5) AT = AU, BU = BS } tangents from external point

∠ATU = ∠AUT

= (180 − 48) ÷ 2 ✓

= 132 ÷ 2

= 66° (angle sum of Δ)

∠USB = ∠BUS ✓

= 110° ÷ 2

= 55°

23

3

$\alpha + 66 + 59 = 180$

$\alpha = 55^\circ$ (straight angle) ✓

$\beta + 59 + 55 = 180^\circ$

$\beta = 66^\circ$ (straight angle) ✓

$\gamma + 55 + 66 = 180^\circ$ ✓

$\gamma = 59^\circ$ (angle sum of Δ)

* Alternatively angle in alternate segment.

2. Given: Let AB be any arc of a circle with ~~the~~ centre O.
Let P + Q be any two points on the circle on the opposite arc.

Aim: To prove $\angle P = \angle Q$

Construction: Join AO + OB.

Proof

Let $\angle APB = \alpha$ 3

Then $\angle AOB = 2\alpha$

(angle at the ~~circle~~ centre of a circle is twice the angle at the circumference standing on the same arc) ✓

$\angle AQB = \alpha$ (angle at the circumference is half the angle at the centre standing on the same arc) ✓

Hence $\angle APB = \angle AQB$
$= \alpha$

Hence $\angle P = \angle Q$. ✓

3. Let $AU = x$
$BU = y$
$CT = z$ ✓
$DS = w$

AB, BC, DC and AD are tangents to the circle.

$AU = AR$
$= x$ (Two tangents from an external point are of equal lengths) ✓

$BU = BT$
$= y$ (as above)

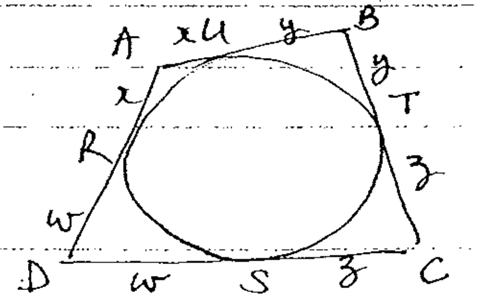
$CT = CS$
$= z$ (as above) ✓

$DS = DR$
$= w$ (as above)

$AB = x + y$
$DC = z + w$ ✓
$AD = x + w$
$BC = y + z$

$AB + DC = x + y + z + w$ ✓
$AD + BC = x + y + z + w$.

Here
$AB + DC = AD + BC$.



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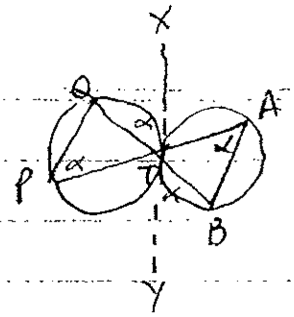
4:



Let $\angle BTY = \alpha$ (angle in the alternate

$\angle XTQ = \angle BTY$ segment) ✓

$= \alpha$ (vertically ✓ opposite angles)

(4)

$\angle QPT = \alpha$ (angle in the alternate segment) ✓

$\angle QPT = \angle PAB$

$= \alpha$ (equal alternate angles) ✓

Hence $QP \parallel AB$

5.

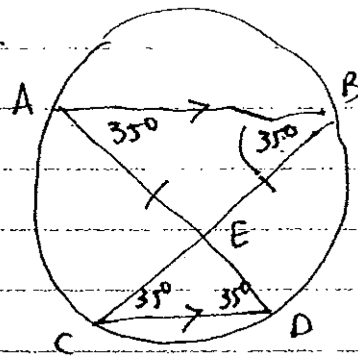


diagram ✓

$\angle BCD = 35°$ (Alternate angles $AB \parallel CD$) ✓

$\angle BAD = 35°$ (angles on the same arc) ✓

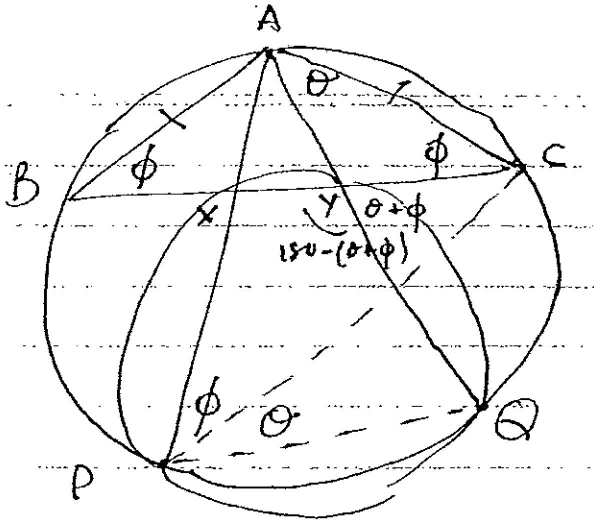
(5)

$\angle EDC = 35°$ (") ✓

ΔABE Isosceles (base angles equal) ✓

$\therefore$ $AE = BE$ (equal sides isosceles)

6.



$AB = AC$ (given)

Construction : Join CP and PQ

$\angle CPQ = \theta$ (angles on the same arc) ✓

ΔABC Isosceles

$\angle ABC = \phi$ (base angles) ✓

$\angle CPA = \phi$ (angles on the same arc) ✓

$\angle CYQ = \theta + \phi$ (exterior angle of ΔAYC) ✓

$\angle XYQ = 180 - (\theta + \phi)$ (straight angle) ✓

$\angle XYQ + \angle QPX$

$= 180 - (\theta + \phi) + \theta + \phi$

$= 180$ ✓

Since opposite angle of the quadrilateral PQYX are supplementary then PQYX must be cyclic.

(6)

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