Functions and logarithms (OPTION 7)

UNIT 1: Functions

QUESTION 1 State whether the following sets of ordered pairs represent a function or not.

a (1, 3), (2, 5), (3, 7), (4, 9), (5, 11) _____ **b** (0, 0), (1, 1), (–1, 1), (2, 4), (–2, 4) _____

c (0, 1), (1, 2), (2, 4), (3, 8) _____ **d** (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) _____

QUESTION 2 If $f(x) = 2x + 3$, find:

a $f(0) =$ _____ **b** $f(4) =$ _____ **c** $f(-1) =$ _____

d $f(1) =$ _____ **e** $f(2) =$ _____ **f** $f(-2) =$ _____

QUESTION 3 If $f(x) = 2^x + 5$, find:

a $f(0) =$ _____ **b** $f(2) =$ _____ **c** $f(4) =$ _____

d $f(-1) =$ _____ **e** $f(-2) =$ _____ **f** $f(3) =$ _____

QUESTION 4 If $g(x) = 5x - 3$, find x if:

a $g(x) = 17$ _____ **b** $g(x) = 32$ _____

QUESTION 5 If $f(x) = 5 - 3x$, find:

a $f(3a) =$ _____ **b** $f\left(\frac{1}{2a}\right) =$ _____

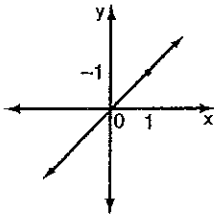
QUESTION 6

a If $f(x) = x^2$, show that $f(-a) = f(a)$. _____

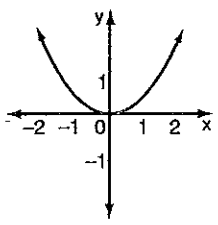
b If $f(x) = x^3$, show that $f(-a) = -f(a)$. _____

QUESTION 7 Which of the graphs given below represent functions?

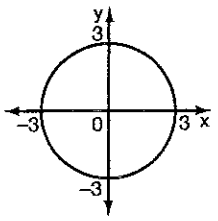
a



b



c



QUESTION 8

a Does $y < 3x + 2$ define a function? _____

b Does a straight line graph always define a function? _____

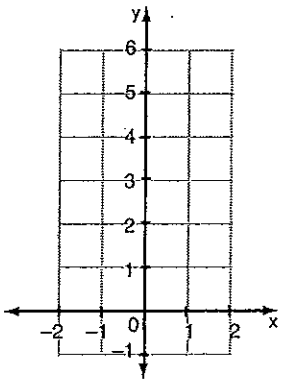
Functions and logarithms (OPTION 7)

UNIT 2: Exponential graphs

QUESTION 1 For the following exponential functions, complete the table and draw the graph.

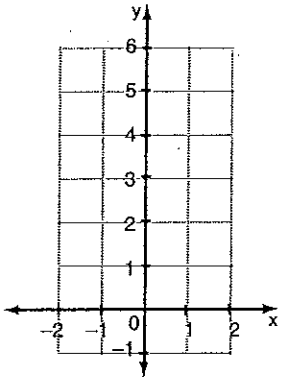
a $y = 2^x$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
$y = 2^x$					



b $y = 2^{-x}$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
$y = 2^{-x}$					



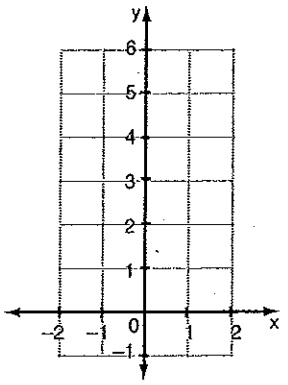
QUESTION 2 For the following exponential functions, complete the tables and, on the same axes, draw the graphs.

a $y = 3^x$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
$y = 3^x$					

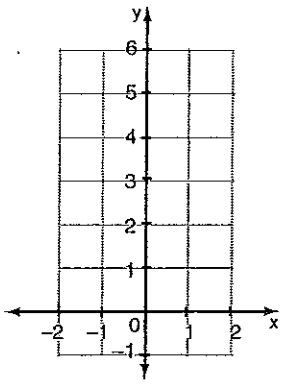
b $y = 3^{-x}$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
$y = 3^{-x}$					



QUESTION 3 Complete the table then draw the graph of $y = \dfrac{2^x + 2^{-x}}{2}$ for $-2 \le x \le 2$.

x	−2	−1	0	1	2
$y = 2^x$					
$y = 2^{-x}$					



Functions and logarithms (OPTION 7)

UNIT 3: Logarithms

QUESTION 1 Write each of the following in logarithmic form.

a $3^2 = 9$ _____ b $64 = 4^3$ _____ c $125 = 5^3$ _____

d $2^5 = 32$ _____ e $3^4 = 81$ _____ f $2^{-3} = \frac{1}{8}$ _____

g $64 = 2^6$ _____ h $343 = 7^3$ _____ i $\frac{1}{9} = 3^{-2}$ _____

j $3^{-1} = \frac{1}{3}$ _____ k $25^{-\frac{1}{2}} = \frac{1}{5}$ _____ l $4 = 8^{\frac{2}{3}}$ _____

QUESTION 2 Write each of the following in index form.

a $\log_2 2 = 1$ _____ b $\log_3 27 = 3$ _____ c $\log_5 \sqrt{5} = \frac{1}{2}$ _____

d $\log_3 9 = 2$ _____ e $\log_8 2 = \frac{1}{3}$ _____ f $\log_{27} 9 = \frac{2}{3}$ _____

g $\log_2 32 = 5$ _____ h $\log_2 16 = 4$ _____ i $\log_3 1 = 0$ _____

j $\log_4 64 = 3$ _____ k $\log_2 \sqrt{2} = \frac{1}{2}$ _____ l $\log_2 128 = 7$ _____

QUESTION 3 Evaluate the following.

a $\log_2 4 =$ _____ b $\log_2 8 =$ _____ c $\log_2 16 =$ _____

d $\log_5 25 =$ _____ e $\log_2 64 =$ _____ f $\log_7 49 =$ _____

g $\log_3 9 =$ _____ h $\log_3 27 =$ _____ i $\log_3 81 =$ _____

j $\log_5 125 =$ _____ k $\log_6 216 =$ _____ l $\log_7 343 =$ _____

QUESTION 4 Solve for x.

a $\log_3 27 = x$ _____ b $\log_2 16 = x$ _____ c $\log_5 125 = x$ _____

d $\log_8 2 = x$ _____ e $\log_7 7 = x$ _____ f $\log_9 3 = x$ _____

g $\log_2 x = 3$ _____ h $\log_3 x = 4$ _____ i $\log_5 x = 2$ _____

j $\log_{\sqrt{2}} x = 6$ _____ k $\log_{\sqrt{3}} x = 4$ _____ l $\log_5 x = 4$ _____

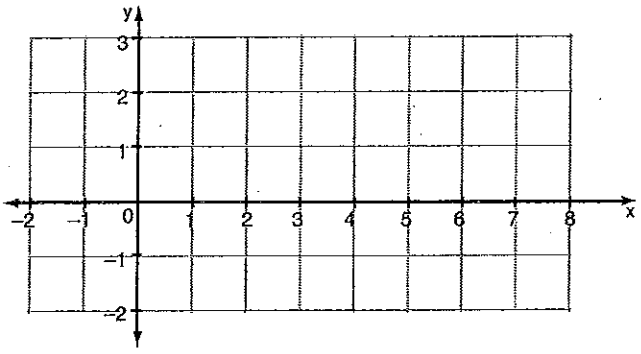
m $\log_x 27 = 3$ _____ n $\log_x 64 = 2$ _____ o $\log_x 81 = 4$ _____

Functions and logarithms (OPTION 7)

UNIT 4: Logarithmic graphs

QUESTION 1 Express $y = \log_2 x$ as an index equation, complete the table and draw its graph.

x						
y	-2	-1	0	1	2	3



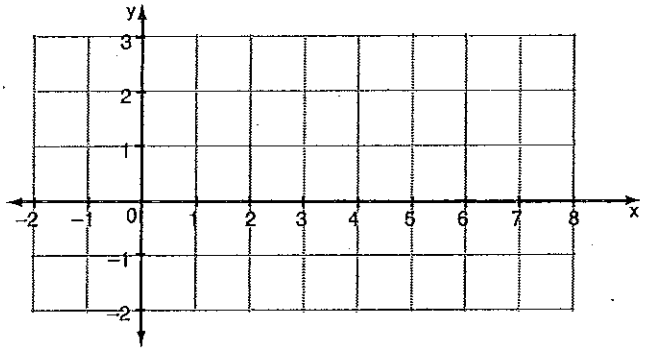
QUESTION 2 Complete the following tables and draw their graphs on the same number plane.

$y = 2^x$

x	-2	-1	0	1	2
y					

$y = \log_2 x$

x					
y	-2	-1	0	1	2



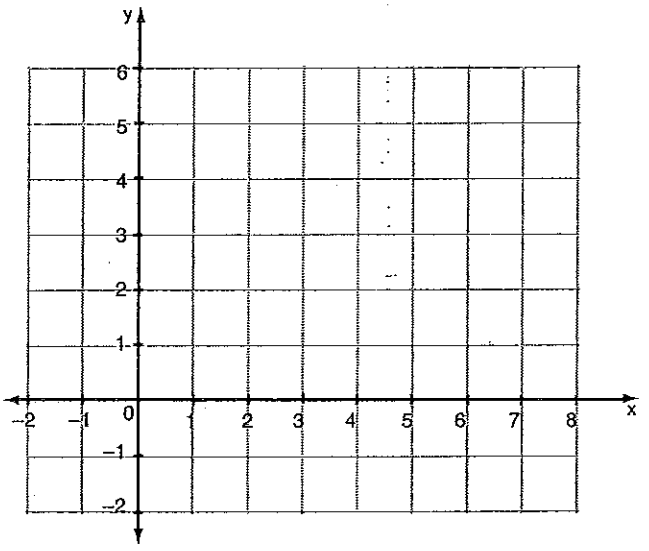
QUESTION 3 On the same number plane, draw the graphs of the following.

$y = \log_2 x$

x					
y					

$y = \log_3 x$

x					
y					



Functions and logarithms

(OPTION 7) UNIT 5: Laws of logarithms

QUESTION 1 Simplify the following.

a $\log_2 32 + \log_2 4 =$ _____

b $\log_{10} 20 + \log_{10} 5 =$ _____

c $\log_3 81 - \log_3 9 =$ _____

d $\log_5 125 - \log_5 25 =$ _____

e $\log_a a^3 - \log_a a^2 =$ _____

f $\dfrac{\log x^3}{\log x} =$ _____

QUESTION 2 Use the logarithm laws to expand the following.

a $\log_a(xy) =$ _____

b $\log_a\left(\dfrac{xy^2}{z}\right) =$ _____

c $\log_a\left(\dfrac{2x}{x-1}\right) =$ _____

d $\log_a\left(x^3\sqrt{y}\right) =$ _____

QUESTION 3 Use the logarithm laws to simplify the following.

a $\log_a x + \log_a y - \log_a z^2 =$ _____

b $3\log_a x - 2\log_a y =$ _____

c $\dfrac{1}{2}\log_a x + 2\log_a y =$ _____

d $2\log_a x - 3\log_a y + \dfrac{1}{2}\log_a z =$ _____

QUESTION 4 If $\log_a 2 = 0.7285$ and $\log_a 3 = 1.0825$, evaluate:

a $\log_a 16 =$ _____

b $\log_a 27 =$ _____

c $\log_a 6 =$ _____

d $\log_a 36 =$ _____

QUESTION 5 Show that $5\log_a x + 3\log_a y - 2\log_a z = \log_a\left(\dfrac{x^5y^3}{z^2}\right)$.

Functions and logarithms (OPTION 7)

UNIT 6: Exponential equations

QUESTION 1 Solve the following exponential equations.

a $2^x = 32$ _____ b $2^x = 64$ _____ c $2^x = 512$ _____

d $3^x = 9$ _____ e $3^x = 81$ _____ f $3^x = 243$ _____

g $4^x = 256$ _____ h $7^x = 2401$ _____ i $9^x = 729$ _____

j $10^x = 10000$ _____ k $5^x = 625$ _____ l $6^x = 216$ _____

QUESTION 2 Solve the following equations.

a $3^x = \frac{1}{9}$ _____ b $4^x = \frac{1}{64}$ _____ c $2^x = \frac{1}{16}$ _____

d $25^x = 5$ _____ e $49^x = 7$ _____ f $128^x = 2$ _____

g $27^x = \frac{1}{3}$ _____ h $8^x = \frac{1}{64}$ _____ i $81^x = \frac{1}{27}$ _____

j $8^x = 64$ _____ k $9^x = 81$ _____ l $25^x = \frac{1}{125}$ _____

QUESTION 3 Solve the following equations.

a $2^{x+1} = 16$ _____ b $2^{2x-1} = 128$ _____ c $2^{3-x} = 512$ _____

d $3^{x-1} = 243$ _____ e $3^{2x+1} = \frac{1}{243}$ _____ f $3^{3x-1} = 9$ _____

g $\left(\frac{1}{4}\right)^x = 64$ _____ h $\left(\frac{1}{2}\right)^{x-1} = 32$ _____ i $\left(\frac{1}{3}\right)^{2x+1} = 27^2$ _____

j $8^{2x-1} = 256$ _____ k $9^{1-x} = 729$ _____ l $4^{x-2} = 64$ _____

m $3^{3x-2} = 2187$ _____ n $4^{2x} = 8$ _____ o $9^{3-2x} = 27^{x-1}$ _____

Functions and logarithms

EXCEL YEARS 9 & 10 ADVANCED MATHS
Ch. 13, 13.5.1, p. 212

(OPTION 7) UNIT 7: Miscellaneous questions

QUESTION 1 If $f(x) = 3x - 4$, find:

a $f(1)$ _____ b $f(5)$ _____ c $f(2)$ _____

d $f(4)$ _____ e $f(-2)$ _____ f $f(3)$ _____

g $f(a)$ _____ h $f(-a)$ _____ i $f\left(\frac{1}{a}\right)$ _____

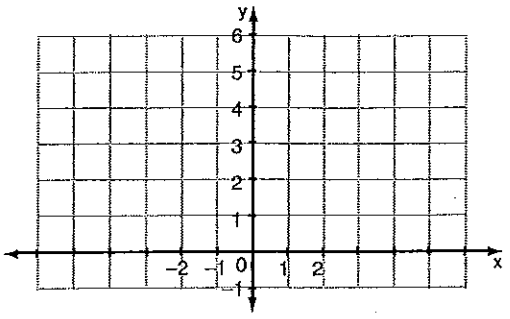
QUESTION 2 For the following exponential functions, complete the tables and, on the same axes, draw the graphs.

a $y = 2^x$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
y					

b $y = 2^{2x}$ for $-2 \le x \le 2$

x	−2	−1	0	1	2
y					



QUESTION 3 Write each of the following in logarithmic form.

a $2^5 = 32$ _____ b $3^3 = 27$ _____ c $4^5 = 1024$ _____

d $5^4 = 625$ _____ e $6^3 = 216$ _____ f $7^3 = 343$ _____

g $9^3 = 729$ _____ h $10^4 = 10000$ _____ i $3^7 = 2187$ _____

QUESTION 4 Write each of the following in index form.

a $\log_3 27 = 3$ _____ b $\log_2 128 = 7$ _____ c $\log_3 81 = 4$ _____

d $\log_5 625 = 4$ _____ e $\log_2 32 = 5$ _____ f $\log_3 243 = 5$ _____

g $\log_3 2187 = 7$ _____ h $\log_6 216 = 3$ _____ i $\log_{\sqrt{x}} 32 = 10$ _____

QUESTION 5 Use logarithm laws to simplify the following.

a $\log_4 64 + \log_4 16$ _____

b $\log_5 125 - \log_5 25$ _____

c $\log_a x^3 - \log_a x^2$ _____

d $\log_7 49 - \log_7 343$ _____

e $\log_6 4 + \log_6 9$ _____

f $\log_8 64 - \log_8 8$ _____

QUESTION 6 Solve the following equations.

a $3^x = 729$ _____

b $5^x = \frac{1}{625}$ _____

c $2^{3x-1} = 32$ _____

Pages 8

UNIT 8: TOPIC TEST

SECTION 1

Functions and logarithms (OPTION 7)

Instructions for SECTION 1

- You have 15 minutes to answer Section 1
- Each question is worth 2 marks
- Attempt ALL questions
- Calculators are NOT to be used
- Fill in only ONE CIRCLE for each question

Marks

1 If $f(x) = 2x - 5$ then $f(-2)$ equals

Ⓐ 9 Ⓑ -9 Ⓒ 1 Ⓓ -1 — 2

2 A graph is a function if it is cut by a vertical line at

Ⓐ 1 point Ⓑ 2 points Ⓒ 3 points Ⓓ 4 points — 2

3 The graph of $y = 2^x$ passes through the point

Ⓐ (0, 1) Ⓑ (1, 0) Ⓒ (0, –1) Ⓓ (–1, 0) — 2

4 If $2^x = 32$ then x equals

Ⓐ 2 Ⓑ 32 Ⓒ 5 Ⓓ $\frac{1}{5}$ — 2

5 $\frac{a^5 \times a^{-3}}{a^{-2}}$ equals

Ⓐ a^5 Ⓑ a^4 Ⓒ a^3 Ⓓ a^2 — 2

6 $\log_2 8$ equals

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 8 — 2

7 $\frac{1}{3}\log_2 64$ equals

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 6 — 2

8 Simplify $\log_a a^2$.

Ⓐ a Ⓑ 2 Ⓒ a^2 Ⓓ $2a$ — 2

9 $y = a^x$ equals

Ⓐ $\log_a x$ Ⓑ $\log_x a$ Ⓒ $\log_a y$ Ⓓ $\log_y a$ — 2

10 $\log_a x + \log_a y$ equals

Ⓐ $\log_a\left(\frac{x}{y}\right)$ Ⓑ $\log_a(xy)$ Ⓒ $\log_a x^y$ Ⓓ $\log_a y^x$ — 2

Total marks achieved for SECTION 1 ___/20

UNIT 8: TOPIC TEST

SECTION 2

Functions and logarithms (OPTION 7)

Instructions for SECTION 2

- You have 20 minutes to answer ALL of Section 2
- Each question is worth 2 marks
- Attempt ALL questions
- Calculators may be used

Questions	Answers	Marks
If $f(x) = \frac{x^2 + x}{2}$ then find:		
1 $f(1)$		2
2 $f(-2)$		2
3 $f(0)$		2
4 If $f(x) = 3x - 7$, find $f\left(\frac{1}{a}\right)$.		2
Solve the following exponential equations.		
5 $3^x = 81$		2
6 $2^{5x-1} = 16$		2
7 $7^{1-x} = 343$		2
8 $4^x = 128$		2
Evaluate the following:		
9 $\log_3 27$		2
10 $\log_3 x = 2$		2
11 $\log_x 125 = 3$		2
12 Simplify $3(\log_2 20 - \log_2 10)$		2
13 $\log mn - \log np + \log p$		2
14 Solve $3^{2x+1} = 27^x$.		2
15 Rewrite the equation $x = a^b$ with b as the subject.		2

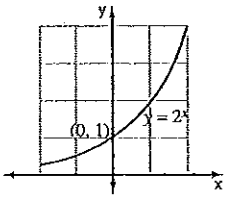
Total marks achieved for SECTION 2 ____/30

Answers

Page 1 1 a Function b Function c Function d Function 2 a 3 b 11 c 1 d 5 e 7 f −1 3 a 6 b 9 c 21 d $5\frac{1}{2}$ e $5\frac{1}{4}$ f 13 4 a 4 b 7 5 a $5-9a$ b $5-\frac{3}{2a}$ 6 a $(-a)^2=a^2$ b $(-a)^3=-a^3$ 7 a, b 8 a no b yes

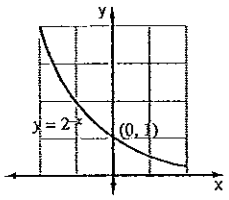
Page 2 1 a

x	−2	−1	0	1	2
$y=2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



b

x	−2	−1	0	1	2
$y=2^{-x}$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

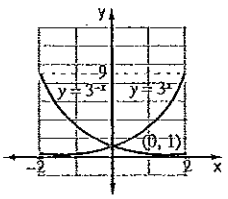


2 a

x	−2	−1	0	1	2
$y=3^x$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

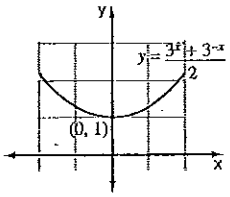
b

x	−2	−1	0	1	2
$y=3^{-x}$	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$



3

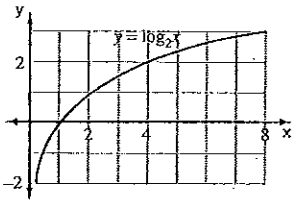
x	−2	−1	0	1	2
$y=2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$y=2^{-x}$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
$y=\frac{2^x+2^{-x}}{2}$	2.13	1.3	1	1.3	2.13



Page 3 1 a $2=\log_3 9$ b $3=\log_4 64$ c $3=\log_5 125$ d $5=\log_2 32$ e $4=\log_3 81$ f $-3=\log_2\left(\frac{1}{8}\right)$ g $6=\log_2 64$ h $3=\log_7 343$ i $-2=\log_3\left(\frac{1}{9}\right)$ j $-1=\log_3\left(\frac{1}{3}\right)$ k $-\frac{1}{2}=\log_{25}\left(\frac{1}{5}\right)$ l $\frac{2}{3}=\log_8 4$ 2 a $2^1=2$ b $3^3=27$ c $5^{\frac{1}{2}}=\sqrt{5}$ d $3^2=9$ e $8^{\frac{1}{3}}=2$ f $27^{\frac{2}{3}}=9$ g $2^5=32$ h $2^4=16$ i $3^0=1$ j $4^3=64$ k $2^{\frac{1}{2}}=\sqrt{2}$ l $2^7=128$ 3 a 2 b 3 c 4 d 2 e 6 f 2 g 2 h 3 i 4 j 3 k 3 l 3 4 a 3 b 4 c 3 d $\frac{1}{3}$ e 1 f $\frac{1}{2}$ g 8 h 81 i 25 j 8 k 9 l 625 m 3 n 8 o 3

Page 4 1 $y=\log_2 x$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	−2	−1	0	1	2	3

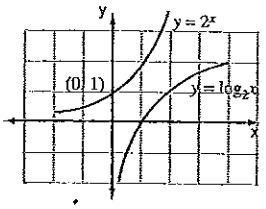


2 $y=2^x$

x	−2	−1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

$y=\log_2 x$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	−2	−1	0	1	2

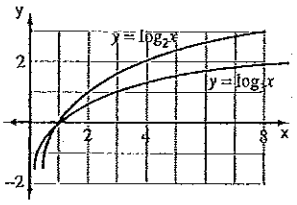


3 $y=\log_2 x$

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
y	−2	−1	0	1	2

$y=\log_3 x$

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	−2	−1	0	1	2



Page 5 1 a 7 b 2 c 2 d 1 e 1 f 3 2 a $\log_a x+\log_a y$ b $\log_a x+2\log_a y-\log_a z$ c $\log_a z+\log_a x-\log_a(x-1)$ d $3\log_a x+\frac{1}{2}\log_a y$ 3 a $\log_a\left(\frac{xy}{z^2}\right)$ b $\log_a\left(\frac{x^3}{y^2}\right)$ c $\log_a\left(\sqrt{x}y^2\right)$ d $\log_a\left(\frac{x^2\sqrt{z}}{y^3}\right)$ 4 a 2.914 b 3.2475 c 1.811 d 3.622

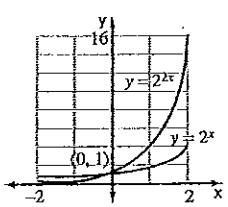
Page 6 1 a $x=5$ b $x=6$ c $x=9$ d $x=2$ e $x=4$ f $x=5$ g $x=4$ h $x=4$ i $x=3$ j $x=4$ k $x=4$ l $x=3$ 2 a $x=-2$ b $x=-3$ c $x=-4$ d $x=\frac{1}{2}$ e $x=\frac{1}{2}$ f $x=\frac{1}{7}$ g $x=-\frac{1}{3}$ h $x=-2$ i $x=-\frac{3}{4}$ j $x=2$ k $x=2$ l $x=-1\frac{1}{2}$ 3 a $x=3$ b $x=4$ c $x=-6$ d $x=6$ e $x=-3$ f $x=1$ g $x=-3$ h $x=-4$ i $x=-3\frac{1}{2}$ j $x=1\frac{5}{6}$ k $x=-2$ l $x=5$ m $x=3$ n $x=\frac{3}{4}$ o $x=1\frac{2}{7}$

Page 7 1 a −1 b 11 c 2 d 8 e −10 f 5 g $3a-4$ h $-3a-4$ i $\frac{3}{a}-4$ 2 $y=2^x$

x	−2	−1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

$y=2^{2x}$

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



3 a $5=\log_2 32$ b $3=\log_3 27$ c $5=\log_4 1024$ d $4=\log_5 625$ e $3=\log_6 216$ f $3=\log_7 343$ g $3=\log_9 729$ h $4=\log_{10} 10000$ i $7=\log_3 2187$ 4 a $3^3=27$ b $2^7=128$ c $3^4=81$ d $5^4=625$ e $2^5=32$ f $3^5=243$ g $3^7=2187$ h $6^3=216$ i $\left(\sqrt{x}\right)^{10}=32$ 5 a 5 b 1 c $\log_a x$ d −1 e 2 f 1 6 a $x=6$ b $x=-4$ c $x=2$

Page 8 1 B 2 A 3 A 4 C 5 B 6 B 7 A 8 B 9 C 10 B

Page 9 1 1 2 1 3 0 4 $\frac{3}{a}-7$ 5 $x=4$ 6 $x=1$ 7 $x=-2$ 8 $x=3\frac{1}{2}$ 9 3 10 $x=9$ 11 $x=5$ 12 3 13 $\log m$ 14 $x=1$ 15 $b=\log_a x$