Outcome 3 – Functions and Logarithms (25 Marks)

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1. Given $f(x)=x^2-x-3$: **2**

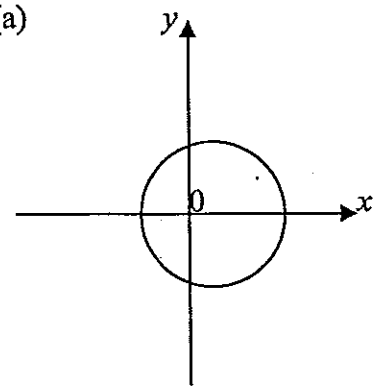
   (a) $f(2)$ (b) $f(y-3)$

2. Sketch $f(x)=x^2+1$ for $x\geq 0$. On the same number plane diagram, sketch $y=f^{-1}(x)$, the inverse function of $y=f(x)$. **2**

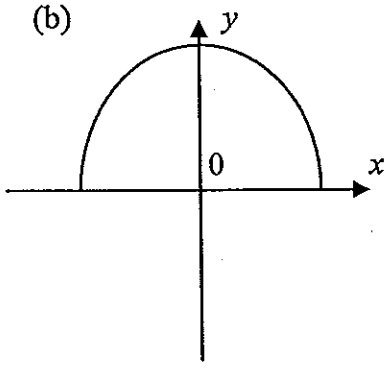
3. Find the inverse of $y=\dfrac{3x-2}{3}$. **2**

4. State whether or not the following diagrams represent the sketch of a function. **3**

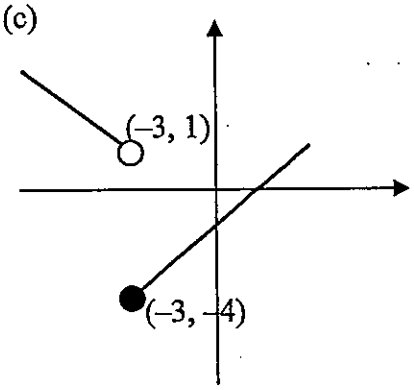
(a)



(b)



(c)



5. Write the statement $3^4=81$ in logarithm form: **1**

6. Write the statement $\log_2 \dfrac{1}{32}=-5$ in index form: **1**

7. Solve each of the following equations: **6**

   (a) $\log_x 9=2$ (b) $\log_{\sqrt{2}} 8=x+2$ (c) $3^{2x}=26$

8. Simplify, fully, each expression. **6**

   (a) $\log_6 9+\log_6 4$ (b) $\log_4 144-\log_4 9$ (c) $\log_{100} 20-\dfrac{1}{2}\log_{100} 4$

9. At the beginning of 2007, David deposited \$150 000 in an account which will pay an interest rate of 6% per annum, compounding monthly. During which year will David's investment be worth twice the original deposit? **2**

Outcome (3)

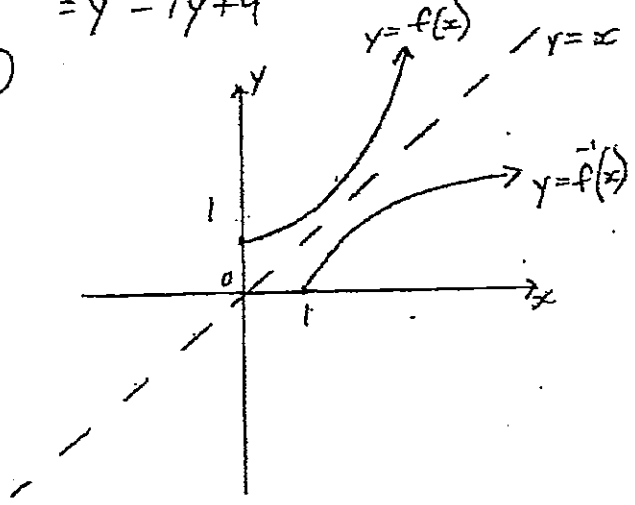
(1) (a) -1

(b) $(y-3)^2-(y-3)-3$

$= y^2-6y+9-y+3-3$

$= y^2-7y+9$

(2)



(3) $x = \frac{3y-2}{3}$

$3x = 3y-2$

$3y = 3x+2$

$y = \frac{1}{3}(3x+2)$

or $f^{-1}(x) = \frac{3x+2}{3}$

(4) (a) No (b) Yes (c) Yes

(5) $\log_3 81 = 4$

(6) $2^{-5} = \frac{1}{32}$

(7) (a) $x^2 = 9$

$x = \pm 3$

but $x \neq -3$

$\therefore x = 3$

(b) $(\sqrt{2})^{x+2} = 8$

$2^{\frac{x+2}{2}} = 2^3$

$\frac{x+2}{2} = 3$

$x = 4$

(c) $\log_3 26 = 2x$

$x = \frac{1}{2}\log_3 26$

(8) (a) $\log_6 36 = 2$

(b) $\log_4\left(\frac{144}{9}\right) = 2$

(c) $\log_{100}\left(\frac{20}{\sqrt{4}}\right)$

$= \log_{100} 10$

$= \frac{1}{2}$

(9) $A = P\left(1+\frac{0.5}{100}\right)^n$

$150000\,(1.005)^n = 300000$

$1.005^n = 2$

$n \log_{10} 1.005 = \log_{10} 2$

$n = \frac{\log_{10} 2}{\log_{10} 1.005}$

≈ 139

After approx. 139 months, the amount doubles

i.e. during 2020.