Use this page to rewrite any answer

CLEARLY INDICATE THE SECTION AND THE QUESTION		
SECTION	QUESTION	

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

DECEMBER 2002

YEAR 10 OPTIONS EXAMINATION

Examiner: F. Jordan

Mathematics

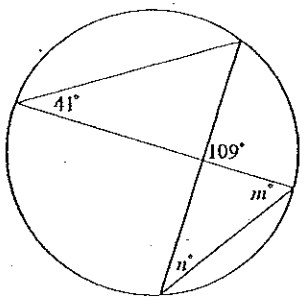
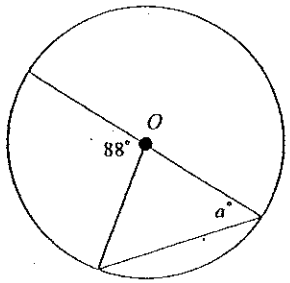
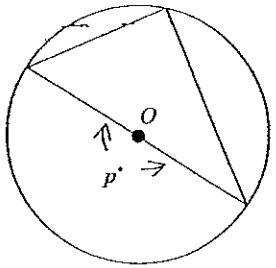
General Instructions

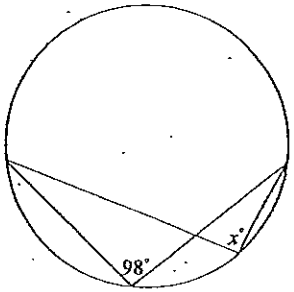
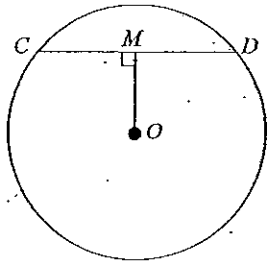
- Working time – 60 minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used
- All necessary working should be shown in every question or full marks may not be awarded.
- Work that is untidy or not set out well may lead to full marks not being awarded.
- The back page may be used to re-write any answer. Clearly indicate the section and question.

NAME:

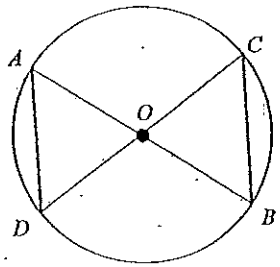
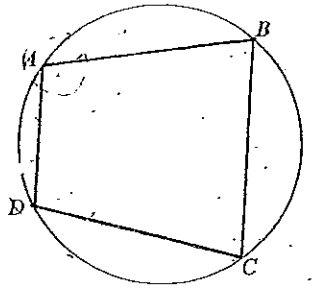
CLASS (Place a cross)		
10 A		E. Choy
10 B		B. Opferkuch
10 C		P. Parker
10 D		F. Nesbitt
10 E		R. Boros
10 F		C. Kourtesis

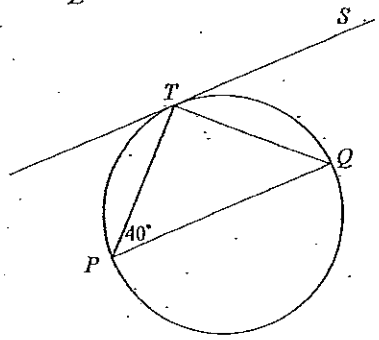
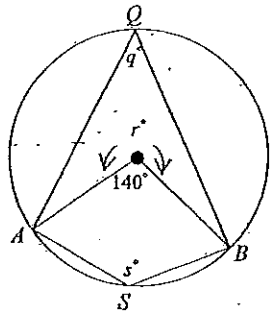
SECTION	MARK
A	/11
B	/12
C	/19
D	/13
E	/20
TOTAL	/75

SECTION A	ANSWERS	Marks
(1) Find the size of m° and n°  		2
(2) O is the centre of the circle. Find the value of a.  		2
(3) O is the centre of the circle. Find the size of p.  		2

(4) Find the value of x, giving reasons.  		3
(5) Given that $CD = 35$ cm, find the length of MD. Give reasons.  		2

END OF SECTION A

SECTION B	ANSWERS	Marks
(6) AB and CD are two diameters of a circle with centre O.  Prove that $\angle DAB = \angle ABC$.		3
(7) Below is cyclic quadrilateral $ABCD$ with $AD \parallel BC$  Prove $\angle BAD = \angle ADC$		3

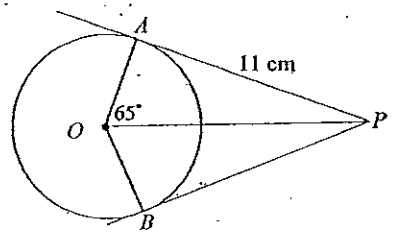
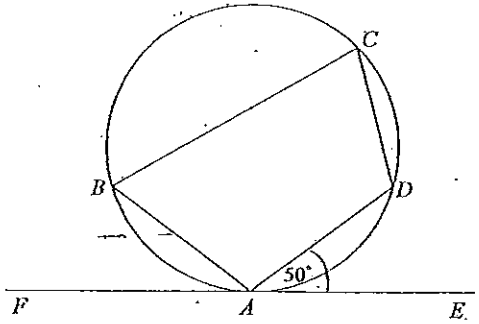
(8) The tangent TS is parallel to the chord PQ. $\angle TPQ = 40^\circ$  Find the size of $\angle PTQ$, giving reasons.		3
(9) The circle below has angles of r° and 140° at its centre. $\angle AQB = q^\circ$ and $\angle ASB = s^\circ$  Find the values of q, r and s.		3

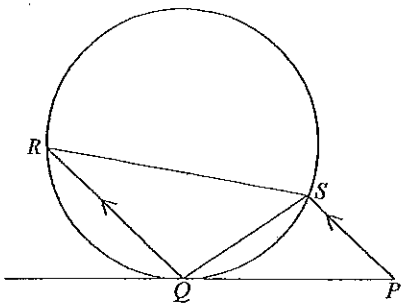
END OF SECTION B

SECTION C	ANSWERS	Marks
(10) The *degree* of the polynomial $y=(x-5)(x-2)^2$ is (A) 1 (B) 3 (C) 4 (D) 6		1
(11) Which of the following is a MONIC polynomial? (A) $4+3x+3x^5$ (B) $\frac{1}{x+3}$ (C) $3-6x+x^7$ (D) $x^2+\sqrt{x}+3$		1
(12) For the polynomial $f(x)=x^4+2x^2-1$ Find: (a) $f(0)$ (b) $f(-2)$		2
(13) $P(x)$ and $Q(x)$ are two polynomials. $P(x)$ has degree 3 and $Q(x)$ has degree 4. What is the degree of: (a) $[P(x)\times Q(x)]$? (b) $[P(x)+Q(x)]$?		2

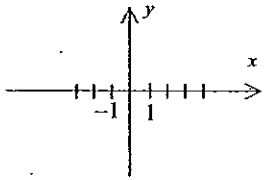
(14) For the polynomial $Q(x)=7x-4x^2-x^3$ State: (a) the leading coefficient; (b) the number of terms; (c) the constant term.		3
(15) (a) Find the centre and radius of the circle $x^2+y^2-10x+8y+32=0$ (b) Hence sketch the circle $x^2+y^2-10x+8y+32=0$		4
(16) Sketch the graph of $f(x)=(x-2)^3+1$ Clearly showing all essential points.		3
(17) Find the roots of the following equation, given that there is at least one integer root. $x^3+4x^2-7x-10=0$		3

END OF SECTION C

SECTION D	ANSWERS	Marks
(18) PA and PB are tangents to the circle with centre O. $AP = 11$ cm, $\angle AOP = 65^\circ$  Find, giving reasons: (a) the length of PB. (b) the measure of $\angle OPB$.		3
(19) $ABCD$ is a cyclic quadrilateral. FAE is a tangent at A. BD is parallel to FE. $\angle DAE = 50^\circ$  Find the size of: (a) $\angle BAF$ (b) $\angle BCD$		3

(20) PQ is a tangent to the circle at Q and PS is parallel to QR.  Prove that $\angle SPQ = \angle QSR$, clearly explaining your reasons at each step.		3
(21) AB and CD are parallel chords of a circle. The tangent at B meets CD produced to E. (a) Draw a diagram to illustrate this information. (b) Prove that the triangles ADB and DBE are similar. Clearly state your reasons.		4

END OF SECTION D

SECTION E	ANSWERS	Marks
22) (a) Show that $f(x)=3x^3+10x^2+11x+4$ is exactly divisible by $3x+4$ (b) Express $f(x)$ in factored form.		4
23) Sketch the graph of $p(x)=(1-x)(1+x)(3+x)$ on the number plane provided.		3
24) Divide $4x^3-10x+18$ by $2x-3$. Clearly indicate the quotient and the remainder.		4

(25) When the polynomial x^3+3x^2-mx+n is divided by $x+2$, the remainder is 9. When divided by $x-3$, the remainder is 49. Find the values of m and n.		4
(26) A polynomial $P(x)$ has a factor of $x-1$ and has a remainder of 4 when divided by $x+2$. (a) If $P(x)$ is divided by x^2+x-2, explain why the remainder will be of the form $ax+b$. (b) Find the remainder when $P(x)$ is divided by x^2+x-2		5

THIS IS THE END OF THE TEST

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FILE

SOLUTIONS

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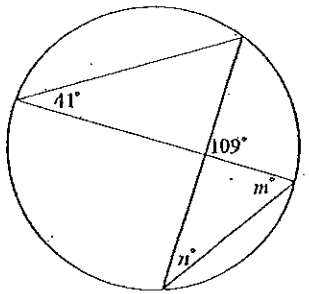
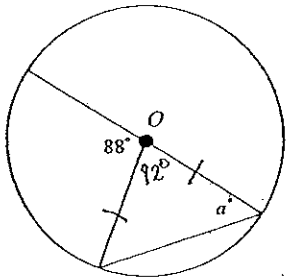
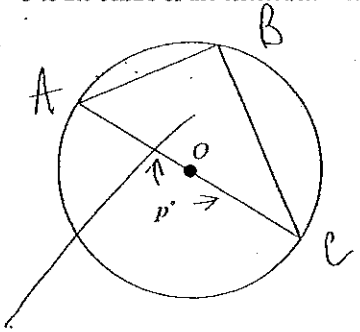
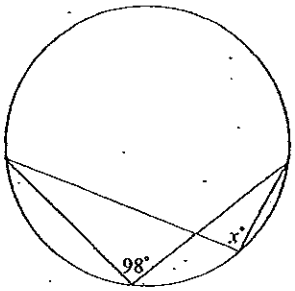
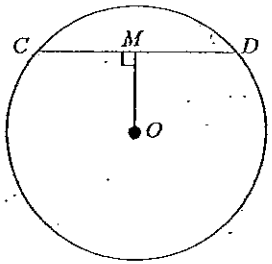
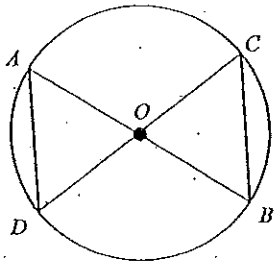
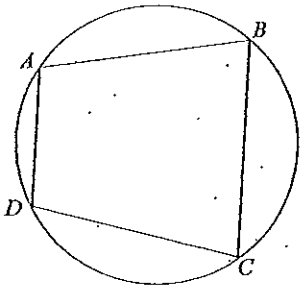
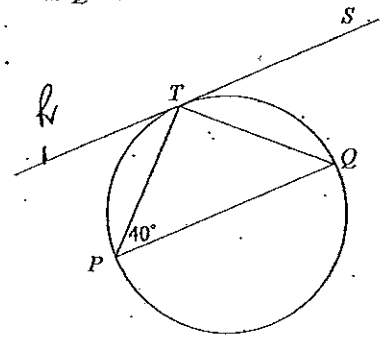
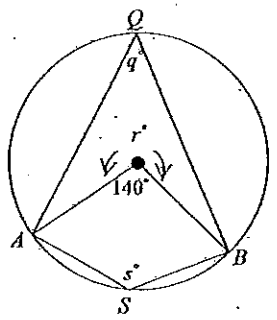
SECTION A	ANSWERS	Marks
(1) Find the size of m° and n° 	$N = 41^\circ$ $\therefore m = 109 - 41$ $= 68^\circ$	2
(2) O is the centre of the circle. Find the value of a. 	$a = \frac{88}{2}$ $= 44^\circ$	2
(3) O is the centre of the circle. Find the size of p. 	$P = 2 \times \angle ABC$ $= 180^\circ$ If AC is a diameter.	2

(4) Find the value of x, giving reasons. 	$x = 98^\circ$ (angles in same segment)	3
(5) Given that $CD = 35$ cm, find the length of MD. Give reasons. 	$CD = 2 \times MD$ (line drawn from centre at right angles to a chord bisects that chord) $\therefore MD = \frac{CD}{2} = \frac{35}{2}$ ie $MD = 17\frac{1}{2}$ cm	

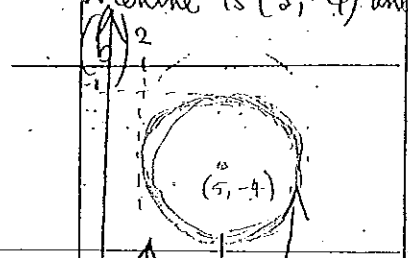
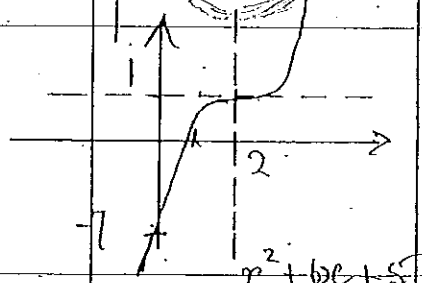
END OF SECTION A

SECTION B	ANSWERS	Marks
(6) *AB* and *CD* are two diameters of a circle with centre *O*.  Prove that $\angle DAB = \angle ABC$.	$\angle DAB = \angle DCB$ (angles in same segment) Also, in $\triangle BOC$: $OB = OC$ (radii of same circle) $\therefore \angle OBC = \angle OCB$ (base angles of isosceles $\triangle$) $\therefore \angle DAB = \angle OBC$ (both $= \angle OCB$) $\therefore \angle DAB = \angle ABC$	3
(7) Below is cyclic quadrilateral *ABCD* with $AD \parallel BC$  Prove $\angle BAD = \angle ADC$	$\angle BAD + \angle DCB = 180°$ (opposite $\angle$'s of cyclic quadrilateral are supplementary) Also $\angle ADC + \angle DCB = 180°$ (co-interior $\angle$'s on parallel lines are supplementary) $\therefore \angle BAD = \angle ADC$ (Both supplements of same angle)	3

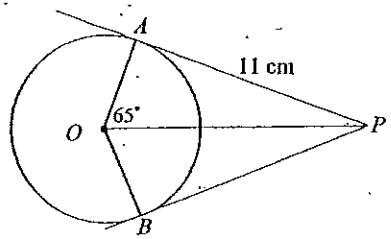
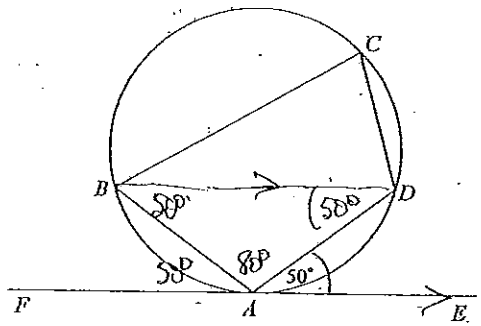
(8). The tangent *TS* is parallel to the chord *PQ*. $\angle TPQ = 40°$  Find the size of $\angle PTQ$, giving reasons.	Mark in R as shown $\angle RTP = \angle TPQ = 40°$ (alternate $\angle$'s on parallel lines) Also $\angle STQ = \angle TPQ = 40°$ (angle between chord & tangent equal to angle in alternate segment) Also $\angle RTP + \angle PTQ + \angle QTS = 180°$ ($\angle$s straight line) ie $40° + \angle PTQ + 40° = 180°$ $\therefore \angle PTQ = 100°$	3
(9) The circle below has angles of $r°$ and $140°$ at its centre. $\angle AQB = q°$ and $\angle ASB = s°$  Find the values of q, r and s.	$r = 360 - 140$ $= 220°$ Now $140° = 2q$ (angle at centre = 2x angle at circumference on same arc) ie $q = 70°$. Similarly, it can be shown that $220° = 2s$ ie $s = 110°$	3

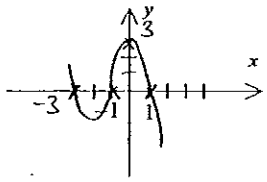
END OF SECTION B

SECTION C	ANSWERS	Marks
(10) The *degree* of the polynomial $y=(x-5)(x-2)^2$ is (A) 1 (B) 3 (C) 4 (D) 6	(B)	1
(11) Which of the following is a MONIC polynomial? (A) $4+3x+3x^5$ (B) $\frac{1}{x+3}$ (C) $3-6x+x^7$ (D) $x^2+\sqrt{x}+3$	(C)	1
(12) For the polynomial $f(x)=x^4+2x^2-1$ Find: (a) $f(0)$ (b) $f(-2)$	(a) $f(0)=0+0-1$ $=-1$ (b) $f(-2)=(-2)^4+2(-2)^2-1$ $=16+8-1$ $=23$	2
(13) $P(x)$ and $Q(x)$ are two polynomials. $P(x)$ has degree 3 and $Q(x)$ has degree 4. What is the degree of (a) $[P(x)\times Q(x)]$? (b) $[P(x)+Q(x)]$?	(a) $3+4=7$ (b) degree 4	2

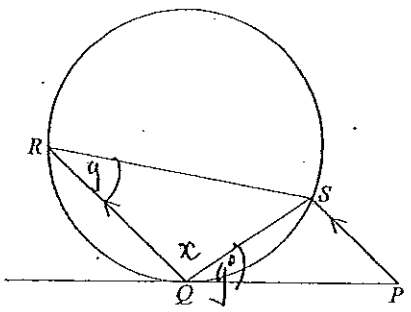
		Marks
(14) For the polynomial $Q(x)=7x-4x^2-x^3$ State: (a) the leading coefficient; (b) the number of terms; (c) the constant term.	(a) -1 (b) 3 (c) 0	3
(15) (a) Find the centre and radius of the circle $x^2+y^2-10x+8y+32=0$ (b) Hence sketch the circle $x^2+y^2-10x+8y+32=0$	(a) $x^2-10x+25+y^2+8y+16+32=25+16$ ie $(x-5)^2+(y+4)^2+32=41$ $(x-5)^2+(y+4)^2=9$ $\therefore$ centre is $(5,-4)$ and radius is 3 	
(16) Sketch the graph of $f(x)=(x-2)^3+1$ Clearly showing all essential points.		3
(17) Find the roots of the following equation, given that there is at least one integer root. $x^3+4x^2-7x-10=0$	$P(1)=1+4-7-10\neq 0$ $P(2)=8+16-14-10=0$ $\therefore$ $x-2$ is a factor x^2+6x+5 $x-2\overline{)x^3+4x^2-7x-10}$ x^3-2x^2 $6x^2-7x-10$ $6x^2-12x$ $5x-10$ $5x-10$ $\therefore P(x)=(x-2)(x^2+6x+5)$ $=(x-2)(x+5)(x+1)$ $\therefore P(x)=0$ for $x=2,-5,-1$.	3

END OF SECTION C

SECTION D	ANSWERS	Marks
(18) PA and PB are tangents to the circle with centre O. $AP = 11$ cm, $\angle AOP = 65^\circ$  Find, giving reasons: (a) the length of PB. (b) the measure of $\angle OPB$.	(a) $PB = AP = 11$cm (Tangents to a circle from the same external point are equal) (b) In $\triangle$s AOP & BOP $AP = BP$ (from above) $AO = OB$ (radii of same circle) OP is common $\therefore \triangle AOP \equiv \triangle BOP$ (SSS) $\therefore \angle OPB = \angle BPO$ (corresponding $\angle$'s in congruent $\triangle$'s) $\therefore \angle OPB = 90 - 65 = 25^\circ$ NB $\angle OAP = 90^\circ$ since tangent $\perp$ to radius drawn to point of contact	3
(19) $ABCD$ is a cyclic quadrilateral. FAE is a tangent at A. BD is parallel to FE. $\angle DAE = 50^\circ$  Find the size of: (a) $\angle BAF$ (b) $\angle BCD$	(a) $\angle DAE = \angle ABD = 50^\circ$ (alternate $\angle$'s on parallel lines) Also $\angle DAE = \angle ABD = 50^\circ$ (angle between chord & tangent equal to angle in alternate segment) $\therefore \angle BAF = \angle ABD = 50^\circ$ (alternate $\angle$'s on parallel lines) (b) $\angle BAF + \angle BAD + \angle DAE = 180^\circ$ (FAE a straight angle) ie $50 + \angle BAD + 50 = 180^\circ$ $\therefore \angle BAD = 80^\circ$ But $\angle BAD + \angle BCD = 180^\circ$ (opposite $\angle$'s of cyclic quad are supplem) $\therefore \angle BCD = 100^\circ$	3

SECTION E	ANSWERS	Marks
(22) (a) Show that $f(x) = 3x^3 + 10x^2 + 11x + 4$ is exactly divisible by $3x+4$	(a) $3x+4 \overline{)3x^3+10x^2+11x+4}$ $x^2 + 2x + 1$ $3x^3 + 4x^2$ $6x^2 + 11x + 4$ $6x^2 + 8x$ $3x + 4$ $3x + 4$ $\therefore 3x+4$ is a factor.	4
(b) Express $f(x)$ in factored form.	(b) $f(x) = (3x+4)(x^2+2x+1)$ $= (3x+4)(x+1)^2$	
(23) Sketch the graph of $p(x) = (1-x)(1+x)(3+x)$ on the number plane provided.	$P(0) = 1.1.3 = 3$  	3
(24) Divide $4x^3 - 10x + 18$ by $2x-3$. Clearly indicate the quotient and the remainder.	$2x^2 + 3x - \frac{1}{2}$ $2x-3 \overline{)4x^3 - 10x + 18}$ $4x^3 - 6x^2$ $6x^2 - 10x + 18$ $6x^2 - 9x$ $-x + 18$ $-x + 1\frac{1}{2}$ $16\frac{1}{2}$ Quotient is $2x^2 + 3x - \frac{1}{2}$ Remainder is $16\frac{1}{2}$	4

(20) PQ is a tangent to the circle at Q and PS is parallel to QR.



Prove that $\angle SPQ = \angle QSR$, clearly explaining your reasons at each step.

3

Let $\angle RQS = x$ and $\angle PQS = y$

In parallel lines QR & PS

$\angle SPQ + \angle PQR = 180°$

(co-interial $\angle$'s are supplementary)

$\therefore \angle SPQ = 180 - \angle PQR$

ie $\angle SPQ = 180 - (x+y)$

Also $\angle PQS = \angle QRS = y$

(angle between chord & tangent equal to $\angle$ in alternate segment)

Also in $\triangle QRS$

$\angle QSR = 180° - (\angle RQS + \angle QRS)$

$= 180 - (x+y)$

$= \angle SPQ$

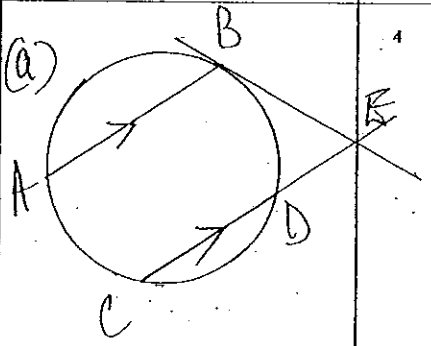
(21) AB and CD are parallel chords of a circle. The tangent at B meets CD produced to E.

(a) Draw a diagram to illustrate this information.

(b) Prove that the triangles ADC and DBE are similar. Clearly state your reasons.

4

(a)



(b) For class discussion.

END OF SECTION D

(25) When the polynomial $x^3 + 3x^2 - mx + n$ is divided by $x+2$, the remainder is 9.

When divided by $x-3$, the remainder is 49.

Find the values of m and n.

4

$P(-2) = 9$

ie $-8 + 12 + 2m + n = 9$

ie $2m + n = 5$

$P(3) = 49$

ie $27 + 27 - 3m + n = 49$

$-3m + n = -5$

Solving Simultaneously

$2m + n = 5$ — A

$-3m + n = -5$ — B

A − B → $5m = 10$

$m = 2$

$\therefore n = 1$

(26) A polynomial $P(x)$ has a factor of $x-1$ and has a remainder of 4 when divided by $x+2$.

(a) If $P(x)$ is divided by $x^2 + x - 2$, explain why the remainder will be of the form $ax + b$.

(b) Find the remainder when $P(x)$ is divided by $x^2 + x - 2$

5

For class discussion.

THIS IS THE END OF THE TEST