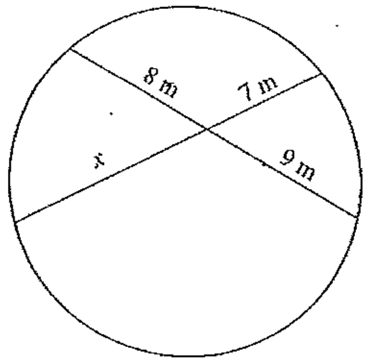
SYDNEY BOYS H.S.

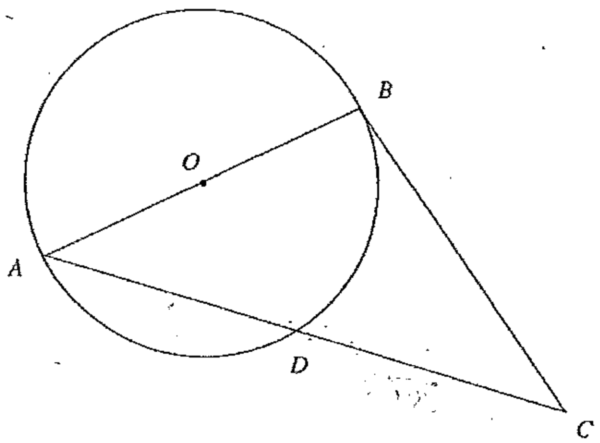
# Year 10 Test
# Algebra and Circle Geometry

Name:_

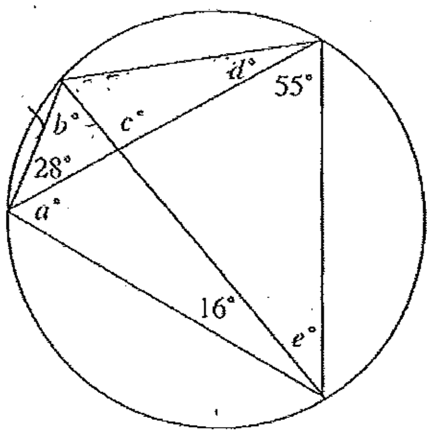
(1) Find the value of $x$. [2 marks]



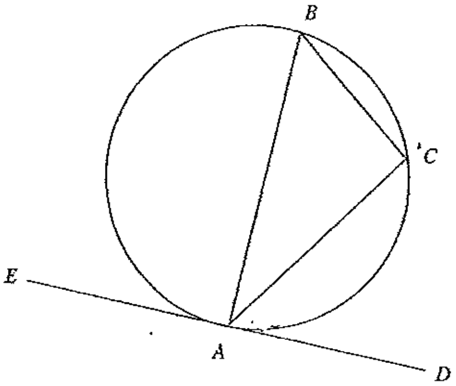

(2) If AD = 4 and DC = $2\sqrt{26}-2$ find BC. [2 marks]




(3) Find the value of all the pronumerals. [5 marks]



(4) What is the incorrect assumption made in this proof of the alternate segment theorem (i.e. that $\angle CAD = \angle ABC$)? [2 marks]




*Proof*

Let $\angle CAD = x$

Then $\angle BAC = 90° - x$ (tangent ⊥ radius)

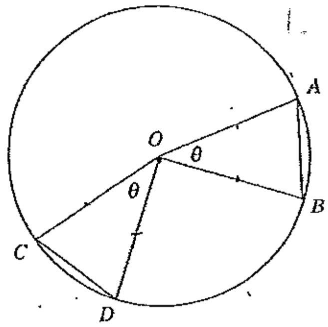
$\angle BCA = 90°$ (∠ in semicircle)

$\therefore \angle ABC = 180° - (90° + 90° - x)$

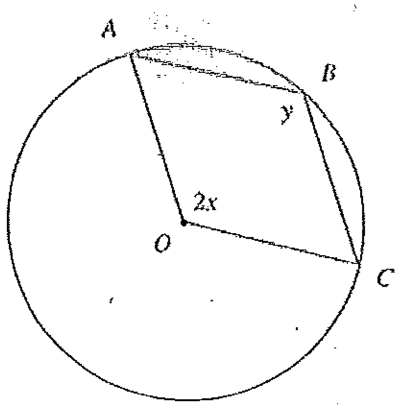
$= x$

$\therefore \angle CAD = \angle ABC$

(5) Prove that if two chords subtend equal angles at the centre then the chords are equal. [4 marks]



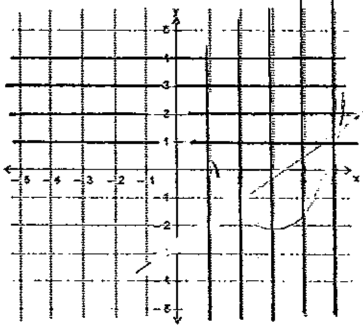

(6) Prove that, in the following figure, $x + y = 180°$. [3 marks]



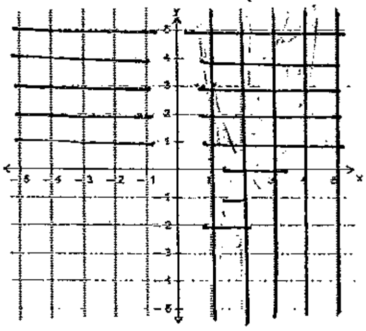
(7) Prove that the ABCD is a cyclic quadrilateral. [4 marks]

(8) Sketch a line and parabola that have: [3 marks]

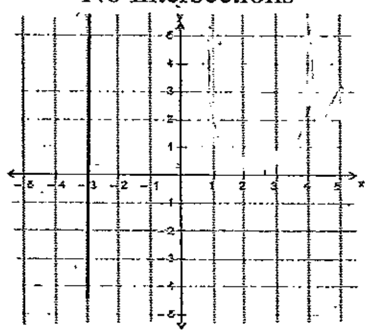
Two Intersections



One Intersection



No Intersections



(9) Find the points of intersection of, [4 marks]

$$y = x^2 + 6x - 21$$
$$y = 15 - 3x$$

(10) Without sketching graphs, show that $y = x^2 + 1$ and $x - 3y - 3 = 0$ do not intersect. [3 marks]

(11) Given the equation of a line $6x + 3y - 18 = 0$, make $y$ the subject and hence find the gradient and $y$-intercept. [4 marks]

(12) Make $t$ the subject of the formula, [3 marks]

$$x = \frac{t-2}{1+t}$$

(13) Substitute $X = \frac{1}{a}$ into $\frac{2}{X} + 3X$ then simplify. [2 marks]

(14)Solve $(3^x)^2 - 10(3^x) + 9 = 0$. [3 marks]

(15)If $n = 8 - x$ solve $n^2 + 2xn = 0$ for $x$. [3 marks]

(16)In the equation $y = -(x+5)^2$, is $y$ negative for all values of $x$? Explain.

[2 marks]

(17)In the formula $M = \sqrt{t-3}$, what values can $t$ possibly take? [2 marks]

(18)In the formula $M = \dfrac{5t}{t-3}$, what value can $t$ not take? [2 marks]

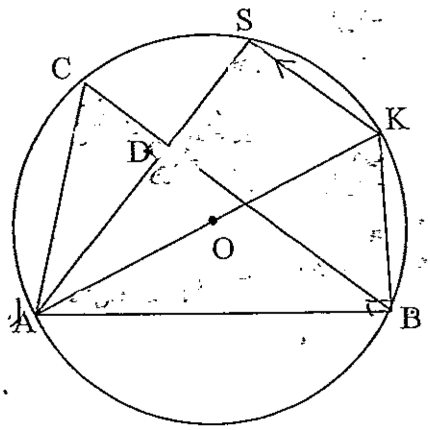
(19)The total resistance in a circuit, $R_T$, is given by the formula, [3 marks]

$\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}$ where $R_1$ and $R_2$ are resistors in parallel. Find $R_2$ in terms of $R_1$ and the total resistance.

(20)In the equation $y=\frac{1}{\sqrt{6-x-x^2}}$, what values can $x$ possibly take? [3 marks]

(21)AK is a diameter of the circle, centre O. SK||CB. [4 marks]

(a) Prove that $AS \perp BC$.



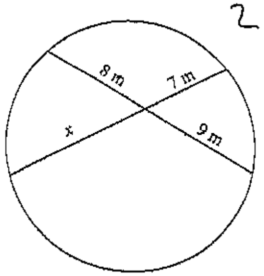

(b) Let $\angle BAK=\alpha$, hence show that $\angle SAC=\alpha$.

# Year 10 Test
## Algebra and Circle Geometry

Name: Rahib

55

(1) Find the value of $x$. [2 marks]




$8 \times 9 = 7x$ (intersecting chords)

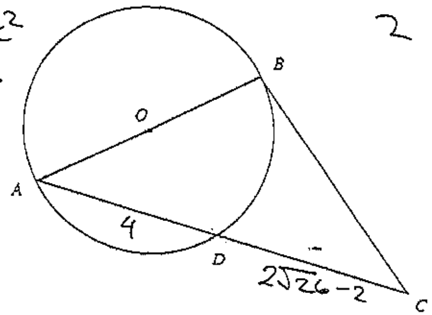
$72 = 7x$

$x = \frac{72}{7}$

$\approx 10.29$

2

(2) If AD = 4 and DC = $2\sqrt{26} - 2$ find BC. [2 marks]




$(2\sqrt{26} + 2) \times (2\sqrt{26} - 2) = BC^2$

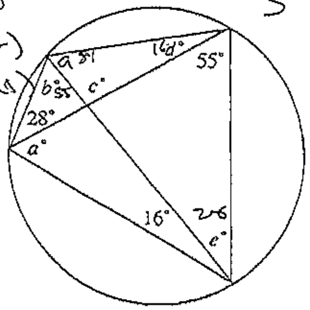
$104 - 4 = BC^2$

$\therefore 100 = BC^2$

$10 = BC$

2

(3) Find the value of all the pronumerals. [5 marks]




$b = 55°$ (∠s in the same segment)

$d = 16$ (∠s in the same segment)

$e = 28$ (∠s in the same segment)

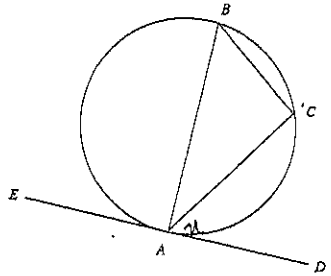
$a = 81$ (∠ sum of Δ)

$c = 83$ (∠ sum of Δ)

5



(4) What is the incorrect assumption made in this proof of the alternate segment theorem (i.e. that $\angle CAD = \angle ABC$)? [2 marks]




*Proof*

Let $\angle CAD = x$

Then $\angle BAC = 90° - x$ (tangent ⊥ radius)

$\angle BCA = 90°$ (∠ in semicircle)

$\therefore \angle ABC = 180° - (90° + 90° - x)$

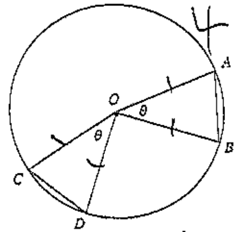
$= x$

$\therefore \angle CAD = \angle ABC$

The assumption that AB passes through the centre of the circle is incorrect.

2

(5) Prove that if two chords subtend equal angles at the centre then the chords are equal. [4 marks]




In Δs OCD and OAB

S OC = OA (radii)

A ∠COD = ∠AOB (given)

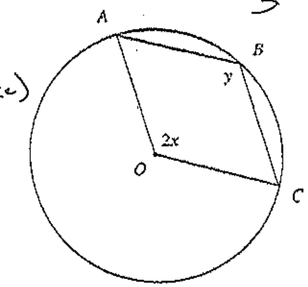
S OD = OB (radii)

∴ ΔOCD ≡ ΔOAB (SAS)

∴ CD = AB (corresponding sides of congruent Δs)

4

(6) Prove that, in the following figure, $x + y = 180°$. [3 marks]




reflex ∠AOC = 360 − 2x (revolution)

∠ABC = 180 − x (∠ at centre twice ∠ at circumference)

∠ABC = y (given)

∴ 180 − x = y

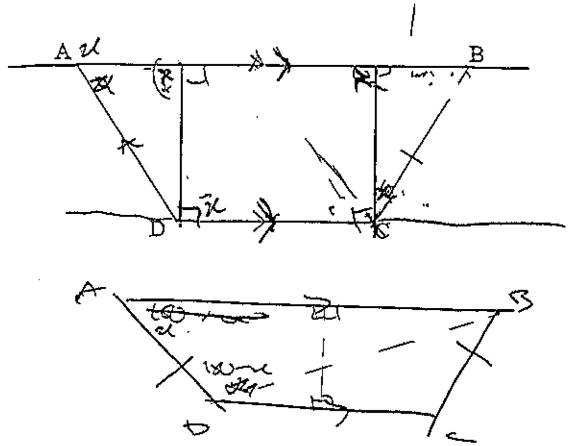
∴ x + y = 180

3

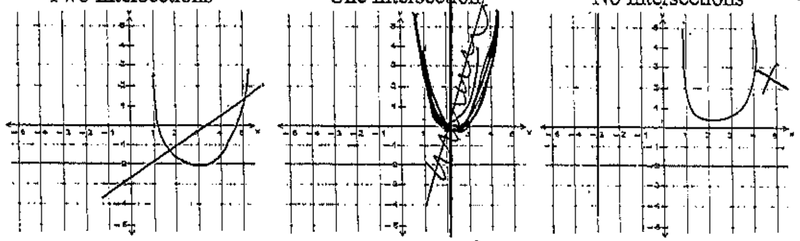
(7) Prove that the ABCD is a cyclic quadrilateral. [4 marks]

Let ∠DAB = x

∴ ∠ADC = 180 − x




(8) Sketch a line and parabola that have: [3 marks]




(9) Find the points of intersection of, [4 marks]

$y = x^2 + 6x - 21$ — ①

$y = 15 - 3x$ — ②

sub ② into ①

$15 - 3x = x^2 + 6x - 21$

$x^2 + 9x - 36 = 0$

$\frac{-9 \pm \sqrt{81 - (-144)}}{2}$

$= \frac{-9 \pm 15}{2}$

$x = -12, 3$

Sub values of x into eq ②

(−12, 51)

(3, 6)



(10) Without sketching graphs, show that $y = x^2 + 1$ and $x - 3y - 3 = 0$ do not intersect. [3 marks]

$y = x^2 + 1$ — ①

$x - 3y - 3 = 0$ — ②

Sub ① into ②

$x - 3(x^2 + 1) - 3 = 0$

$x - 3x^2 - 3 - 3 = 0$

$-3x^2 + x - 6 = 0$

$\Delta = b^2 - 4ac$

$= 1 - 4(-6 \times -3)$

$= 1 - 72$

$\Delta = -71$ = no roots.

∴ they do not intersect

(11) Given the equation of a line $6x + 3y - 18 = 0$, make $y$ the subject and hence find the gradient and $y$-intercept. [4 marks]

$3y = 18 - 6x$

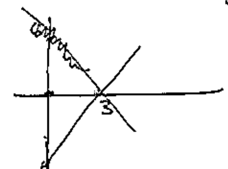
$y = 6 - 2x$

x = 6x − 18 = 0

6x = 18

x = 3

x int = 3



gradient = 6/3 = −2

x int = 3

(12) Make $t$ the subject of the formula, [3 marks]

$x = \frac{t-2}{1+t}$

$x + xt = t - 2$

$xt - t = -2 - x$

$t(x - 1) = -2 - x$

$t = \frac{-2 - x}{x - 1}$

(13) Substitute $X = \frac{1}{a}$ into $\frac{2}{X} + 3X$ then simplify. [2 marks]

$\frac{2}{\frac{1}{a}} + \frac{3}{a}$

$= \frac{2a + 3}{a}$

$= \frac{2a^2 + 3}{a}$

2 ÷ 1/a

2a

(14)Solve $(3^x)^2 - 10(3^x) + 9 = 0$. [3 marks]

Let $3^x = a^2$
$a^2 - 10a + 9 = 0$
$(a-9)(a-1) = 0$
$a = 9, 1$
$3^x = 9, 1$
$x = 3, 0$

2

(15)If $n = 8 - x$ solve $n^2 + 2xn = 0$ for $x$. [3 marks]

$(8-x)^2 + 2(8-x)x = 0$
$64 + x^2 - 16x + 2(8x - x^2) = 0$
$64 + x^2 - 16x + 16x - 2x^2 = 0$
$64 - x^2 = 0$
$(8-x)(8+x) = 0$
$x = \pm 8$

3

(16)In the equation $y = -(x+5)^2$, is $y$ negative for all values of $x$? Explain. [2 marks]

Yes, since squaring any integer may it be positive or negative will result in a positive integer. By multiplying -1 to any positive integer will result in a negative integer. Therefore y is negative for all values of x

(17)In the formula $M = \sqrt{t-3}$, what values can $t$ possibly take? [2 marks]

$t \nless 3$

ans = $t \geq 3$

2

(18)In the formula $M = \frac{5t}{t-3}$, what value can $t$ not take? [2 marks]

$t - 3 \neq 0$

$t \neq 3$

2



(19)The total resistance in a circuit, $R_T$, is given by the formula, [3 marks]

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ where $R_1$ and $R_2$ are resistors in parallel. Find $R_2$ in terms of $R_1$ and the total resistance.

$\frac{1}{R_T} - \frac{1}{R_1} = \frac{1}{R_2}$

$\frac{R_1 - R_T}{R_T \times R_1} = \frac{1}{R_2}$

$R_2(R_1 - R_T) = R_T \cdot R_1$

$\therefore R_2 = \frac{R_T \cdot R_1}{R_1 - R_T}$

3

(20)In the equation $y = \frac{1}{\sqrt{6-x-x^2}}$, what values can $x$ possibly take? [3 marks]

$\sqrt{6-x-x^2} = 0$
$6 - x - x^2 = 0$
$x^2 + x = 6$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$

$-x - x^2 \ngtr 6$
$x^2 + x > -6$
$x^2 + x - 6 > 0$
$(x+3)(x-2) > 0$

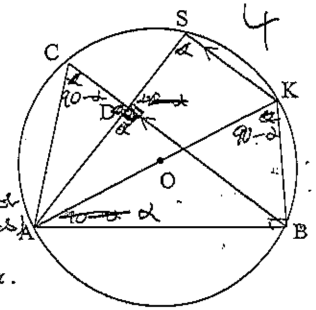
$x \neq -3$
$x \neq 2$

$-3 < x < 2$

3

(21)AK is a diameter of the circle, centre O. SK||CB. [4 marks]

(a) Prove that AS ⊥ BC.




Let $\angle ASK = \alpha$
$\therefore \angle SDB = 180 - \alpha$ (cointerior ∠s of // lines SK//CB)
$\therefore \angle ADB = \alpha$ (∠ sum of straight line)
$\angle ABK = 90$ (∠ in a semicircle)
$\therefore \angle ASK + \angle ABK = 180$ (opp ∠s of cyclic quad)
$\alpha + 90 = 180$
$\alpha = 90$
$\therefore \angle ADB = 90$
$\therefore$ AS ⊥ BC
Since $\angle ADB = 90$

4

(b) Let $\angle BAK = \alpha$, hence show that $\angle SAC = \alpha$.

Let $\angle BAK = \alpha$
& $\angle ABK = 90$ (∠ in a semicircle)
$\therefore \angle AKB = 90 - \alpha$ (∠ sum of Δ)
$\angle ACB = \angle AKB = 90 - \alpha$ (∠ in the same segment)
Since $\angle ADB = 90$ (from (a))
$\angle ADC = 90$ (∠ sum of straight line)
$\therefore \angle SAC = \alpha$ (∠ sum of Δ)