Non-right-angled Triangles

The trigonometric methods given earlier apply only to triangles containing a right angle.

For triangles without a right angle, the sine rule, the cosine rule and the area formula can be used to solve triangles and find their areas.
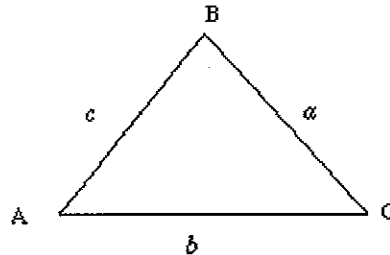
<u>Sine Rule</u>   <u>Cosine Rule</u>   <u>Area Formula</u>

Sine Rule

The sine rule is concerned with triangles where pairs of angles and their opposite sides are given.

The sine rule states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or, in an alternative form for use when finding angle sizes:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Examples	Answers
(a) Find the value of a 	Use the formula: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ $\dfrac{a}{\sin 57^\circ} = \dfrac{8}{\sin 68^\circ}$ $a = \dfrac{8 \times \sin 57^\circ}{\sin 68^\circ}$ $a = \dfrac{8 \times 0.8387}{0.9272}$ $a = 7.2 \quad (\text{to 2 sig.fig.})$
(b) Find the size of angle y 	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ $\dfrac{\sin y}{6} = \dfrac{\sin 51^\circ}{7}$ $\sin y = \dfrac{\sin 51^\circ \times 6}{}$

$$\frac{}{7}$$

$$\sin y = \frac{0.7771 \times 6}{7}$$

$$\sin y = 0.6661$$
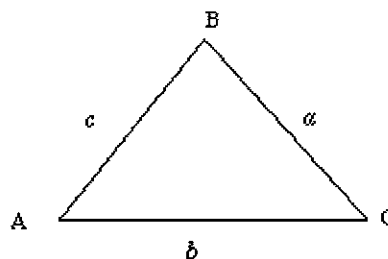
$$y = 41.8°$$

(to 1 d.p.)

Cosine Rule

The cosine rule is concerned with triangles where the lengths of two sides and the angle between them, or the lengths of three sides, are given.
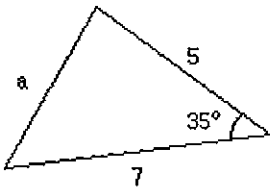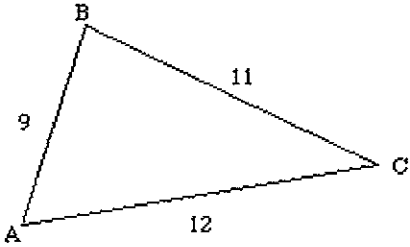
The cosine rule states:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

or , in alternative form for finding angles:

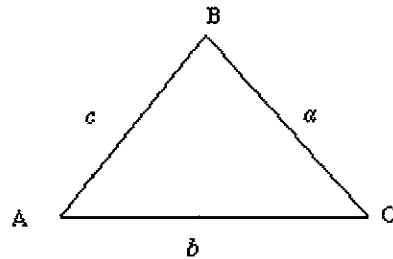$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

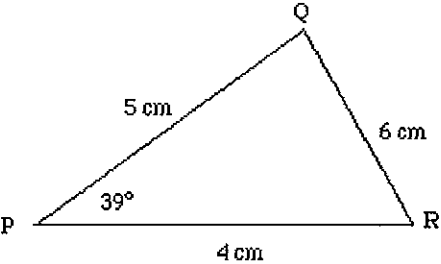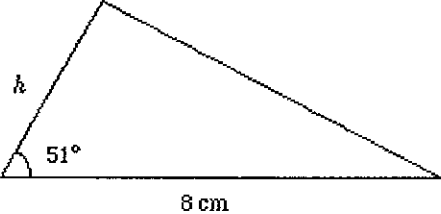| Examples	Answers
(a) Find the value of a	Use the formula: $a^2 = b^2 + c^2 - 2bc \cos A$ $a^2 = 7^2 + 5^2 - 2 \times 7 \times 5 \times \cos 35°$ $a^2 = 49 + 25 - 70 \times 0.8192$ $a^2 = 74 - 57.344$ $a^2 = 16.656$ $a = 4.1$ (to 2 sig.fig.)
(b) Find the size of angle BAC	Use the formula: (to 1 d.p.) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{12^2 + 9^2 - 11^2}{2 \times 12 \times 9}$ $\cos A = \frac{144 + 81 - 121}{216}$ $\cos A = \frac{104}{216}$ $\cos A = 0.4815$ (to 4 sig.fig.) $A = 61.2°$

Area Formula

The area of a triangle can be found given the lengths of two sides and the angle between them.

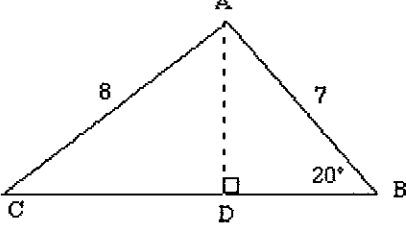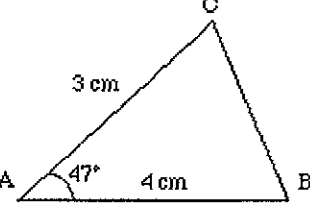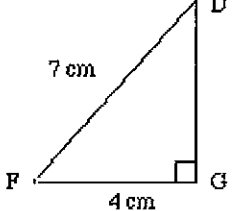The area formula states:

Area of triangle $= \frac{1}{2}bc\sin A$

Examples	Answers
(a) Find the area of $\triangle$PQR 	Area of PQR $= \frac{1}{2}qr\sin P$ Area $= \frac{1}{2} \times 4 \times 5 \times \sin 39^{\circ}$ Area $= 10 \times 0.6293$ Area $= 6.29 \, cm^2$ (to 3 sig.fig.)
(b) The area of the triangle is 27cm^2. Find the value of h. 	Area $= \frac{1}{2}ab\sin C$ $27 = \frac{1}{2} \times h \times 8 \times \sin 51^{\circ}$ $h = \dfrac{27 \times 2}{8 \times 0.7771}$ $h = 8.7 \, cm$ (to 2 sig.fig.)

Non-right-angled Triangles

Unit Test #44

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "**Submit Test**" button. Clicking the "Begin Test Again" button will clear all the answers.

Q1:	Given three sides of a non-right-angled triangle, which of the following would you use to find one of the angles?	A. Sine Rule B. Cosine Rule C. Area Formula D. Laws of Indices	Answer 1:	
Q2:	Fred is using the cosine rule to find an angle P. He writes 36 = 296 - 280cosP A correct statement that follows this is:	A. $332 = 280\cos P$ B. $260 = -280\cos P$ C. $36 = 16\cos P$ D. $-260 = -280\cos P$	Answer 2:	
Q3:	 The value of sin C is:	A $56 \sin 20°$ B. $\dfrac{7 \sin 20°}{8}$ C. $\dfrac{8 \sin 20°}{7}$ D. $\dfrac{8}{7 \sin 20°}$	Answer 3:	
Q4:	 The best expression to use to find the length of side BC is	A. $\dfrac{a}{\sin B} = \dfrac{b}{\sin C}$ B. $a^2 + b^2 = c^2$ C. $a^2 = b^2 + c^2 - 2bc \cos A$ D. $a = b \tan A$	Answer 4:	
Q5:	In using the Cosine Rule, a student wrote $x^2 = 36 + 49 - 84\cos 40°$ The statement that follows from this is:	A. $x^2 = 36 - 35\cos40°$ B. $x^2 = 85 - 84\cos40°$ C. $x^2 = 1\cos40°$ D. $x^2 = 6+7 - 9.2\cos40°$	Answer 5:	
Q6:	If $x = \dfrac{2.3 \times \sin 43}{\sin 59}$ find x (to 2 s.f.)	A. 1.8 B. 1.83 C. 1.4 D. 3.3	Answer 6:	

Q7:	A line in a calculation using the Cosine Rule is $\cos A = \frac{104}{216}$ What is the size of angle A (to 1 d.p.)?	A. 64.3° B. 28.8° C. 25.7° D. 61.2°	**Answer 7:** ⬚▾
Q8:	If $x^2 = 36 - 35\cos 35°$, what is x? (to 2 sig.fig.)	A. 2.7 B. 0.82 C. 7.3 D. 54	**Answer 8:** ⬚▾
Q9:	Find the square root of 105 to 3 significant figures.	A. 10.2 B. 10.25 C. 10 D. 11025	**Answer 9:** ⬚▾
Q10:	 7 cm, 4 cm, D, F, G Which of the following would you use to find the length of DG?	A. Sine Rule B. Cosine Rule C. Pythagoras' Theorem D. Area Formula	**Answer 10:** ⬚▾

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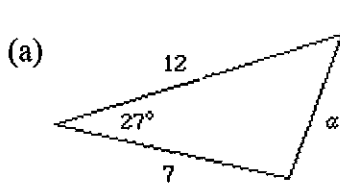
[Submit Test] [Begin Test Again]

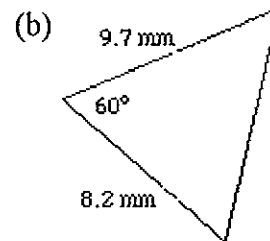# Non-right-angled Triangles
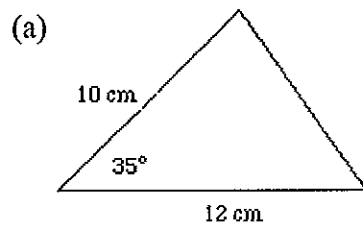
Give answers correct to 3 significant figures.

1.Use the sine rule to find the side or angle marked by a letter.

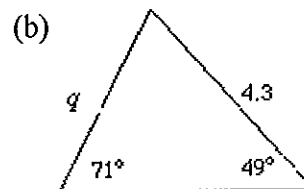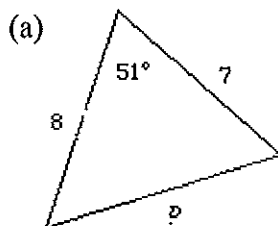(a)

(b)
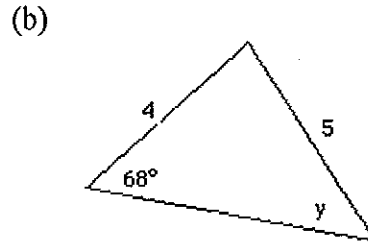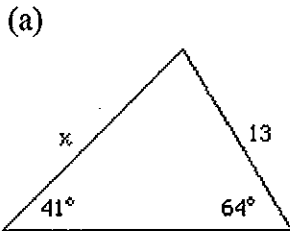
2. Use the cosine rule to find a and b.

(a)

(b)
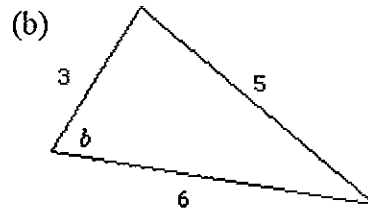
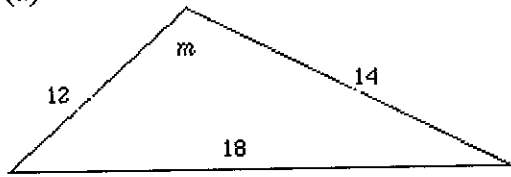3. Find the area of the triangles.

(a)

(b)

4. Find the lengths of the sides marked by letters.
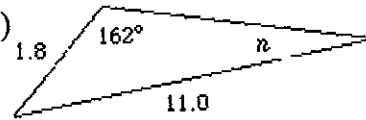
(a)

(b)

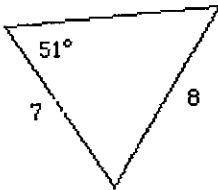5. Find the size of the angles marked by letters.
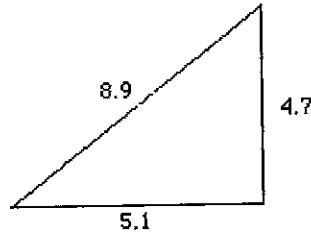
(a)

(b)

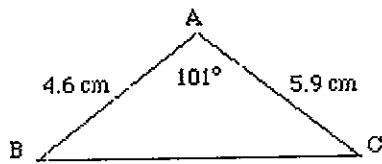6. Find the sizes of all of the unmarked sides and interior angles of the triangles.

(a)

(b)
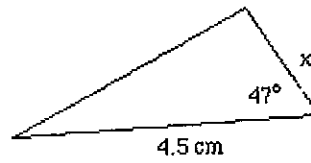

7.

(a) Find the area of the triangle ABC

(b) Find the value of x, if the area of the triangle is 15 cm^2.

Non-right-angled Triangles

1. (a) 17.8 (b) 47.9^o

2. (a) 6.58 (b) 56.3^o

3. (a) 34.4 cm^2 (b) 34.4 mm^2

4. (a) 6.52 (b) 3.43

5. (a) 87.3^o (b) 2.90^o

6. (a) 42.8^o, 86.2^o, 10.3
(b) 23.7^o, 25.9^o, 130.4^o

7. (a) 13.3 cm^2 (b) 9.12 cm

Section (A) - Multiple Choice

1. B   2. D   3. B

4. C   5. B   6. B

7. D   8. A   9. A

10. C