

## Trigonometric Ratios

Trigonometry is concerned with the measurement and calculation of the lengths of sides and the sizes of angles of triangles. There are three basic ratios called the sine, the cosine and the tangent of the angle. These are abbreviated to sin, cos and tan.

Greek letters are often used to stand for angles in triangles.

Right-angled Triangles	Trigonometric Ratios
Finding Ratios	Solution of Triangles

### Right-angled Triangles

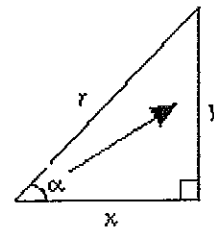
Given a right-angled triangle with an angle marked as shown:

In the diagram:

$y$  is called the **opposite** side (to the angle  $\alpha$ )

$x$  is called the **adjacent** side (to the angle  $\alpha$ )

$r$  is called the **hypotenuse** side



Greek letters, such as  $\alpha$  are often used in trigonometry.

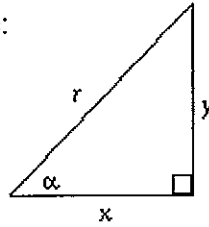
There are several methods used for finding angles and sides in right-angled triangles. You may have been taught and use a different one to that shown below. They all produce the same answers if done correctly!

For example, some people call the opposite side, the **separated** side and the adjacent side, the **connected** side.

Examples		Answers
<p>In the triangle shown, how long is:</p> <p>(a) The side opposite to <math>\theta</math> ?</p> <p>(b) The side adjacent to <math>\theta</math> ?</p> <p>(c) The hypotenuse?</p>		<p>(a) The opposite side is 5 cm long.</p> <p>(b) The adjacent side is 6 cm long.</p> <p>(c) The hypotenuse is 8 cm long.</p>

## Trigonometric Ratios

For a right-angled triangle:



### REMEMBER

Sine of angle $\alpha$ =	$\frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{y}{r}$ i.e. $\sin \alpha = \frac{y}{r}$	SOH
Cosine of angle $\alpha$ =	$\frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{x}{r}$ i.e. $\cos \alpha = \frac{x}{r}$	CAH
Tangent of angle $\alpha$ =	$\frac{\text{length of opposite side}}{\text{length of adjacent side}} = \frac{y}{x}$ i.e. $\tan \alpha = \frac{y}{x}$	TOA

These can be rearranged to:

$$y = r \sin \alpha$$

$$x = r \cos \alpha$$

$$y = x \tan \alpha$$


## Circular Functions

The trigonometric ratios are also known as **circular functions** and the sine, cosine and tangent of any angle, positive or negative can be found.

To see an illustration of these circular functions click below:

<a href="#">sine x</a>	<a href="#">cosine x</a>	<a href="#">tangent x</a>
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## Finding Ratios

The sine, cosine and tangent  ratios are the same for a particular angle regardless of the size of the triangle.

The ratios can be found using a calculator or from a book of trigonometric tables.

## Calculators.

Scientific calculators such as Texas Instruments' TI-30X IIB have trigonometric function keys. These will give the sine, cosine, and tangent of an angle as well as the inverse of these functions. These decimals will be given to a large number of significant figures, which will often need to be rounded off (4 significant figures is usually accurate enough).

e.g To find sine  $53^\circ$  on a calculator, press  $\sin$   $53$   $=$   $0.79863551$  which could then be rounded as necessary.

The other trigonometric buttons are  $\cos$  and  $\tan$ .

Care must be taken to ensure that the calculator is set to work in values in **degrees (DEG)**, as there are other ways of measuring angles such as radians (**RAD**) and gradians (**GRAD**). Use the **mode** button to change these settings.

### Tables.

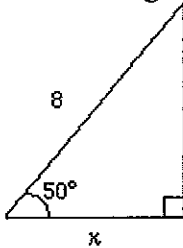
Before the days of calculators, everyone used printed tables giving the trigonometric ratios and these can be used if you do not have a calculator available. These tables give the ratios for angles from  $0^\circ$  to  $90^\circ$  to four or five significant figures, with angles being given to 1 decimal place.



### Solution of Right-angled Triangles

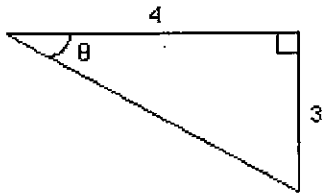
The sides and angles of right-angled triangles can be found using the trigonometric ratios. For each problem the information given should be written down and substituted into one of the ratios, which can then be solved as an equation.

Remember to use Pythagoras' Theorem if only sides are involved. The final answer should be rounded off to a similar degree of accuracy to that given to measurements in the question.

Examples	Answers
<p>(a) Find the length of <math>x</math></p> 	<p> <math>8 = \text{hypotenuse (H)}</math>  <math>x = \text{adjacent (A)}</math>  <math>50^\circ = \text{angle}</math>                      Use cosine <math>x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H}</math>  <math>\cos 50^\circ = \frac{x}{8}</math>  <math>8 \times \cos 50^\circ = x</math>  <math>8 \times 0.6428 = x</math>  <math>5.1424 = x</math>  <math>5.1 = x</math> (to 2 sig. fig.)                 </p> <p>On a calculator the step of writing out <math>\cos 50^\circ</math> can be missed out:</p>

**3** **×** **cos** **5** **0** **=** **5.142300878** which rounds to 5.1

(b) Find angle  $\theta$



3 = opposite

4 = adjacent

$\theta$  = angle

Use the tangent ratio  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A}$

$$\tan \theta = \frac{3}{4}$$

$$\tan \theta = 0.75$$

$$\theta = 36.9^\circ$$

On a calculator this last step is done as follows:

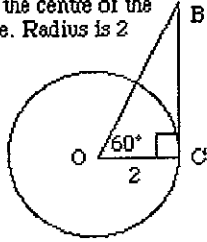
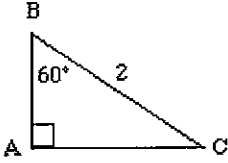
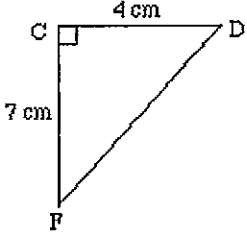
**SHIFT** **tan** **0** **.** **7** **5** **=** **36.86989765**

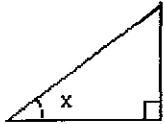
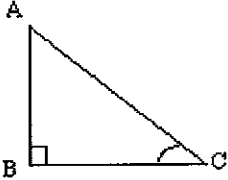
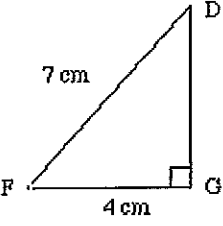
which rounds to  $36.9^\circ$

## Trigonometric Ratios

### Unit Test #42

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "Submit Test" button. Clicking the "Begin Test Again" button will clear all the answers.

<p><b>Q1:</b></p>	<p>O is the centre of the circle. Radius is 2</p>  <p>The length of BC is:</p>	<p>A. <math>\tan 60^\circ</math>                  B. <math>2 \tan 60^\circ</math>                  C. <math>4 \tan 60^\circ</math>                  D. <math>\frac{2}{\tan 60^\circ}</math></p>	<p><b>Answer 1:</b></p>	<input type="text"/>
<p><b>Q2:</b></p>	<p>If <math>\sin A = 0.4</math>, what is the size of angle A (to 1 d.p.)?</p>	<p>A. <math>23.6^\circ</math>                  B. <math>113.4^\circ</math>                  C. <math>23.5^\circ</math>                  D. <math>66.4^\circ</math></p>	<p><b>Answer 2:</b></p>	<input type="text"/>
<p><b>Q3:</b></p>	 <p>What is the length of AB (to 2 s.f.)?</p>	<p>A. 1.0 cm                  B. 2.0 cm                  C. 0.5 cm                  D. 1.5 cm</p>	<p><b>Answer 3:</b></p>	<input type="text"/>
<p><b>Q4:</b></p>	 <p>Find the size of angle CDF to nearest degree.</p>	<p>A. <math>60^\circ</math>                  B. <math>30^\circ</math>                  C. <math>35^\circ</math>                  D. <math>55^\circ</math></p>	<p><b>Answer 4:</b></p>	<input type="text"/>
<p><b>Q5:</b></p>	<p>The longest side of a right angled triangle is 20 cm and the opposite side to angle A is 16 cm.</p> <p>What is the value of the sine of angle A?</p>	<p>A. 1.25                  B. 0.8                  C. 0.75                  D. 0.25</p>	<p><b>Answer 5:</b></p>	<input type="text"/>

<p><b>Q6:</b></p>	 <p>If <math>\cos x^\circ = \frac{4}{5}</math>, the value of <math>\tan x^\circ</math> is (Hint: Use Pythagoras' Theorem)</p>	<p>A. 0.75 B. 0.6 C. 0.8 D. 1.25</p>	<p>Answer 6:</p>	<input type="text"/>
<p><b>Q7:</b></p>	 <p>Which side is the Hypotenuse?</p>	<p>A. AB B. BC C. AC D. None of these.</p>	<p>Answer 7:</p>	<input type="text"/>
<p><b>Q8:</b></p>	<p>In the triangle above, what name is given to side <b>AB</b> in relation to angle <b>C</b>?</p>	<p>A. adjacent B. opposite C. hypotenuse D. None of these.</p>	<p>Answer 8:</p>	<input type="text"/>
<p><b>Q9:</b></p>	<p>In the triangle above, which side is <b>adjacent</b> to angle <b>C</b>?</p>	<p>A. AB B. BC C. AC D. None of these.</p>	<p>Answer 9:</p>	<input type="text"/>
<p><b>Q10:</b></p>	 <p>What is the size of angle <b>DFG</b> to nearest degree?</p>	<p>A. 35° B. 30° C. 55° D. 60°</p>	<p>Answer 10:</p>	<input type="text"/>

Enter your initial and surname here:

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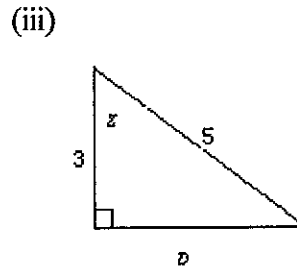
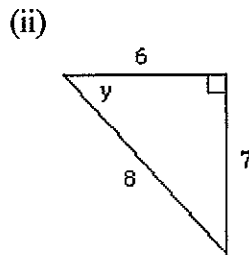
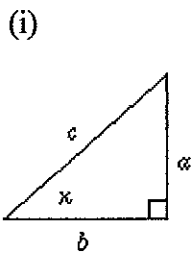
## Trigonometric Ratios

1. In the following triangles, give the size or letter of:

(a) The adjacent side

(b) The opposite side

(c) The hypotenuse



2. Use a calculator to find the following (give the ratios to 4 sig.fig. and the angles to 1 decimal place):

(a)  $\tan 61^\circ$

(b)  $\cos 24.3^\circ$

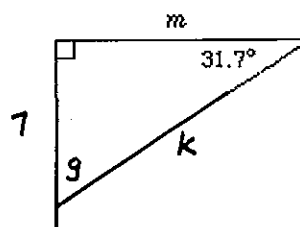
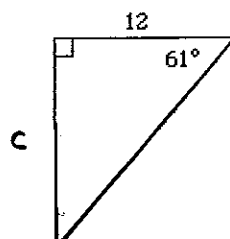
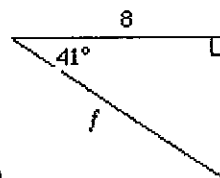
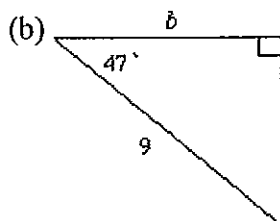
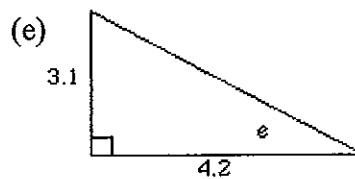
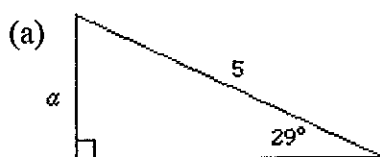
(c)  $\sin 87.6^\circ$

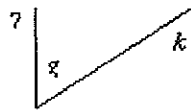
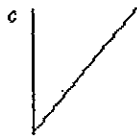
(d)  $x$  if  $\sin x = 0.5$

(e)  $y$  if  $\cos y = 0.3821$

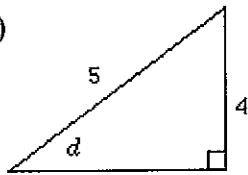
(f)  $z$  if  $\tan z = 2.34$

3. Use the trigonometric ratios to find the values of the sides and angles marked by letters. Round off your answers to 1 decimal place.

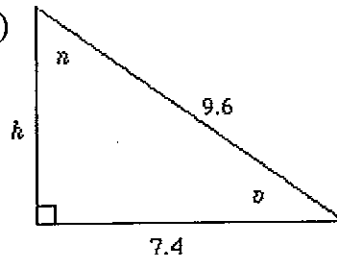




(d)



(h)

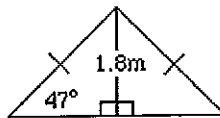


4. A man runs 70 m in a straight line up a hill that is inclined at an angle of  $23^\circ$ . How much has he risen by at the end of his run?

5. A rectangular paddock has a diagonal 150 m long. The diagonal makes an angle of  $30^\circ$  with the long side. Sketch a diagram of the paddock and find the length of the short side.

6. A vertical television mast is 100 m high and is held up by wires attached to the top of the mast and to the flat ground. The wires are 120 m long. What angle do they make with the mast?

7. Harry and Betty buy a tent. It has the cross-section shown in the diagram.



The centre pole is 1.8 m high.  
How wide is their tent?



## Trigonometric Ratios

1. (i)	(a) b	(b) a	(c) c
(ii)	(a) 6	(b) 7	(c) 8
(iii)	(a) 3	(b) p	(c) 5
2	(a) 1.804	(b) 0.9114	(c) 0.9991
	(d) $30.0^\circ$	(e) $67.5^\circ$	(f) $66.9^\circ$
3	(a) 2.4	(b) 6.1	(c) 21.6
	(d) $53.1^\circ$	(e) $36.4^\circ$	(f) 10.6

(g)  $g = 58.3^\circ$ ,  $m = 11.3$ ,  $k = 13.3$

(h)  $h = 6.1$ ,  $n = 50.4^\circ$ ,  $p = 39.6^\circ$

4. 27.4 metres (to 3 sig.fig.)

5. Short side = 75 metres

6.  $33.6^\circ$

7. 3.4 m (to 2 sig. fig.)

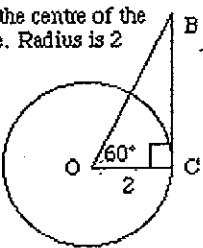
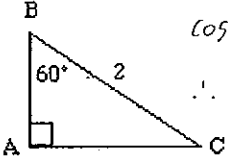
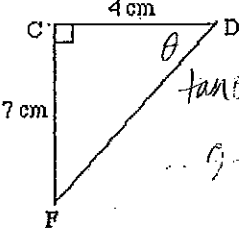
# ANS

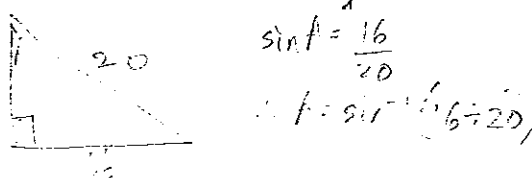
Notes | Test | Summary | FAQ | Exercise | Answers

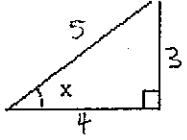
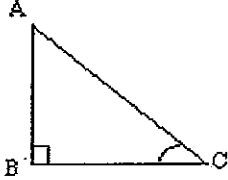
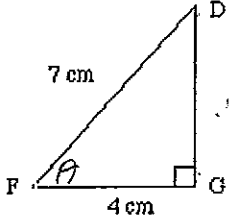
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<p>Q2:</p>	<p>If <math>\sin A = 0.4</math>, what is the size of angle A (to 1 d.p.)?  <math>A = \sin^{-1} 0.4</math></p>	<p>A. <math>23.6^\circ</math>            B. <math>113.4^\circ</math>            C. <math>23.5^\circ</math>            D. <math>66.4^\circ</math></p>	<p>Answer 2: <input checked="" type="checkbox"/> A ✓</p>
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<p>Q10:</p>	 <p>What is the size of angle DFG to nearest degree?</p> <p><i><math>\cos D = \frac{4}{7}</math></i> <i><math>\therefore \theta = \cos^{-1}(\frac{4}{7})</math></i></p>	<p>A. <math>35^\circ</math> B. <math>30^\circ</math> C. <math>55^\circ</math> D. <math>60^\circ</math></p>	<p>Answer 10:</p>	<p><input checked="" type="radio"/> C</p>

Enter your initial and surname here:

*V. Good effort!*

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