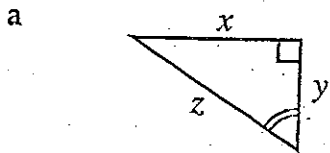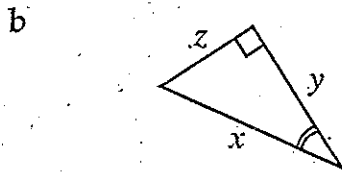# CHAPTER 9
Trigonometry of the right angled triangle

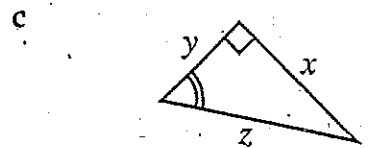## UNIT 1: Naming the sides of a right angled triangle

QUESTION 1   In each of the following triangles, state whether x, y and z are the opposite side, adjacent side or hypotenuse with reference to the angle marked.
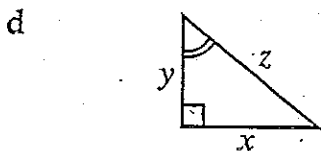
a

$x = $ ___ , $y = $ ___ , $z = $ ___

b

$x = $ ___ , $y = $ ___ , $z = $ ___

c

$x = $ ___ , $y = $ ___ , $z = $ ___

d

$x = $ ___ , $y = $ ___ , $z = $ ___

e

$x = $ ___ , $y = $ ___ , $z = $ ___

f
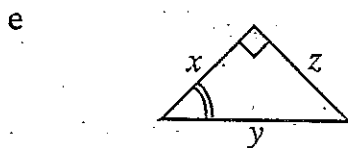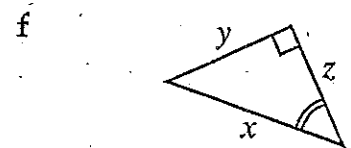
$x = $ ___ , $y = $ ___ , $z = $ ___

QUESTION 2   Name the sides in the following right angled triangles with reference to the angle marked.

a

$p = $ ___ , $q = $ ___ , $r = $ ___

b

$a = $ ___ , $b = $ ___ , $c = $ ___

c

$d = $ ___ , $e = $ ___ , $f = $ ___

d

$a = $ ___ , $b = $ ___ , $c = $ ___

e

$p = $ ___ , $q = $ ___ , $r = $ ___

f
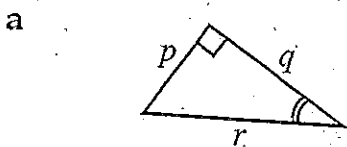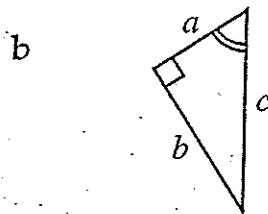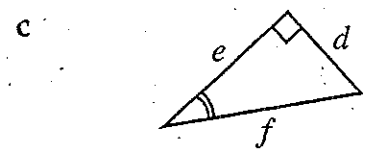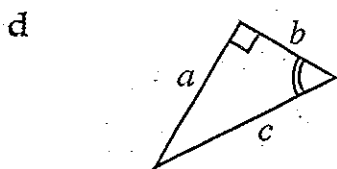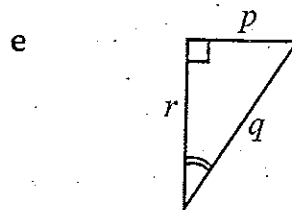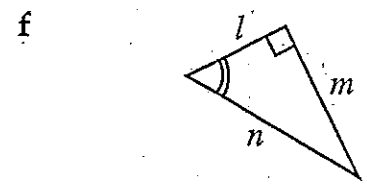
$l = $ ___ , $m = $ ___ , $n = $ ___

QUESTION 3   Name the hypotenuse in each triangle given below.

a

A

B   C

b

D

E   F

c

P

Q

R
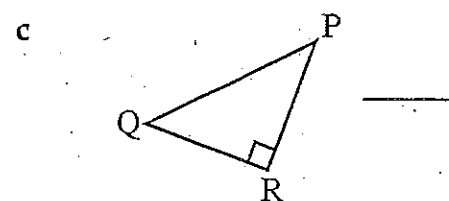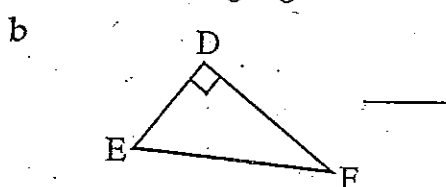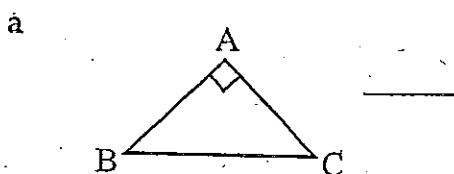
Trigonometry of the right angled triangle

UNIT 2: The trigonometric ratios

QUESTION 1   Write the trigonometric ratios for the following triangles.

a

_____

b

_____

c

_____

d

_____

e

_____

f

_____
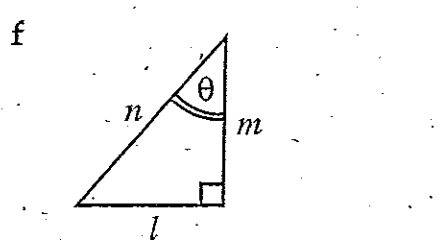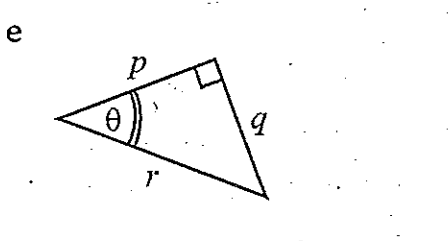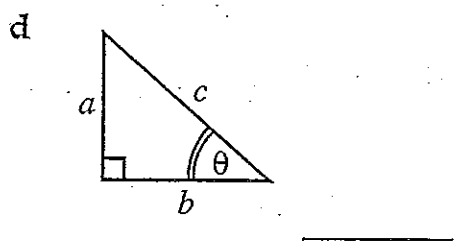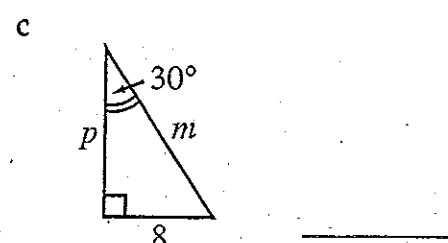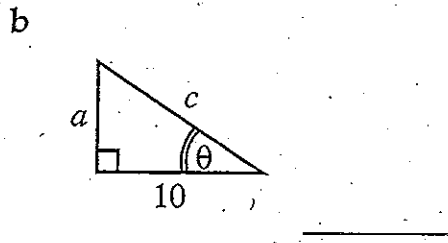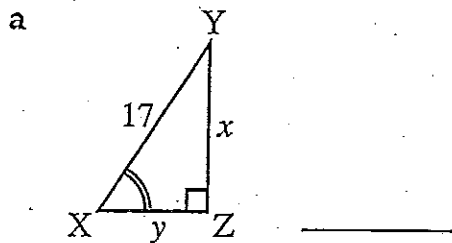
QUESTION 2   Find $\sin \theta$, $\cos \theta$ and $\tan \theta$ in the following triangles.

a

_____

b

_____

c

_____

d

_____

e

_____

f

_____
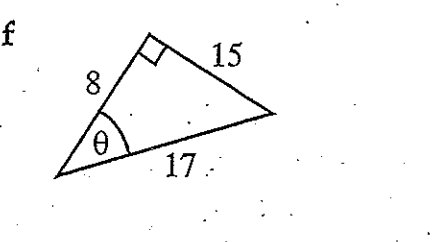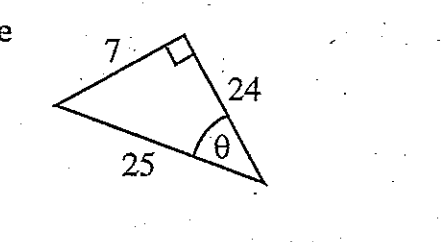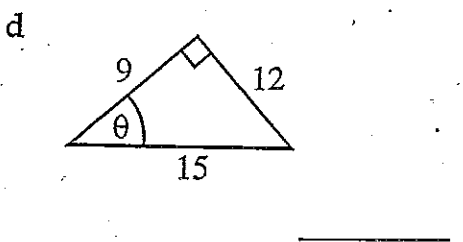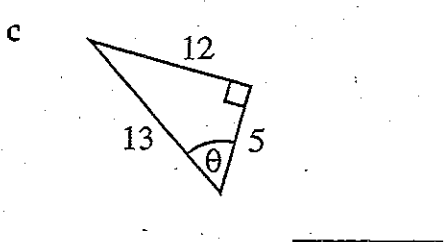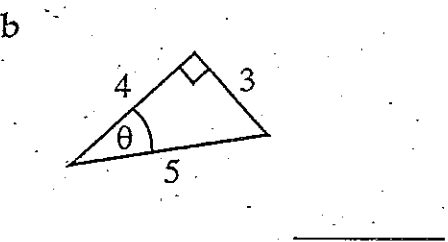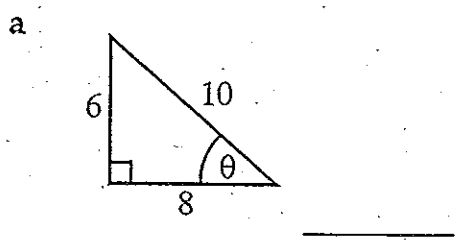
QUESTION 3   Use Pythagoras' theorem to find the unknown side and then find $\sin \theta$, $\cos \theta$ and $\tan \theta$.

a

_____

b

_____

c

_____
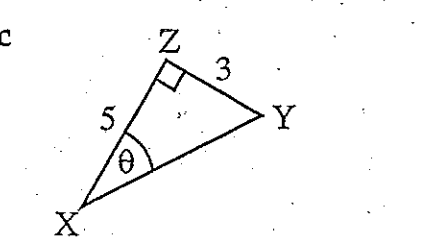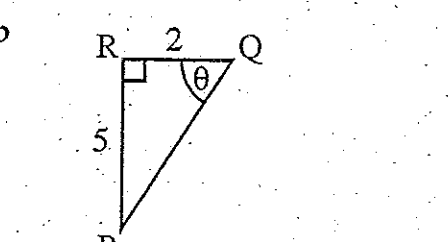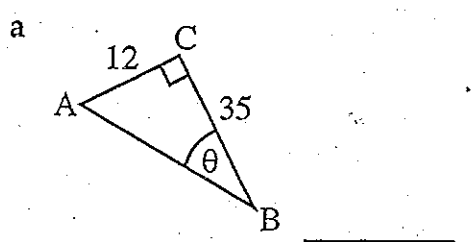
Trigonometry of the right angled triangle

UNIT 3: Use of a calculator in trigonometry

QUESTION 1   Find the value of the following correct to three decimal places.

a $\sin 69° =$ _____

b $\cos 60° =$ _____

c $\tan 21° =$ _____

d $\cos 82° =$ _____

e $\tan 28° =$ _____

f $\sin 58° =$ _____

g $\tan 31° =$ _____

h $\sin 35° =$ _____

i $\cos 43° =$ _____

QUESTION 2   Find the value of the following correct to three significant figures.

a $2.8 \sin 42° =$ _____

b $\tan 58°4' =$ _____

c $\sin 27°15' =$ _____

d $8 \cos 19° =$ _____

e $\sin 53°27' =$ _____

f $\cos 28°35' =$ _____

g $\sin 59°28' =$ _____

h $30.6 \cos 65°12' =$ _____

i $\tan 31°49' =$ _____

QUESTION 3   Find the value of the following correct to two decimal places.

a $\dfrac{\tan 58°}{6} =$ _____

b $\dfrac{\cos 63°}{5} =$ _____

c $\dfrac{14.3}{\sin 54°} =$ _____

d $\dfrac{\sin 39°41'}{4.7} =$ _____

e $\dfrac{\sin 54°28'}{2.5} =$ _____

f $\dfrac{18.6}{\cos 37°15'} =$ _____

g $\dfrac{\tan 25°54'}{8.25} =$ _____

h $\dfrac{\tan 38°29'}{8.6} =$ _____

i $\dfrac{359}{\tan 75°36'} =$ _____

QUESTION 4   A is an acute angle. Find its size to the nearest degree.

a $\sin A = 0.5736$ _____

b $\tan A = 0.7836$ _____

c $\cos A = 0.8126$ _____

d $\cos A = 0.5990$ _____

e $\sin A = 0.7587$ _____

f $\tan A = 1.491$ _____

g $\tan A = 2.5583$ _____

h $\cos A = 0.2935$ _____

i $\sin A = 0.9941$ _____

QUESTION 5   A is an acute angle. Find its size in degrees and minutes.

a $\sin A = 0.5$ _____

b $\cos A = 0.3568$ _____

c $\tan A = 1.326$ _____

d $\cos A = 0.4836$ _____

e $\tan A = 0.7983$ _____

f $\sin A = 0.4839$ _____

QUESTION 6   Find the size of the acute angle in degrees and minutes.

a $\cos A = \dfrac{1}{2}$ _____

b $\sin A = \dfrac{13}{18}$ _____

c $\tan A = \dfrac{15.7}{12.85}$ _____

d $\tan A = \dfrac{15}{22}$ _____

e $\cos A = \dfrac{8.5}{11.9}$ _____

f $\sin A = \dfrac{1}{2}$ _____

Trigonometry of the right angled triangle

UNIT 4: Finding an unknown side

QUESTION 1   Find the value of the unknown side correct to one decimal place.

a

30° / 13.8 cm   x

b

a / 60° / 8.9 cm

c

48° / 18.6 cm / m
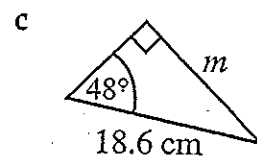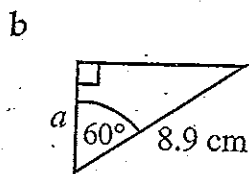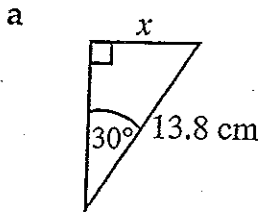
QUESTION 2   Find the value of x in the following triangles, correct to three decimal places.

a

x / 25° / 8 cm

b

x / 36° / 12.5 cm

c

24.8 cm / 60° / x
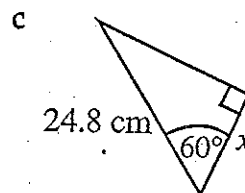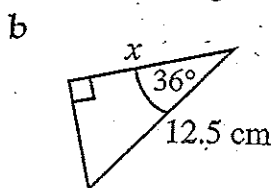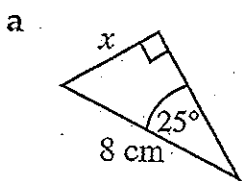
QUESTION 3   Find the value of the pronumeral correct to two decimal places.

a

a / 20° / 8.9 cm

b

y / 50° / 16.5 cm

c

m / 30° / 12.9 cm

d

d / 75° / 30.8 cm

e

25.6 cm / 30° / l

f

70° / 18.2 cm / c
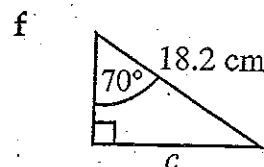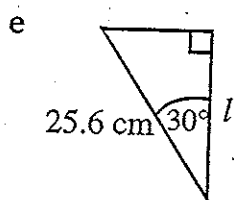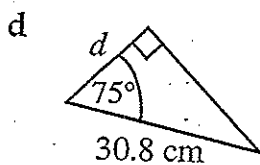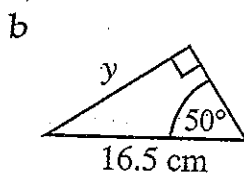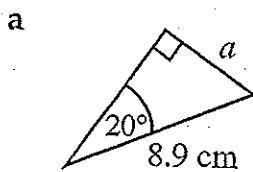
QUESTION 4   Michelle is flying a kite on a 55 metre string that makes an angle of 56° with the horizontal. Calculate the height of the kite to the nearest metre.

_____

_____

_____

QUESTION 5   In $\triangle ABC$, $\angle C = 90°$, $\angle B = 34.5°$ and $BC = 3.6$ cm. Find AB.

_____

_____

_____

Trigonometry of the
right angled triangle

UNIT 5: Finding the hypotenuse

QUESTION **1** Find the length of the hypotenuse correct to one decimal place.

a

5 cm
25°
h

b

60°
8 cm
h

c

12 cm
38°
h

_____   _____   _____
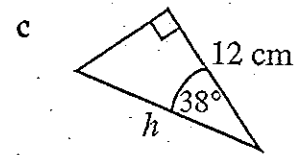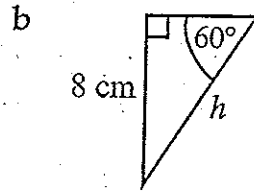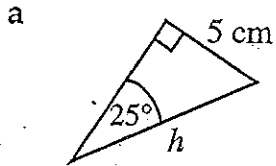
QUESTION **2** Find the length of the hypotenuse correct to one decimal place.

a

3 cm
40°
h

b

7 cm
70°
h

c

h
29°
10 cm

_____   _____   _____
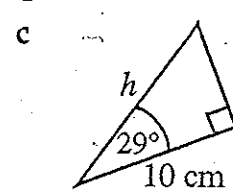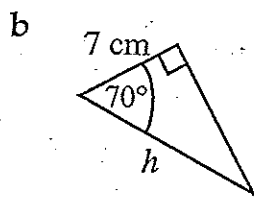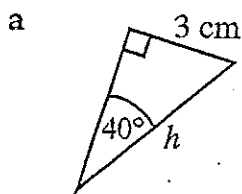
QUESTION **3** Find the length of the hypotenuse correct to one decimal place.

a

50°
h
24 cm

b

29 cm
34°
h

c

9 cm
55°
h

_____   _____   _____

d

13.5 cm
48°
h

e

21.6 cm
h
81°

f

15.7 cm
72°
h

_____   _____   _____
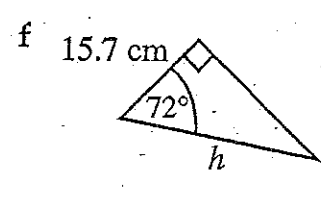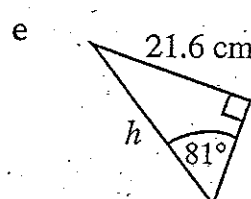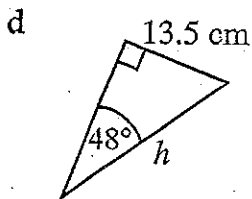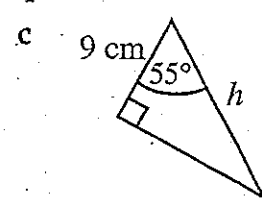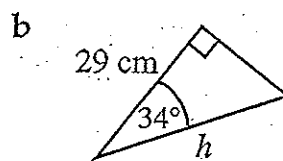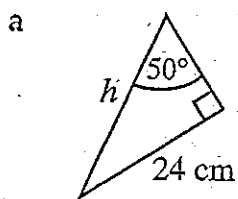
QUESTION **4** The diagram shown opposite is a trapezium with
DC parallel to AB. Calculate the length of BD and AB.

D   10 cm   C
60°
A   B

_____

_____

_____
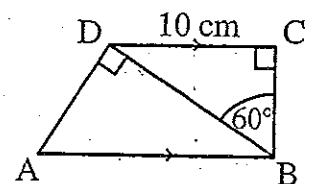
QUESTION **5** Find the length of the diagonal of a rectangle if the length of the rectangle is 10.7 cm
and the diagonal makes an angle of 30° with the longer side.

_____

_____

_____

# Trigonometry of the right angled triangle
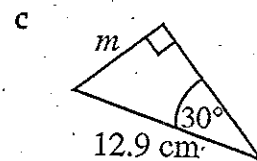
UNIT 6: Finding the unknown angle.

QUESTION 1   Find the size of the angle marked with a pronumeral.

a

7.5 θ

3.2

b

6.3 α 10.5

c

5.6

12.7 β
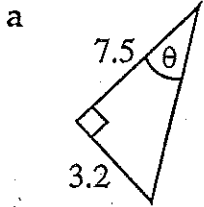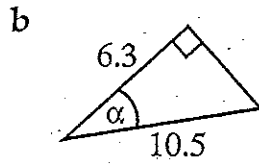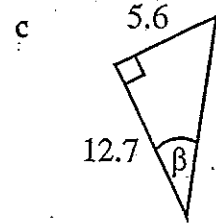
QUESTION 2   Find the size of the angle marked.

a

8.9 α 20.1

b

3.5   11.5 β

c

7.8   16.2 θ
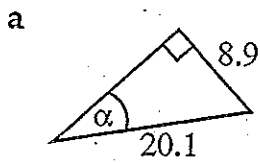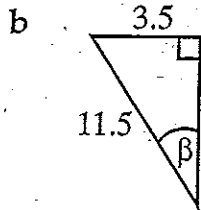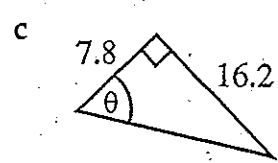
QUESTION 3   Find the size of the angle marked.

a

5.9 α 19.6

b

8.7   17.3 β

c

18.9 θ 28.75

d

3.8   15.6 β

e

20.8 α 33.6

f

11.7 θ 23.8
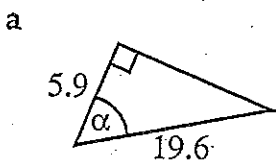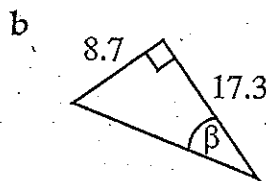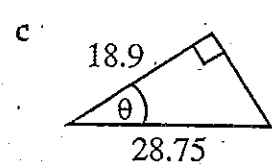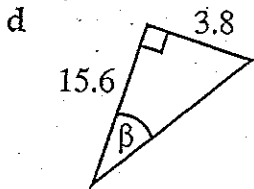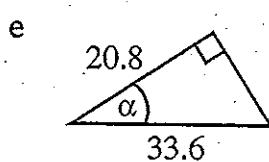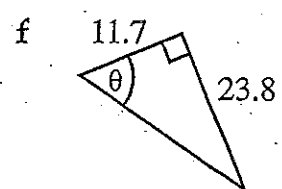
QUESTION 4   An 18 m ladder standing on level ground reaches 14 m up a vertical wall. Find the angle that the ladder makes with the ground (give your answer to the nearest degree).

QUESTION 5   *ABCD* is a rectangle with *AC* = 25 cm and *AD* = 14 cm. Find ∠*ACD* correct to the nearest degree.

Trigonometry of the right angled triangle

UNIT 7: Exact trigonometric ratios

QUESTION 1   Evaluate the following using the exact values of the trigonometric ratios.

a $\sin 30° =$ _____

b $\sin 60° =$ _____

c $\sin 30° \cdot \cos 45° =$ _____

d $\cos 60° =$ _____

e $\cos 45° =$ _____

f $\sin 45° \cdot \cos 45° =$ _____

g $\sin 45° =$ _____

h $\tan 60° =$ _____

i $\cos 30° =$ _____

j $\tan 30° =$ _____

k $\sin 30° \cdot \cos 60° =$ _____

l $\tan 45° =$ _____

m $\dfrac{\sin 45°}{\cos 45°} =$ _____

n $\dfrac{\sin 30°}{\cos 30°} =$ _____

o $\dfrac{\sin 60°}{\tan 60°} =$ _____

p $\dfrac{\cos 30°}{\cos 60°} =$ _____

q $\dfrac{\cos 60°}{\sin 45°} =$ _____

r $\dfrac{\sin 30°}{\sin 60°} =$ _____

QUESTION 2   Prove the following identities.

a $\dfrac{\sin 30°}{\cos 30°} = \tan 30°$

b $\dfrac{\sin 45°}{\cos 45°} = \tan 45°$

c $\dfrac{\sin 60°}{\cos 60°} = \tan 60°$

d $2\sin 30° \cos 30° = \sin 60°$

e $2\sin 45° \cos 45° = \sin 90°$

f $2\sin 60° \cos 60° = \sin 120°$

QUESTION 3   Prove the following results.

a $\sin 30° \times \cos 60° = \dfrac{1}{4}$

b $\sin 60° + \cos 30° = \sqrt{3}$

c $\sin 30° + \cos 30° + \cos 45° = \dfrac{1 + \sqrt{3} + \sqrt{2}}{2}$

QUESTION 4   A 12 m ladder standing on level ground makes an angle of 60° with the ground. How far up the vertical wall does it reach?

Trigonometry of the right angled triangle

UNIT 8: Angle of elevation, angle of depression and bearings

QUESTION 1

a The angle of elevation of the top of a tower AB is 58° from a point C on the ground at a distance of 200 metres from the base of the tower. Calculate the height of the tower to the nearest metre.

_____

_____

_____
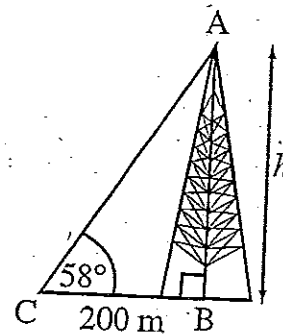
b A man 1.65 m tall is 18 metres away from a tower 25 m high. What is the angle of elevation of the top of the tower from his eyes?

_____

_____

_____

c From the top of a building 80 metres high, the angle of depression of a car parked on the ground is 52°. Find the distance of the car from the base of the building. (Write your answer correct to two decimal places.)

_____

_____

_____

QUESTION 2

a A ship sets out from a point A and sails due north to a point B, a distance of 150 km. It then sails due east to a point C. If the bearing of C from A is 048°37', find:

i   the distance BC.   ii   the distance AC.

_____   _____

_____   _____

b A ship leaves port for a destination 80 km east and 70 km north. In which direction should it sail?

_____

_____

c A ship starts from a port P, sails S46°W for a distance of 120 km. Find:

i   how far south of P it is.   ii   how far west of P it is.

_____   _____

_____   _____

_____   _____

Trigonometry of the right angled triangle

UNIT 9: Problems involving two right angled triangles

QUESTION **1**   From the diagram given opposite, find:

a   the length of side AB.

_____

_____

b   the length of side CD.

_____

_____
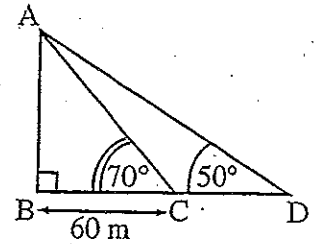
c   the angle DAC.

_____

_____

QUESTION **2**   In the diagram given opposite, calculate:

a   the length a (correct to one decimal place).

_____

_____

b   the size of angle θ.

_____

_____
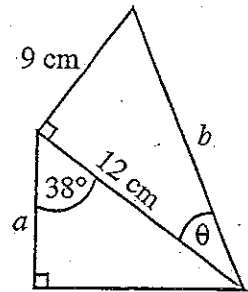
c   the length b.

_____

_____

QUESTION **3**   Find the value of the unknowns in the given diagram.

_____

_____

_____

_____
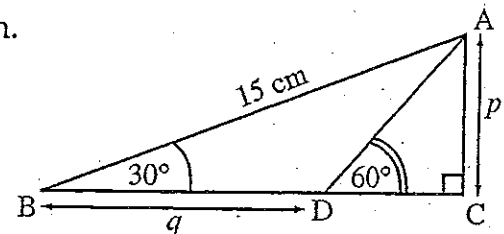
QUESTION **4**   The angle of elevation of the top of a cliff from a boat 600 m out to sea is 37°. If the boat then travels a further d metres out to sea, the angle of elevation of the cliff is now 25°. Find:

a   the height of the cliff above sea level to the nearest metre.

_____

_____

_____

b   the value of d to the nearest metre.

_____

_____

_____

Trigonometry of the right angled triangle

				Marks

1 Use your calculator to find $\sin 36°$ correct to two decimal places.

   Ⓐ 0.58   Ⓑ 0.57   Ⓒ 0.59   Ⓓ 0.81   `2`

2 Evaluate $12 \sin 85°$ correct to two decimal places.

   Ⓐ 12.05   Ⓑ 11.95   Ⓒ 1.05   Ⓓ 137.16   `2`

3 Find the value of $\dfrac{\sin 38° - \cos 55°}{\tan 36°}$ correct to one decimal place.

   Ⓐ 0.2   Ⓑ 0.5   Ⓒ 0.05   Ⓓ 0.1   `2`

4 If $\sin\theta = \dfrac{4}{7}$, calculate the size of the angle θ to the nearest degree.

   Ⓐ 55°   Ⓑ 30°   Ⓒ 35°   Ⓓ 45°   `2`

5 A 3 metre ladder leans against a building with its top reaching a height of 2.6 metres. What angle, to the nearest degree, does the ladder make with the wall?

   Ⓐ 35°   Ⓑ 40°   Ⓒ 30°   Ⓓ None of these   `2`

6 In the triangle ABC, the angle B is 90°, AB is 4 m and AC is 5 m. Find the size of angle A correct to the nearest degree.

   Ⓐ 37°   Ⓑ 53°   Ⓒ 39°   Ⓓ 27°   `2`

7 Jane is flying a kite on a 100 m string that makes an angle of 48° with the horizontal. How high is the kite above Jane's hand? Give your answer correct to the nearest metre.

   Ⓐ 65 m   Ⓑ 82 m   Ⓒ 78 m   Ⓓ 74 m   `2`

8 The diagonal of a rectangle makes an angle of 42° with one of the shorter sides. If the length of the rectangle is 12 cm, find the length of the diagonal correct to one decimal place.

   Ⓐ 15.8 m   Ⓑ 22.5 m   Ⓒ 10.5 m   Ⓓ 17.9 m   `2`

9 From the top of a tower the angle of depression of a boat is 30°. If the tower is 20 m high, how far is the boat from the foot of the tower?

   Ⓐ 40 m   Ⓑ $10\sqrt{3}$ m   Ⓒ $20\sqrt{2}$ m   Ⓓ $20\sqrt{3}$ m   `2`

10 If $\cos\theta = \dfrac{1}{2}$, find the size of angle θ.

   Ⓐ 30°   Ⓑ 60°   Ⓒ 45°   Ⓓ 55°   `2`

Total marks achieved for SECTION 1   `/20`

Trigonometry of the right angled triangle

Instructions for SECTION 2

- You have 20 minutes to answer ALL of Section 2
- Each question is worth 2 marks
- Attempt ALL questions
- Calculators may be used

Questions	Answers	Marks

Use your calculator to find correct to two decimal places:

1 $\tan 58°$. — 2

2 $\sin 63°$. — 2

3 $19.7 \cos 78°$. — 2

4 $\dfrac{28.67}{\sin 46°}$. — 2

5 $\dfrac{\sin 35° + \cos 35°}{\tan 34°}$. — 2

6 $\tan 48° - \sin 30° + \cos 73°$ — 2

Calculate the size of each angle to the nearest degree if:

7 $\cos\theta = \dfrac{4}{5}$. — 2

8 $\sin\theta = \dfrac{12}{13}$. — 2

9 $\tan\theta = 0.6781$. — 2

For the triangle ABC given opposite, calculate:

10 the value of h. — 2

11 the size of $\angle A$. — 2

12 the length of BC. — 2

From the diagram given opposite find:

13 the length of the side AB. — 2

14 the length of the side CD. — 2

15 the angle DAC. — 2
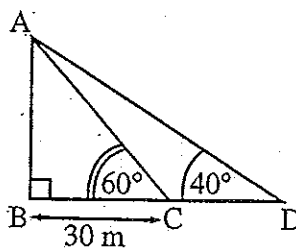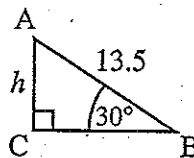
Total marks achieved for SECTION 2 /30

PAGE 1. 1 a x = opp., y = adj., z = hyp. b x = hyp., y = adj., z = opp. c x = opp., y = adj., z = hyp. d x = opp., y = adj., z = hyp. e x = adj., y = hyp., z = opp. f x = hyp., y = opp., z = adj. 2 a p = opp., q = adj., r = hyp. b a = adj., b = opp., c = hyp. c d = opp., e = adj., f = hyp. d a = opp., b = adj., c = hyp. e p = opp., q = hyp., r = adj. f l = adj., m = opp., n = hyp. 3 a BC b EF c PQ

PAGE 2. 1 a $\sin X = \dfrac{x}{17}, \cos X = \dfrac{y}{17}, \tan X = \dfrac{x}{y}$ b $\sin\theta = \dfrac{a}{c}, \cos\theta = \dfrac{10}{c}, \tan\theta = \dfrac{a}{10}$ c $\sin 30° = \dfrac{8}{m}, \cos 30° = \dfrac{p}{m}, \tan 30° = \dfrac{8}{p}$

d $\sin\theta = \dfrac{a}{c}, \cos\theta = \dfrac{b}{c}, \tan\theta = \dfrac{a}{b}$ e $\sin\theta = \dfrac{q}{r}, \cos\theta = \dfrac{p}{r}, \tan\theta = \dfrac{q}{p}$ f $\sin\theta = \dfrac{l}{n}, \cos\theta = \dfrac{m}{n}, \tan\theta = \dfrac{l}{m}$ 2 a $\sin\theta = \dfrac{6}{10}, \cos\theta = \dfrac{8}{10}, \tan\theta = \dfrac{6}{8}$

b $\sin\theta = \dfrac{3}{5}, \cos\theta = \dfrac{4}{5}, \tan\theta = \dfrac{3}{4}$ c $\sin\theta = \dfrac{12}{13}, \cos\theta = \dfrac{5}{13}, \tan\theta = \dfrac{12}{5}$ d $\sin\theta = \dfrac{12}{15}, \cos\theta = \dfrac{9}{15}, \tan\theta = \dfrac{12}{9}$ e $\sin\theta = \dfrac{7}{25}, \cos\theta = \dfrac{24}{25}, \tan\theta = \dfrac{7}{24}$

f $\sin\theta = \dfrac{15}{17}, \cos\theta = \dfrac{8}{17}, \tan\theta = \dfrac{15}{8}$ 3 a $AB = 37, \sin\theta = \dfrac{12}{37}, \cos\theta = \dfrac{35}{37}, \tan\theta = \dfrac{12}{35}$ b $PQ = \sqrt{29}, \sin\theta = \dfrac{5}{\sqrt{29}}, \cos\theta = \dfrac{2}{\sqrt{29}}, \tan\theta = \dfrac{5}{2}$

c $XY = \sqrt{34}, \sin\theta = \dfrac{3}{\sqrt{34}}, \cos\theta = \dfrac{5}{\sqrt{34}}, \tan\theta = \dfrac{3}{5}$

PAGE 3. 1 a 0.934 b 0.500 c 0.384 d 0.139 e 0.532 f 0.848 g 0.601 h 0.574 i 0.731 2 a 1.87 b 1.60 c 0.458 d 7.56 e 0.803 f 0.878 g 0.861 h 12.8 i 0.620 3 a 0.27 b 0.09 c 17.68 d 0.14 e 0.33 f 23.37 g 0.06 h 0.09 i 92.18 4 a 35° b 38° c 36° d 53° e 49° f 56° g 69° h 73° i 84° 5 a 30° b 69°06′ c 52°59′ d 61°05′ e 38°36′ f 28°56′ 6 a 60° b 46°14′ c 50°42′ d 34°17′ e 44°25′ f 30°

PAGE 4. 1 a 6.9 cm b 4.5 cm c 13.8 cm 2 a 3.381 cm b 10.113 cm c 12.400 cm 3 a 3.04 cm b $y = 12.64$ cm c $m = 6.45$ cm d 7.97 cm e 22.17 cm f 17.10 cm 4 a 46 m 5 4.37 m

PAGE 5. 1 a 11.8 cm b 9.2 cm c 15.2 cm 2 a 4.7 cm b 20.5 cm c 11.4 cm 3 a 31.3 cm b 35.0 cm c 15.7 cm d 18.2 cm e 21.9 cm f 50.8 cm 4 $BD = 11.5$ cm, $AB = 13.3$ cm 5 12.36 cm

PAGE 6. 1 a 23°06′ b 53°07′ c 23°47′ 2 a 26°17′ b 17°43′ c 64°17′ 3 a 72°29′ b 26°42′ c 48°54′ d 13°41′ e 51°45′ f 63°49′ 4 51° 5 34°

PAGE 7. 1 a $\dfrac{1}{2}$ b $\dfrac{\sqrt{3}}{2}$ c $\dfrac{1}{2\sqrt{2}}$ d $\dfrac{1}{2}$ e $\dfrac{1}{\sqrt{2}}$ f $\dfrac{1}{2}$ g $\dfrac{1}{\sqrt{2}}$ h $\sqrt{3}$ i $\dfrac{\sqrt{3}}{2}$ j $\dfrac{1}{\sqrt{3}}$ k $\dfrac{1}{4}$ l 1 m 1 n $\dfrac{1}{\sqrt{3}}$ o $\dfrac{1}{2}$

p $\sqrt{3}$ q $\dfrac{\sqrt{2}}{2}$ r $\dfrac{1}{\sqrt{3}}$ 2 Answers will vary 3 Answers will vary 4 10.39 m

PAGE 8. 1 a 320 m b 52°22′ c 62.50 m 2 a i $BC = 170.24$ km ii $AC = 226.90$ km b N48°49′E c i 83.36 km ii 86.32 km

PAGE 9. 1 a 164.85 m b 78.33 m c 20° 2 a 9.5 cm b 36°52′ c 15 cm 3 $p = 7.5$ cm, $q = 8.7$ cm 4 a 452 m b 369 m

PAGE 10. 1 C 2 B 3 D 4 C 5 C 6 A 7 D 8 D 9 D 10 B

PAGE 11. 1 1.60 2 0.89 3 4.10 4 39.86 5 2.06 6 0.90 7 37° 8 67° 9 34° 10 6.75 11 60° 12 11.69 13 51.96 m 14 31.93 m 15 20