BRIGIDINE RANDWICK

FORTITER ET SUAVITER

JULY 26 2013

Year 11

Task 2 EXAMINATION

Mathematics Extension 1

General Instructions

- Working time – 45 minutes
- Write using black or blue pen only
- Board-approved calculators may be used
- All necessary working should be shown in every question

Q1. Differentiate : $(4x^2-5)^4$

A. $4(4x^2-5)$ B. $4(4x^2-5)^3$

C. $32x(4x^2-5)$ D. $32x(4x^2-5)^3$

Q2. The equation $2x+3x^2=1$ has roots α and β. The value of $2\alpha+2\beta$ is :

$3x^2+2x-1=0$

A. 10 B. $-\frac{1}{3}$ C. $-\frac{2}{3}$ D. $-\frac{4}{3}$

Q3. At which points is the tangent of the curve $y=-2x^3+9x^2$ parallel to the line $y=12x+1$

A. (1,7) and (2,20)

B. (1,12) and (2,12)

C. (-1,7) and (-2,20)

D. (1,7) and (-2,12)

Show all necessary working for the following :

Q4. Differentiate the following with respect to x leaving your answer in surd form where applicable.

i. $y=\sqrt{4-x^2}$ 2

ii. $y=\dfrac{x^2}{x^2+1}$ 2

Q5. Find the equation of the normal of $y=x\sqrt{x+1}$ at $x=3$. 3

Q6. Given that $f(x)=x^2+2x$, find the value(s) of b for which $f'(b)=f(b)$. 2

Q7. i. Find the discriminant of $x^2-2kx+6k$. 1

ii. For what values of k is $x^2-2kx+6k$ positive definite? 2

Q8. Consider the quadratic equation $x^2-(k+2)x+2k=0$.

i. Find the value of k if the roots are reciprocals of each other. 1

ii. Show that the roots are always real for all values of k. 3

Q9. i. Sketch the parabola $(y-1)^2=x$, showing the intercepts. 1

ii. Write down the coordinates of the focus of the parabola. 2

Q10. The volume, V litres, of a tank after t minutes is given by $V=60+4t-t^2$.

i. At what time(s) will the tank be empty. 1

ii. Find the maximum volume of the tank. 2

iii. At what rate is the tank being emptied when $t=4$. 1

Q 11. Solve for x:

$$(\log_{10} x)^2-\log_{10} x-12=0 \qquad 3$$

Q12. The roots of the quadratic equation $px^2-x+q=0$ are -1 and 3. Find p and q. 2

Q13. Use the method of mathematical induction to show that if x is a positive integer then $(1+x)^n-1$ is divisible by x for all positive integers $n\geq 1$. 4

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(1) $\frac{d}{dx}(4x^2-5)^4$

$= 4(4x^2-5)^3 \times 8x$

$= 32x(4x^2-5)^3$

$\therefore D$

Q2 $3x^2+2x-1=0$

$2(\alpha+\beta)$

$2 \times \frac{-2}{3} = -\frac{4}{3}$ $\therefore D$

Q3 $y' = -6x^2+18x$

parallel $\therefore m = 12$

$-6x^2+18x = 12$

$0 = 6x^2-18x+12$

$0 = x^2-3x+2$

$0 = (x-2)(x-1)$

$x = 2$ $x = 1$

$y = 20$ $y = 7$

$\therefore A$

Q4 i)

$y = \sqrt{4-x^2}$

$y = (4-x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times -2x$

$y' = \frac{-x}{\sqrt{4-x^2}}$

2 marks: $\frac{-x}{\sqrt{4-x^2}}$

1 mark: $\frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times -2x$

ii)

$y = \frac{x^2}{x^2+1}$

$y' = \frac{2x(x^2+1)-2x(x^2)}{(x^2+1)^2}$

$y' = \frac{2x^3+2x-2x^3}{(x^2+1)^2}$

$y' = \frac{2x}{(x^2+1)^2}$

2 marks: $\frac{2x}{(x^2+1)^2}$

1 mark: $\frac{2x(x^2+1)-2x(x^2)}{(x^2+1)^2}$

Q5 $y = x\sqrt{x+1}$

$y = x(x+1)^{\frac{1}{2}}$

$y' = 1(x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}} \times x$

$y' = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

at $x = 3$

$m = \sqrt{3+1} + \frac{3}{2\sqrt{3+1}} = \frac{11}{4}$

$m_{Norm} = -\frac{4}{11}$

when $x = 3$

$y = 3\sqrt{3+1} = 6$

3 marks:

$y = -\frac{4}{11}x + 7\frac{1}{11}$

2 marks:

$m_{norm} = -\frac{4}{11}$

or 1 mistake

1 mark: $y' = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$

$m = -\frac{4}{11}$, $(3, 6)$

$y - 6 = -\frac{4}{11}(x - 3)$

$11y - 66 = -4x + 12$

$11y = -4x + 78$

$y = -\frac{4}{11}x + 7\frac{1}{11}$

or $4x + 11y - 78 = 0$

Q6 $f(x) = x^2 + 2x$

$f'(x) = 2x + 2$

$f'(b) = 2b + 2$

$f(b) = b^2 + 2b$

$\therefore f'(b) = f(b)$

$2b + 2 = b^2 + 2b$

$2 = b^2$

$\therefore b = \pm\sqrt{2}$

2marks: $b = \pm\sqrt{2}$

1mark: $2b + 2 = b^2 + 2b$

Q7 i) $\Delta = b^2 - 4ac$

$= (-2k)^2 - 4 \times 1 \times 6k$

$= 4k^2 - 24k$

1mark: $4k^2 - 24k$

ii) positive defenite : $\Delta < 0$

$4k^2 - 24k < 0$

$k^2 - 6k < 0$

$k(k - 6) < 0$

$0 < k < 6$

2marks: $0 < k < 6$

1mark: $\Delta < 0$

Q8

i) α , $\frac{1}{\alpha}$

$\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$1 = \frac{2k}{1}$

$k = \frac{1}{2}$

i) mark: $k = \frac{1}{2}$

ii) real roots $\Delta \geq 0$

$b^2 - 4ac \geq 0$

$(k+2)^2 - 4 \times 1 \times 2k \geq 0$

$(k+2)^2 - 8k \geq 0$

$k^2 + 4k + 4 - 8k \geq 0$

$k^2 - 4k + 4 \geq 0$

$(k - 2)^2 \geq 0$

since ( ) is squared then always ≥ 0 so roots are real for any k.

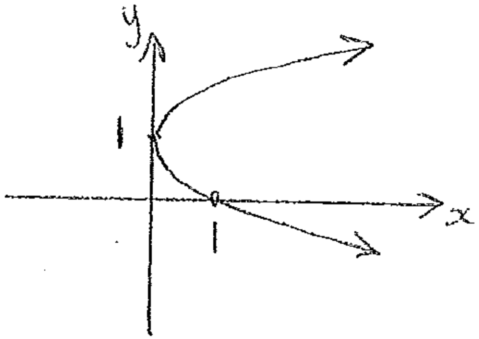
ii) 3marks see solution with a conclusion

2marks: $(k - 2)^2 \geq 0$

1mark: $\Delta \geq 0$

or $k^2 - 4k + 4$

Q9 i) $(y-1)^2 = x$



ii) $(y-1)^2 = x$

$\therefore 4a = 1$

$a = \frac{1}{4}$

$\therefore$ focus $= (\frac{1}{4}, 1)$

Q10 $V = 60 + 4t - t^2$

empty $\rightarrow V = 0$

$0 = 60 + 4t - t^2$

$t^2 - 4t - 60 = 0$

$t = \dfrac{4 \pm \sqrt{(4)^2 - 4 \times 1 \times -60}}{2 \times 1}$

$t = \dfrac{4 \pm \sqrt{256}}{2}$

$t = \dfrac{4 \pm 16}{2}$

1 mark: see solutions

2 marks: $(\frac{1}{4}, 1)$

1 mark: $a = \frac{1}{4}$

(wrong focal length, 1)

must be found.

1 mark: $t = 10$

$t = \frac{20}{2}$ or $\frac{-12}{2}$

$t = 10$ or $\cancel{-6}$

can't have negative time

$\therefore t = 10$

ii) Max Volume:

$t = -\frac{b}{2a} = -\frac{4}{2 \times -1}$

$= 2$

$\therefore V = 60 + 4 \times 2 - 2^2$

$\underline{\underline{V = 64}}$

iii) rate empty: $\frac{dV}{dt} = ?$

$\frac{dV}{dt} = 4 - 2t$

when $t = 4$

$\frac{dV}{dt} = 4 - 2 \times 4 = -4$

2 marks: $V = 64$

1 mark $t = 2$

1 mark: -4 L/min

or 4 L/min

Q11

$(\log_{10} x)^2 - \log_{10} x - 12 = 0$

let $M = \log_{10} x$

$M^2 - M - 12 = 0$

$(M-4)(M+3) = 0$

$M = 4 \qquad M = -3$

$\log_{10} x = 4 \qquad \log_{10} x = -3$

$\underline{\underline{x = 10^4}} \qquad \underline{\underline{x = 10^{-3}}}$

Q12 $px^2 - x + q = 0$

roots are $\alpha = -1, \beta = 3$

$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{-1}{p}$

$-1 + 3 = \frac{1}{p}$

$2 = \frac{1}{p}, \quad \underline{\underline{p = \frac{1}{2}}}$

$\alpha\beta = \frac{c}{a}$

$-1 \times 3 = \frac{q}{p}$

$q = -3p \qquad \therefore \underline{\underline{q = \frac{-3}{2}}}$

3 marks: $x = 10^4, 10^{-3}$

2 marks:

$\log_{10} x = 4, \log_{10} x = -3$

1 mark: $(m-4)(m+3) = 0$

2 marks: $p = \frac{1}{2}$ and $q = -\frac{3}{2}$

1 mark: $p = \frac{1}{2}$ or $q = -\frac{3}{2}$

or finding one letter then correctly attempting to find the other.

Q13 if $n = 1$

$(1+x)^1 - 1 = 1 + x - 1$

$= x$

which is divisible by x

so true for $n = 1$

Assume true for $n = k$

$(1+x)^k - 1 = xP$ where P is some positive integer

Prove true for $n = k+1$

$(1+x)^{k+1} - 1 = xQ$

(where Q is some integer)

$(1+x)^1 \times (1+x)^k - 1$

from assumption

$(1+x)^k = xP + 1$

$\therefore (1+x)(1+xP) - 1$

$1 + xP + x + x^2P - 1$

$x^2P + xP + x$

$x(xP + P + 1)$

which is divisible by x

$\therefore$ statement true proven by mathematical induction.

4 marks: see soln.

3 marks:

extensive progress

2 marks:

Some progress.

1 mark: proving true for $n = 1$