



SCEGGS Darlinghurst

2013

Preliminary Course  
Semester 1 Examination

# Mathematics Extension 1

## General Instructions

- Time allowed – 1 hour
- This paper has **two** sections
- Attempt **all** questions
- Write using black or blue pen
- Answer Section I on the multiple-choice answer sheet provided
- Answer Section II questions in the writing booklets provided
- Begin each question in a new writing booklet
- Draw all diagrams using a pencil and ruler
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

Total marks – 36

### Section I

4 marks

- Attempt Questions 1–4
- Allow about 6 minutes for this part

### Section II

32 marks

- Attempt Questions 5–7
- Allow about 54 minutes for this section

Question	Basic Arithmetic & Algebra	Equations	Trigonometry	TOTAL
1–4	/2	/1	/1	/4
5	/2	/3	/5	/10
6	/2	/6	/3	/11
7	/3	/3	/5	/11
<b>TOTAL</b>	<b>/9</b>	<b>/13</b>	<b>/14</b>	<b>/36</b>

## Section I – Multiple Choice

4 marks

Attempt Questions 1–4

Allow about 6 minutes for this section

Use the multiple-choice answer sheet provided.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:  $2 + 4 =$  (A) 2 (B) 6 (C) 8 (D) 9

A  B  C  D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A  B  C  D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A  B  C  D   
correct

1  $(2\sqrt{6} - \sqrt{2})^2 =$

- (A) 26
- (B)  $26 - 4\sqrt{3}$
- (C) 22
- (D)  $26 - 8\sqrt{3}$

2 The solution to  $|3x + 2| = 4x + 1$  is

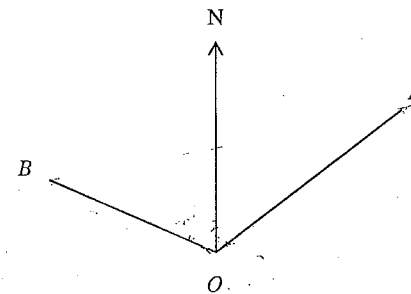
- (A)  $x = 1$  and  $x = -\frac{3}{7}$
- (B)  $x = 1$  only
- (C)  $x = -\frac{3}{7}$  only
- (D) no value of  $x$

3 The simplified form of  $12^n \times 4^{n+1}$  is

- (A)  $48^{2n+1}$
- (B)  $48^{n^2+n}$
- (C)  $2^{4n+1} \times 3^n$
- (D)  $4^{2n+1} \times 3^n$

4 The bearing of  $A$  from  $O$  is  $20^\circ$  and the bearing of  $B$  from  $O$  is  $290^\circ$ .

If  $A$  is 10 km from  $O$  and  $B$  is 7 km from  $O$ , find the bearing of  $B$  from  $A$ .



- (A)  $55^\circ$
- (B)  $75^\circ$
- (C)  $235^\circ$
- (D)  $255^\circ$

**End of Section II**

# Mathematics Extension 1

## Section II

32 marks

Attempt Questions 5–7

Allow about 54 minutes for this section

Answer the question in a writing booklet. Extra writing booklets are available.  
Answer each question in a SEPARATE writing booklet.

### Question 5 (10 marks)

- (a) Make
- $x$
- the subject of the formula

2

$$y = \frac{1-x}{3+x}$$

- (b) Solve
- $x^4 - 2x^2 - 15 = 0$

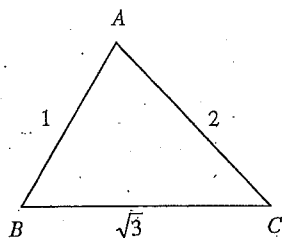
3

- (c) Given
- $\sin x = -\frac{3}{4}$
- and
- $\tan x > 0$
- , evaluate
- $\sec x$
- in surd form.

3

- (d) Find
- $\angle BCA$
- .

2



NOT  
TO  
SCALE

- Start a NEW writing booklet

### Question 6 (11 marks)

- (a) Solve simultaneously

3

$$\begin{aligned} 3x - y - 2 &= 0 \\ y &= 2x^2 - 5x + 4 \end{aligned}$$

- (b) Simplify fully

2

$$\frac{x^{-1} - y^{-1}}{x - y}$$

- (c) Solve

3

$$\frac{3-x}{x} \geq 1$$

- (d) Solve for
- $-180^\circ \leq \theta \leq 180^\circ$
- ,

3

$$\tan 2\theta = \frac{-1}{\sqrt{3}}$$

- Start a NEW writing booklet

**Question 7 (11 marks)**

- (a) Solve simultaneously

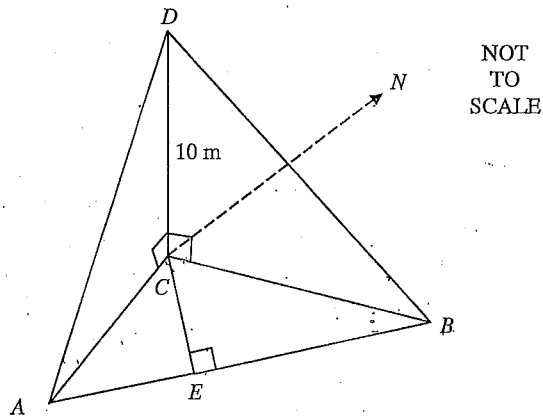
3

$$2a + b - 4c = 6$$

$$a + 3b + 2c = 7$$

$$a - b - 6c = 3$$

- (b)



$CD$  is a vertical flagpole of height 10 metres. It stands with its base on horizontal ground.

$A$  and  $B$  are points on the ground due South and East of  $C$  respectively.

The angle of elevation of  $D$  is  $45^\circ$  from  $A$  and  $30^\circ$  from  $B$ .

$E$  is the foot of the perpendicular from  $C$  to  $AB$ .

- (i) Show  $BC = 10\sqrt{3}$  m. 1
- (ii) By finding  $AC$ , show  $\angle ABC = 30^\circ$ . 2
- (iii) Find the angle of elevation of  $D$  from  $E$ , correct to the nearest degree. 2

Question 7 continues on the next page

**Question 7 (continued)**

- (c) Consider the expression  $x^6 - 1$ .

Factorise this expression fully

- (i) as a difference of 2 squares. 1
- (ii) as a difference of 2 cubes. 1
- (iii) Hence express  $x^4 + x^2 + 1$  in factorised form. 1

End of paper

Mathematics Extension 1



Centre Number

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Student Number

Section I  
4 marks

Write your Student Number at the top of this page.

Multiple Choice Answer Sheet

- |          |   |   |                                  |   |                                  |   |                                  |   |                                  |   |
|----------|---|---|----------------------------------|---|----------------------------------|---|----------------------------------|---|----------------------------------|---|
| Question | 1 | A | <input type="radio"/>            | B | <input type="radio"/>            | C | <input type="radio"/>            | D | <input checked="" type="radio"/> | ✓ |
|          | 2 | A | <input type="radio"/>            | B | <input checked="" type="radio"/> | C | <input type="radio"/>            | D | <input type="radio"/>            | ✓ |
|          | 3 | A | <input checked="" type="radio"/> | B | <input type="radio"/>            | C | <input type="radio"/>            | D | <input type="radio"/>            | X |
|          | 4 | A | <input type="radio"/>            | B | <input type="radio"/>            | C | <input checked="" type="radio"/> | D | <input type="radio"/>            | ✓ |

B 1  
E 1  
T 1

(3)

Student Number: \_\_\_\_\_

$$5.a) \quad y(3+x) = 1-x \quad \begin{matrix} x+xy = 1-3y \\ x(1+y) = 1-3y \\ x = \frac{1-3y}{1+y} \end{matrix}$$

$$3y + xy = 1-x \quad \begin{matrix} xy = 1-x-3y \\ \text{Rearrange } x \text{ to same side} \\ \text{for first mark.} \end{matrix} \quad X$$

$$\therefore x = \frac{1-x-3y}{y}$$

$$\therefore x = \frac{1-x}{y} - 3$$

x can't be in two places.

X  
(B0)

b) ~~x^4 - 2x^2 - 15 = 0~~

$(x^2-5)(x^2+3) = 0$  ✓  
What conclusion do you make about  $x^2+3=0$ ?  
~~x = ±√5 or x = ±i√3~~  $\therefore x = \pm\sqrt{5}$

(E2)

c)  $\sin x = -\frac{3}{4}$  (Quad 3)  
 $\tan x > 0$

$$4^2 = (-3)^2 + y^2$$

$$y^2 = \sqrt{4^2 - (-3)^2}$$

$$y = \pm\sqrt{7} \text{ (only positive for pythagoras' rule length.)}$$

state: since pythagoras' rule length must be positive

$$\therefore \sec x = -\frac{4}{\sqrt{7}}$$

(T3)

Student Number: \_\_\_\_\_

$$5. d) \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{(\sqrt{3})^2 + 2^2 - 1^2}{2 \times \sqrt{3} \times 2}$$

$$\cos C = \frac{3 + 4 - 1}{4\sqrt{3}}$$

$$\cos C = \frac{3}{2\sqrt{3}}$$

~~XXXXXX~~ ✓

$$C = \cos^{-1}\left(\frac{3}{2\sqrt{3}}\right)$$

$$= 30$$

$$\therefore \angle BCA = 30^\circ$$

(T2)

Student Number: \_\_\_\_\_

$$6. a) 3x - (2x^2 - 5x + 4) - 2 = 0$$

$$3x - 2x^2 + 5x - 4 - 2 = 0$$

$$-2x^2 + 8x - 6 = 0$$

$$-x^2 + 4x - 3 = 0$$

$$(-x+1)(x-3) = 0 \quad \checkmark$$

$$\therefore x = 1 \text{ or } x = 3$$

$$y = 2(1)^2 - 5(1) + 4$$

$$= 2 - 5 + 4$$

$$y = 2(3)^2 - 5(3) + 4$$

$$= 2 \times 9 - 15 + 4$$

$$\therefore y = \text{or } 1$$

$$= \text{or } 7$$

~~XXXX~~ ✓

$$\therefore \{x=1, y=1\}, \{x=3, y=7\}$$

$$b) \frac{x^{-1} - y^{-1}}{x - y}$$

$$= \left(\frac{1}{x} - \frac{1}{y}\right) \div (x - y) \quad \checkmark$$

$$= \left(\frac{y-x}{xy}\right) \times \frac{1}{x-y} \quad \checkmark$$

$$= -\frac{1}{xy}$$

Student Number: \_\_\_\_\_

$$6.c) \frac{3-x}{x} \geq 1 \quad (D: x \neq 0)$$

$$x(3-x) \geq x^2 \quad \checkmark$$

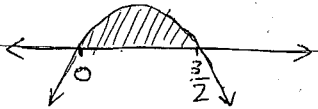
$$3x - x^2 \geq x^2$$

$$3x - x^2 - x^2 \geq 0$$

$$3x - 2x^2 \geq 0 \quad \checkmark$$

$$x(3-2x) \geq 0$$

$$x = 0, x = \frac{3}{2}$$



$$\therefore 0 \leq x \leq \frac{3}{2} \quad \checkmark$$

$$d) \tan 2\theta = \frac{1}{\sqrt{3}}$$

Student Number: \_\_\_\_\_

$$6.d) \tan 2\theta = \frac{1}{\sqrt{3}} \quad (\text{quad } 2 \text{ \& } 4) \leftarrow$$

$$2\theta = 30^\circ$$

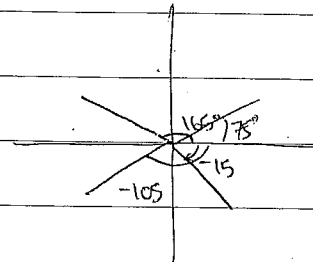
$$-360^\circ \leq 2\theta \leq 360^\circ \quad \checkmark$$

$$2\theta = 30^\circ, (180^\circ - 30^\circ), (180^\circ + 30^\circ), (360^\circ - 30^\circ), -30^\circ, -(180^\circ - 30^\circ),$$

Cancel  
not in  
quad 2 & 4  
= 30, 150, 210, 330, -30, -150, -210, -330

$$\theta = \cancel{15^\circ}, \cancel{75^\circ}, \cancel{105^\circ}, \cancel{165^\circ}, -15^\circ, \cancel{-105^\circ}, \cancel{-165^\circ}$$

$$\therefore \theta = 165^\circ, -15^\circ, -105^\circ \quad \checkmark \quad ! \quad \& \quad 75^\circ$$



Student Number: \_\_\_\_\_

$$7a) 2a + b - 4c = 6 \quad (1)$$

$$a + 3b + 2c = 7 \quad (2)$$

$$a - b - 6c = 3 \quad (3)$$

$$(1) - (2) - (3)$$

$$-b = -4$$

$$b = 4$$

sub in (3)

$$a - 4 - 6c = 3$$

$$a - 6c = 3 + 4$$

$$a = 7 + 6c \quad (3)$$

sub in (2)

$$7 + 6c + 3(4) + 2c = 7$$

$$8c = 7 - 7 - 12$$

$$8c = -12$$

$$\therefore c = -\frac{3}{2}$$

$$a = 7 + 6\left(-\frac{3}{2}\right)$$

$$\therefore a = -2$$

$$\therefore a = -2, b = 4, c = -\frac{3}{2}$$

Student Number: \_\_\_\_\_

$$7.b) i) \tan 30^\circ = \frac{10}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{BC}$$

$$BC = \frac{10}{\frac{1}{\sqrt{3}}}$$

$$\therefore BC = 10\sqrt{3} \text{ m}$$

$$ii) \tan 45^\circ = \frac{10}{AC}$$

$$1 = \frac{10}{AC}$$

$$\therefore AC = 10 \text{ m}$$

$$\tan \angle ABC = \frac{10}{10\sqrt{3}}$$

$$\tan \angle ABC = \frac{1}{\sqrt{3}}$$

$$\angle ABC = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore \angle ABC = 30^\circ$$

$$iii) \sin 30^\circ = \frac{CE}{10\sqrt{3}}$$

$$10\sqrt{3} \sin 30^\circ = CE$$

$$CE = 8.66 \text{ m}$$

In  $\triangle DCE$

$$\tan \angle DEC = \frac{CD}{CE}$$

$$\tan \angle DEC = \frac{10}{8.66} \quad (\text{PTO})$$



$$7.b) iii) \angle DEC = \tan^{-1} \left( \frac{10}{8.66} \right)$$

5

$$\angle DEC = 49^{\circ} 6' 23.78''$$

$\therefore$  angle of elevation of D from E

$$= 49^{\circ} \text{ (nearest degree)}$$

$$c) x^6 - 1$$

$$i) (x^2 - 1)(x^3 + 1) = (x^2 - 1)(x^3 + 1)$$

$$ii) (x^3 - 1)(x^3 + 1)$$

$$iii) x^2(x^2 + 1) = -1 \quad X$$

1

$$i) (x^3 - 1)(x^3 + 1) = (x+1)(x^2 - x + 1)(x-1)(x^2 + x + 1)$$

$$ii) (x^2 - 1)(x^4 + x^2 + 1) = (x+1)(x-1)(x^4 + x^2 + 1)$$

$$iii) (x+1)(x^2 - x + 1)(x-1)(x^2 + x + 1) = (x+1)(x-1)(x^4 + x^2 + 1)$$

$$(x^2 - x + 1)(x^2 + x + 1) = x^4 + x^2 + 1$$