



SCEGGS Darlinghurst

2013

Preliminary Course
Semester 1 Examination

Mathematics

Outcomes Assessed: P2, P3 and P4
Task Weighting: 25%

General Instructions

- Time allowed – 1½ hours
- This paper has two sections
- Attempt all questions and show all working
- Write using black or blue pen
- Answer Section I on the multiple-choice answer sheet provided
- Answer Section II in the writing booklets provided
- Begin each question in a new writing booklet
- Marks will be deducted for careless or badly arranged work
- Approved scientific calculators and mathematical templates may be used

Total marks – 50

Section I

5 marks

- Attempt Questions 1–5
- Allow about 10 minutes for this section

Section II

45 marks

- Attempt Questions 6–8
- Allow about 80 minutes for this section

Section	Questions	Algebra & Equations	Trigonometry	Geometry	Marks
I – Multiple Choice	1–5	/3	/2		/5
II	6	/8	/4	/2	/14
	7	/6	/6	/3	/15
	8	/4	/12		/16
TOTAL		/21	/24	/5	/50

Centre Number

Student Number

1. What is 4.09784 correct to three significant figures.

- (A) 4.09
(B) 4.10
(C) 4.097
(D) 4.098

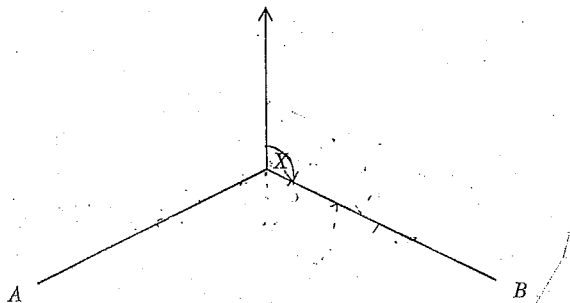
2. Which of the following is equal to $\frac{1}{2\sqrt{5}-\sqrt{3}}$?

- (A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$
(B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$
(C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$
(D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

3. $\frac{1}{4} + \frac{x}{8} =$

- (A) $\frac{1+x}{8}$
(B) $\frac{2+x}{8}$
(C) $\frac{1+x}{12}$
(D) $\frac{8+x}{32}$

- 4 Alex leaves point X and walks on a bearing of 230° . Brooke leaves point X and walks on a bearing of $S70^\circ E$.



What is the size of $\angle AXB$?

- (A) 50°
 (B) 120°
 (C) 160°
 (D) 300°
- 5 What is the solution to the equation

$$2\cos\beta = -\sqrt{3} \text{ for } 0^\circ \leq \beta \leq 360^\circ?$$

- (A) $\beta = 30^\circ$ or 330°
 (B) $\beta = 60^\circ$ or 300°
 (C) $\beta = 150^\circ$ or 210°
 (D) $\beta = 120^\circ$ or 240°

End of Section I

Section II

45 marks

Attempt Questions 6–8

Allow about 80 minutes for this section

Answer each question in a writing booklet. Extra writing booklets are available.
 Start each question in a NEW book.

Question 6 (14 marks)

- (a) Evaluate, correct to two decimal places. 1

$$\frac{195.32}{4.6^2 + 5.79}$$

- (b) Factorise fully.

(i) $16 - x^2$ 1

(ii) $3x^3 - 24$ 2

- (c) Simplify $\frac{3}{x+3} - \frac{1}{x-3}$ 2

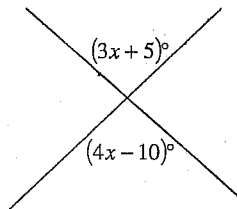
- (d) Write $x^{-\frac{1}{2}}$ without negative or fractional indices. 1

- (e) Find the exact value of $\tan 300^\circ$. 2

Question 6 continues on the next page

Question 6 (continued)

(f)



2

Find the value of x , giving reasons.

(g)

Simon is standing on the ground. He measures the angle of elevation to the top of a 150m tower to be 87° .

2

Calculate the distance to the nearest metre between Simon and the base of the tower.

(h)

Susan bought a Prada bag in Myer, and later sold it on Ebay for \$2016, making a 12% profit. How much did she originally pay for the bag?

1

End of Question 6

Start a NEW writing booklet

Question 7 (15 marks)

(a) Solve $17 - 4n > 33$ and graph the result on a number line.

2

(b) Find the exact ratio of $\cos 30^\circ + \cos 60^\circ$.

1

(c) Show that $(\sqrt{15} - 2\sqrt{3})^2 + 12\sqrt{5}$ is rational.

2

(d) If $\cos \theta = \frac{2}{5}$ and $\tan \theta < 0$, find the exact ratio of $\cot \theta$.

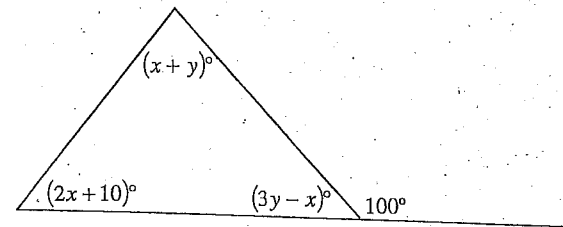
2

(e) Solve $|2x + 1| = 3x - 2$

2

(f) Find x and y .

3



(g) Simplify $\frac{1 + \cos \theta}{1 - \sin \theta} \times \frac{1 - \cos \theta}{1 + \sin \theta}$

3

End of Question 7

- Start a NEW writing booklet

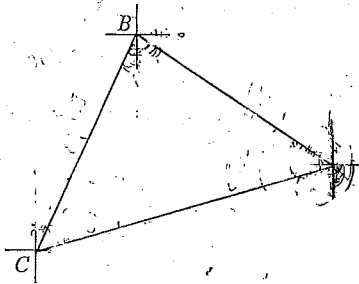
Question 8 (16 marks)

- (a) Jenny began to solve a quadratic equation using the quadratic formula. She wrote correctly 2

$$x = \frac{3 \pm \sqrt{9 + 20}}{2}$$

What was the equation?

- (b) Amy travelled 12 km on a bearing of 290° from her home (A) to Bondi Beach (B). She then travelled on a bearing of 210° to her friend Cathy's place (C). After spending some time at Cathy's place she travelled 20 km straight back home.



- (i) Copy the diagram above, adding the information given and find the size of angle ABC . 1
- (ii) Calculate the bearing of Cathy's place from Amy's home (to the nearest degree). 3
- (iii) Calculate the total distance Amy travelled. Give your answer correct to the nearest km. 2

Question 8 continues on the next page

Question 8 (continued)

- (c) Simplify $\frac{1}{1 - \frac{1}{m}}$ 2

- (d) Solve $\tan^2 \theta + 3 \tan \theta - 4 = 0$, for $0^\circ \leq \theta \leq 360^\circ$ 3

- (e) Show that $\frac{2 \cos^3 \theta - \cos \theta}{\sin \theta \cos^2 \theta - \sin^3 \theta} = \cot \theta$ 3

End of paper

Mathematics

Centre Number

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Student Number

Section I
5 marks

Write your Student Number at the top of this page.

Multiple Choice Answer Sheet

Question

- | | | | | | | | | | |
|---|---|-----------------------|---|----------------------------------|---|----------------------------------|---|----------------------------------|---|
| 1 | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> | ✓ |
| 2 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input checked="" type="radio"/> | ✓ |
| 3 | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> | ✓ |
| 4 | A | <input type="radio"/> | B | <input checked="" type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> | ✓ |
| 5 | A | <input type="radio"/> | B | <input type="radio"/> | C | <input checked="" type="radio"/> | D | <input type="radio"/> | ✓ |

A T
3 2

Student Number: _____

6.a) $\frac{195 \cdot 32}{2 \cdot 16 \cdot 5 \cdot 79}$

$\therefore = 7 \cdot 28 (2 \text{ d.p.})$ X

b)i) $16 - x^2$

$= (4+x)(4-x)$ ✓

ii) $3x^3 - 24$

$= 3(x^3 - 8)$ ✓

$= 3(x^3 - 2^3) = 3(x-2)(x^2+2x+4)$

$\therefore = 3(x-2)(x^2+2x+4)$ ✓

c) $\frac{3}{x+3} - \frac{1}{x-3}$

$= \frac{3(x-3) - (x+3)}{(x+3)(x-3)}$ ✓

$= \frac{3x-9-x-3}{x^2-9}$

$= \frac{2x-12}{x^2-9}$ ✓

$\therefore = \frac{2(x-6)}{(x+3)(x-3)}$

d) $x^{-\frac{1}{2}}$

$= \frac{1}{x^{\frac{1}{2}}}$

$\therefore = \frac{1}{\sqrt{x}}$ ✓

6.e) $\tan 300^\circ$ (Quadrant 4)

relative $\angle = \tan 60^\circ$

$\tan 60^\circ = \sqrt{3}$

$\therefore \tan 300^\circ = -\sqrt{3}$ ✓✓

f) $(4x - 10^\circ) = (3x + 5)^\circ$ (vertically opposite \angle s are equal)

$4x - 3x = 5 + 10$

$\therefore x = 15^\circ$ ✓

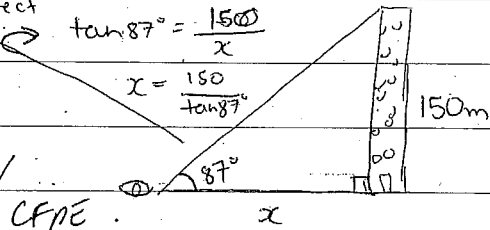
g) let x = distance between Simon & the base of the tower

$\tan 87^\circ = \frac{x}{150}$ ~~incorrect~~

$\tan 87^\circ = \frac{150}{x}$

$x = 150 \times \tan 87^\circ$

$\therefore x = 2862 \text{ m (nearest m)}$ ✓

h) let x = original price

$x(1 + 0.12) = \$2016$

$x(1 - 0.12) = \$2016$

$x = \frac{\$2016}{1 - 0.12}$

$\therefore x = \$1800$ ✓

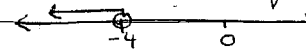
7.a) $17 - 4n > 33$

$-4n > 33 - 17$

$-4n > 16$

$n < \frac{16}{4}$

$\therefore n < -4$ ✓✓



b) $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\cos 30^\circ + \cos 60^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$

$\therefore \cos 30^\circ + \cos 60^\circ = \frac{\sqrt{3} + 1}{2}$ ✓

c) $(\sqrt{5} - 2\sqrt{3})^2 + 12\sqrt{5}$

$= 15 - 4\sqrt{45} + 12 + 12\sqrt{5}$

$= 27 - 4\sqrt{5 \times 9} + 12\sqrt{5}$

$= 27 - 12\sqrt{5} + 12\sqrt{5}$ ✓✓

$\therefore = 27$ (rational number)

$$7.d) \cos \theta = \frac{2}{5} \quad (\text{odd } \theta)$$

$$\tan < 0$$

$$5^2 = 2^2 + x^2$$

$$x = \sqrt{5^2 - 2^2}$$

$$x = \sqrt{21}$$

$$\therefore \cot \theta = \frac{-2}{\sqrt{21}} \quad \checkmark \checkmark$$

$$e) |2x+1| = 3x-2$$

$$-(2x+1) = 3x-2 \quad \text{or} \quad 2x+1 = 3x-2$$

$$2x+1 = -3x+2$$

$$2x-3x = -2-1$$

$$2x+3x = 2-1$$

$$-x = -3$$

$$5x = 1$$

$$x = 3 \quad \checkmark$$

$$x = \frac{1}{5}$$

$$\text{Test } x=3$$

$$\text{Test } x = \frac{1}{5}$$

$$\text{LHS} = |2(3)+1|$$

$$\text{LHS} = |2(\frac{1}{5})+1|$$

$$= |6+1|$$

$$= |\frac{2}{5}+1|$$

$$= 7$$

$$= \frac{7}{5}$$

$$\text{RHS} = 3(3)-2$$

$$\text{RHS} = 3(\frac{1}{5})+2$$

$$= 9-2$$

$$= \frac{3}{5}-2$$

$$= 7$$

$$= -\frac{7}{5}$$

$$\therefore \text{LHS} = \text{RHS} \quad \therefore x=3$$

$$\therefore \text{LHS} \neq \text{RHS}$$

FINAL SOLUTION

$$7.f) (x+y) + (2x+10) = 100$$

$$3x + y + 10 = 100$$

$$3x + y = 90$$

$$y = 90 - 3x$$

~~$$3(90-3x) - x = 180 - 100$$~~

~~$$3(90-3x) - x = 180 - 100$$~~

~~$$270 - 9x - x = 80$$~~

~~$$-10x = 80 - 270$$~~

~~$$-10x = -190$$~~

~~$$x = \frac{-190}{-10}$$~~

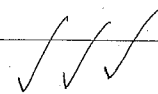
~~$$\therefore x = 19$$~~

~~$$y = 90 - 3(19)$$~~

~~$$y = 90 - 57$$~~

~~$$\therefore y = 33$$~~

~~$$\therefore x = 19, y = 33$$~~



$$\begin{aligned}
 7.g) \quad & \frac{1+\cos\theta}{1-\sin\theta} \times \frac{1-\cos\theta}{1+\sin\theta} \\
 & = \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\sin\theta)(1+\sin\theta)} \\
 & = \frac{1-\cos^2\theta}{1-\sin^2\theta} \\
 & = \frac{\sin^2\theta}{\cos^2\theta}
 \end{aligned}$$

$$\therefore = \tan^2\theta$$

$$\begin{aligned}
 8.a) \quad x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 x &= \frac{3 \pm \sqrt{9 + 20}}{2}
 \end{aligned}$$

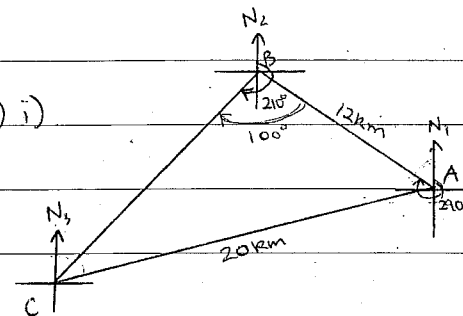
$$a = 1$$

$$b = -3$$

$$c = -5$$

$$\therefore x^2 - 3x - 5 = 0$$

b) i)



$$\begin{aligned}
 \angle N_1AB &= 360^\circ - 290^\circ \\
 &= 70^\circ
 \end{aligned}$$

$$\begin{aligned}
 \angle N_2BA &= 180^\circ - 70^\circ \\
 &= 110^\circ
 \end{aligned}$$

$$\begin{aligned}
 \angle ABC &= \angle N_2BC - \angle N_2BA \\
 &= 210^\circ - 110^\circ
 \end{aligned}$$

$$\therefore \angle ABC = 100^\circ$$

$$8.b) ii) \frac{\sin \angle BCA}{12} = \frac{\sin 100^\circ}{20}$$

$$\angle BCA = \frac{12 \times \sin 100^\circ}{20}$$

$$\therefore \angle BCA = 36^\circ 13'$$

$$180^\circ - (100^\circ + 36^\circ 13') = \angle BAC$$

$$\angle BAC = 43^\circ 47'$$

$$\text{Bearing} = 290^\circ \text{ (BAC ~~170^\circ~~)}$$

$$\text{Bearing} = 290^\circ - \angle BAC$$

$$= 290^\circ - 43^\circ 47'$$

$$= 246^\circ 13'$$

\therefore Bearing of Cathy's Place from Amy's home

$$= 246^\circ \text{ (nearest degree)}$$

$$iii) BC^2 = AB^2 + AC^2 - (2 \times AB \times AC \times \cos A)$$

$$BC^2 = 12^2 + 20^2 - (2 \times 12 \times 20 \times \cos 44^\circ)$$

$$BC = \sqrt{544 - (480 \times \cos 44^\circ)}$$

$$14.096 \text{ km}$$

$$\therefore BC = 14 \text{ km (nearest km)}$$

$$\therefore \text{Distance} = 12 + 20 + 14.096 \dots \text{ km} + 12 + 20$$

$$= 46 \text{ km (nearest km)}$$

$$8.c) \frac{1}{1 - \frac{1}{m}}$$

$$= \frac{1}{\frac{m-1}{m}}$$

$$= \frac{1}{\frac{m-1}{m}}$$

$$= 1 \div \frac{m-1}{m}$$

$$= 1 \times \frac{m}{m-1}$$

$$\therefore = \frac{m}{m-1}$$

$$d) \tan^2 \theta + 3 \tan \theta - 4 = 0$$

$$\text{let } \tan \theta = u$$

$$u^2 + 3u - 4 = 0$$

$$(u+4)(u-1) = 0$$

$$u = -4, u = 1$$

$$\tan \theta = -4 \text{ or } \tan \theta = 1$$

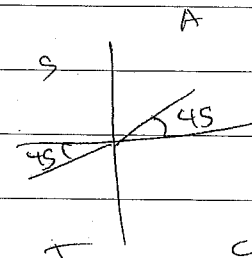
$$\theta = -75^\circ 58'$$

$$\theta = 45^\circ$$

$$\text{since } 0^\circ \leq x \leq 360^\circ$$

$$\therefore \theta = 45^\circ, (180^\circ + 45^\circ)$$

$$\therefore \theta = 45^\circ, 225^\circ$$



you must locate θ

in the correct

quadrants

Student Number:

$$8.e) \frac{2\cos^3\theta - \cos\theta}{\sin\theta\cos^2\theta - \sin^3\theta} = \cot\theta$$

$$\text{LHS} = \frac{\cos\theta(2\cos^2\theta - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$$

$$= \frac{\cos\theta((\cos^2\theta + (1 - \sin^2\theta)) - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)} \quad \checkmark$$

$$= \frac{\cos\theta(\cos^2\theta - \sin^2\theta + 1 - 1)}{\sin\theta(\cos^2\theta - \sin^2\theta)}$$

$$= \frac{\cos\theta(\cancel{\cos^2\theta} - \cancel{\sin^2\theta})}{\sin\theta(\cancel{\cos^2\theta} - \cancel{\sin^2\theta})}$$

$$= \frac{\cos\theta}{\sin\theta} \quad \checkmark$$

$$= \cot\theta \quad \checkmark$$

$$\therefore \text{LHS} = \text{RHS} \quad \checkmark$$