Sydney Girls' High School

2013
MATHEMATICS EXTENSION 1

YEAR 11- Preliminary Course
HALF YEARLY ASSESSMENT

Time Allowed: 60 minutes (plus 5 mins reading time)

TOPICS: Real Functions, Trigonometry & Harder Advanced Mathematics Topics.

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 60 marks

NAME:.. TEACHER:...L.............................

QUESTION 1 (15 marks)   Marks

a) Express $\dfrac{12\sin 60°}{\cos 45°}$ in simplest exact form   2

b) Sketch the following showing all relevant features:

   i) $y = x^3 + 3$ 2

   ii) $y = -\sqrt{16 - x^2}$ 2

   iii) $y = \dfrac{1}{|2-x|}$ 2

c) Given $f(x) = 7x^2 - 6x - 4$ determine whether the function is odd, even or neither.   1

d) Solve the following inequality

   $$\frac{2}{x-1} \geq 1$$ 3

e) Simplify $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta}$ 3

QUESTION 2 (15 marks)

a) If $\sec\theta = \dfrac{8}{5}$ and θ is acute, find the exact value of $\operatorname{cosec}\theta$ and $\tan\theta$ 3

b) State the domain of the following:

$$f(x) = \sqrt{x-3} - \sqrt{5-x}$$ 2

c) A function is defined by the following:

$$f(x) = \begin{cases} 0 & \text{if } x \le -2 \\ -1 & \text{if } -2 < x < 0 \\ x & \text{if } x \ge 0 \end{cases}$$

    i)   Sketch the curve   3

    ii)   Find $f(-2) + f(-1) + f(0)$ 2

d)   i) Expand and Simplify $(a+b)^4$ 1

    ii) Hence expand and simplify $(x^2+3)^4$ 2

e) Find the value of x if 4^x equals one quarter of 2^{88}.   2

QUESTION 3 (15 marks)

a) State the domain and the range of $y = \dfrac{1}{\sqrt{1-x^2}}$ 2

b) On the same number plane shade the region given by the two conditions

$$x^2 + y^2 \le 9 \quad \text{and} \quad 2x + y - 2 > 0.$$ 3

c) Solve the following for $0° \le \theta \le 360°$

    i) $\tan\theta = \sqrt{3}$ 2

    ii) $2\sin^2\theta - 3\sin\theta + 1 = 0$ 2

d)   i)   Show $\dfrac{x+4}{x+3} = 1 + \dfrac{1}{x+3}$ 1

    ii)   Sketch the graph of $y = \dfrac{x+4}{x+3}$ showing all relevant features.   3

e) Eliminate θ given $x = 5\sin^2\theta$, $y = 4\cos\theta$ and form an equation in terms of x and y.   2

a)   i)   Sketch the graph of $y = \cos x$ for $0° \leq x \leq 360°$.   2

     ii) On the same axes, sketch $y = \sec x$ for $0° \leq x \leq 360°$.   2

b) Ricky and Delta depart from the same location O.
    Ricky drives in a direction 050° at a speed of 80km/h.
    Delta leaves 15minutes after Ricky but travels in a direction SE of O at a speed of 100km/h.

   i)   How far does Ricky travel in 45 minutes?   1
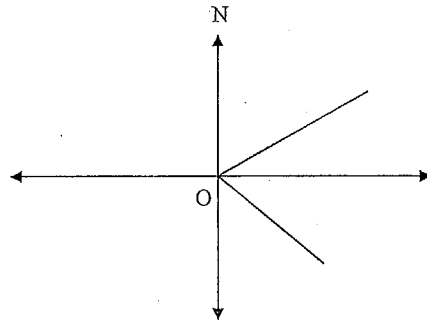
    ii) Copy and complete the diagram given above showing the position of Ricky
         and Delta, 30 minutes after Delta's departure.   1

    iii) Hence, find the distance between them at this time (to 2 significant figures).   2

    iv) What is the bearing of Ricky as seen from Delta?   2

c) A, B are two points 600m apart on the ground and D is the top of a tower.
    The angles DAB and DBA are 59° and 54° respectively.
    The angle of elevation of D from A is 5°.

    i)   Copy the diagram below and label all information given.   1
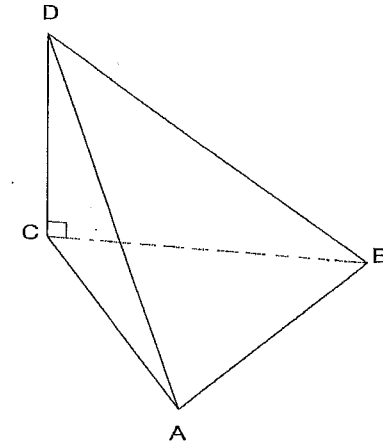
    ii) Show that the height 'h' of the tower is given by :

    $$h = \frac{600 \sin 5°. \sin 54°}{\sin 67°}$$ 3
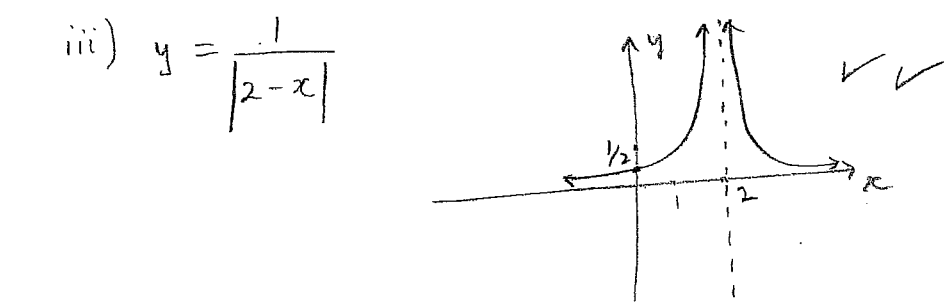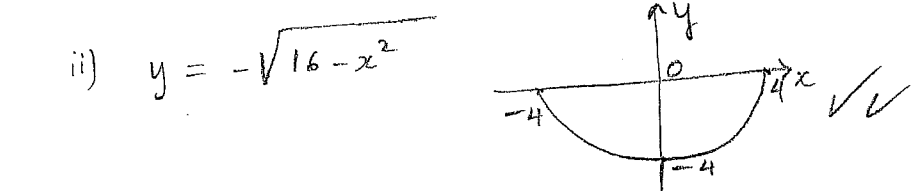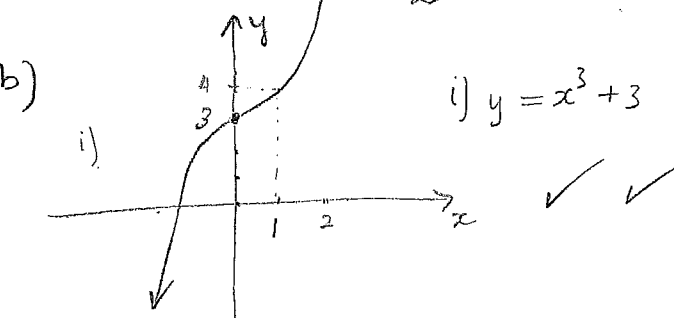
    iii) Hence, find the height of the tower to the nearest metre.   1
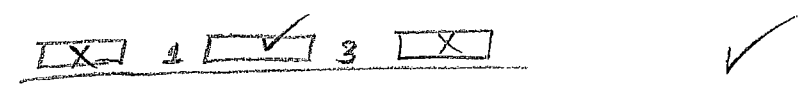
THE END

Question 1

a) $\dfrac{12\sin 60°}{\cos 45°} = \dfrac{12 \times \frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}}$ ✓ $= \dfrac{12\sqrt{3}}{\sqrt{2}} = \dfrac{12\sqrt{6}}{2} = 6\sqrt{6}$ ✓

b)

i)

i) $y = x^3 + 3$ ✓ ✓

ii) $y = -\sqrt{16 - x^2}$

 ✓ ✓

iii) $y = \dfrac{1}{|2-x|}$

 ✓ ✓

c) $f(x) = 7x^2 - 6x - 4$

$f(-x) = 7(-x)^2 - 6(-x) - 4$

$\qquad = 7x^2 + 6x - 4$

$\qquad \neq f(x)$ Neither odd Nor even ✓

$\qquad \neq -f(x)$

d) $\dfrac{2}{x - 1} \geqslant 1$

• $x - 1 = 0 \quad \therefore \quad x = 1$

• $\dfrac{2}{x-1} = 1 \quad \therefore \quad 2 = x - 1 \quad \therefore \quad x = 3$

$\boxed{✗} \quad 1 \quad \boxed{✓} \quad 3 \quad \boxed{✗}$ ✓

Try $x = 0 \quad \therefore \quad \dfrac{2}{-1} \geqslant 1$ Wrong

Try $x = 2 \quad \therefore \quad \dfrac{2}{2-1} \geqslant 1$ True

Try $x = 4 \quad \therefore \quad \dfrac{2}{3} \geqslant 1$ Wrong

✓✓

$\therefore$ Final Solution : $\boxed{1 < x \leqslant 3}$

e) Simplify: $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta}$

$\dfrac{\frac{1}{\cos\theta}}{\cos\theta} - \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$ ✓

$\dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1 - \sin^2\theta}{\cos^2\theta}$ ✓

$= \dfrac{\cos^2\theta}{\cos^2\theta} = 1$ ✓

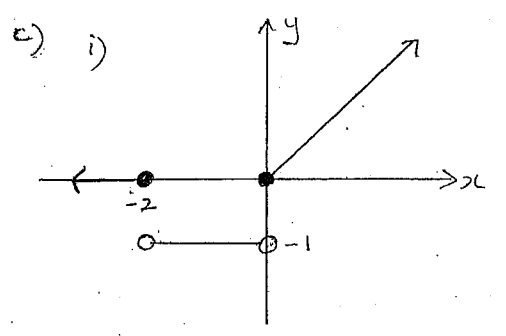**2 a)**

$\sec\theta = \dfrac{8}{5}$

$x = \sqrt{8^2 - 5^2}$

$= \sqrt{39}$
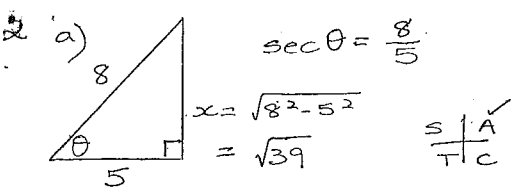
$\therefore \quad \text{cosec}\,\theta = \dfrac{8}{\sqrt{39}}$

$\tan\theta = \dfrac{\sqrt{39}}{5}$

b) $\quad x - 3 \geq 0$

$\quad x \geq 3 \quad$ and

$\quad 5 - x \geq 0$

$\quad -x \geq -5$

$\quad x \leq 5$

$\therefore \quad 3 \leq x \leq 5 \quad$ Domain

c) i)

ii) $f(-2) + f(-1) + f(0)$

$= 0 - 1 + 0$

$= -1$

d)

$\qquad 1 \qquad\qquad n = 0$

$\qquad 1 \quad 1 \qquad\quad n = 1$

$\qquad 1 \quad 2 \quad 1 \qquad n = 2$

$\quad 1 \quad 3 \quad 3 \quad 1 \qquad n = 3$

$\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad n = 4$

i) $(a+b)^4 = a^4 + 4a^3 b + 6a^2 b^2$
$\qquad\qquad + 4ab^3 + b^4$

ii) $(x^2 + 3)^4 = (x^2)^4 + 4(x^2)^3 \cdot$
$\qquad + 6(x^2)^2 (3)^2 + 4(x^2)(3)^3$
$\qquad + 3^4$

$= x^8 + 12x^6 + 54x^4 + 108x^2$
$\qquad + 81$

e) $\quad 4^x = \dfrac{1}{4} \times 2^{88}$

$2^{2x} = \dfrac{1}{2^2} \times 2^{88}$

$2^{2x} = 2^{86}$
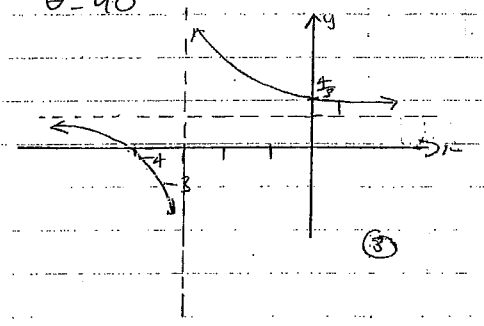
$2x = 86$

$x = 43$

Solutions Question 3 Ext 1 Yr 11

a) $\quad y = \sqrt{\dfrac{1}{1 - x}}$

$\quad D: -1 \leq x \leq 1 \qquad R: y \geq 1$ ②

b)

 ③
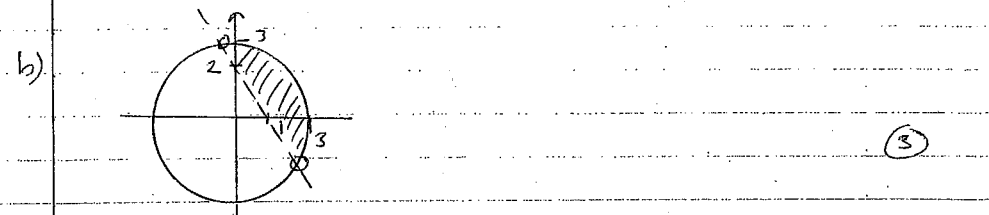
c) i) $\tan\theta = \sqrt{3}$
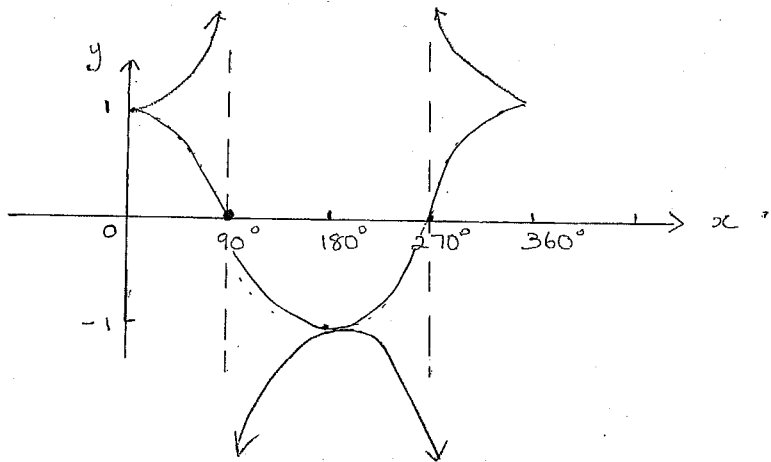

$\theta = 60°,\ 240°$ ②

ii) $2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\sin\theta = \dfrac{1}{2} \qquad$ or $\qquad \sin\theta = 1$

$\theta = 30°,\ 150° \qquad\qquad \theta = 90°$ ②

d) i) $\text{LHS} = \dfrac{x+4}{x+3}$

$= \dfrac{x + 3 + 1}{x + 3}$

$= 1 + \dfrac{1}{x+3}$ ①

$= \text{RHS}$

ii)

 ③

e) $\quad x = 5\sin^2\theta \qquad\qquad y = 4\cos\theta$

$\quad \sin^2\theta = \dfrac{x}{5} \qquad\qquad y^2 = 16\cos^2\theta$

$\qquad\qquad\qquad\qquad\qquad \cos^2\theta = \dfrac{y^2}{16}$

$\cos^2\theta + \sin^2\theta = 1$

$\dfrac{y^2}{16} + \dfrac{x}{5} = 1 \quad \Rightarrow \quad 5y^2 + 16x = 80$ ②

T. a) i) $y = \cos x$
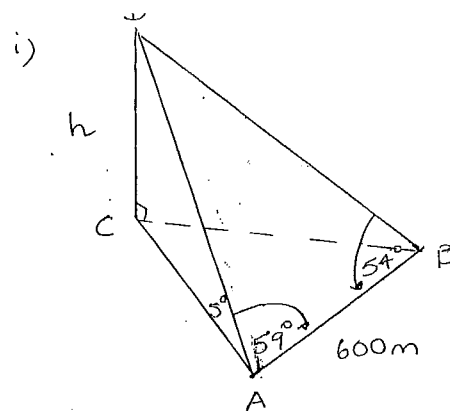

ii) $y = \sec x$.

b) i) Ricky 45 mins : $\frac{3}{4} \times 80 \text{km/hr} = 60 \text{km}$

∴ distance travelled $= 60 \text{km}$

ii)
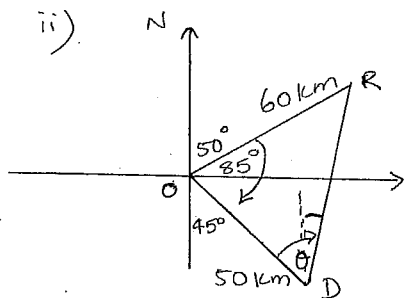
Delta $= \frac{1}{2} \times 100 \text{km/hr}$
$= 50 \text{km}$

iii) $RD^2 = 50^2 + 60^2 - 2(50)(60) \cos 85°$

$RD = \sqrt{5577.065}$

∴ distance $RD = 75 \text{km}$.

iv) $\dfrac{75}{\sin 85°} = \dfrac{60}{\sin \theta}$

$\sin \theta = \dfrac{60 \times \sin 85°}{75}$

∴ $\theta \doteq 53°$

∴ bearing
is $53 - 45°$
$= 008° T$ or $N8°E$

i)

$\angle ADB = 180 - (59 + 54)$
$= 67°$

ii) In $\triangle DAB$, $\dfrac{DA}{\sin 54°} = \dfrac{600}{\sin 67°}$

$DA = \dfrac{600 \times \sin 54°}{\sin 67°}$

In $\triangle DCA$, $\sin 5° = \dfrac{DC}{DA}$

∴ $DC = DA \cdot \sin 5°$

$DC = \dfrac{600 \times \sin 54° \times \sin 5°}{\sin 67°}$

∴ $h = \dfrac{600 \times \sin 5° \times \sin 54°}{\sin 67°}$

iii) Height $= 46 \text{ m}$ (nearest m)