Sydney Girls High School

YEAR 11

MATHEMATICS

Half Yearly Examination 2013

Time Allowed: 60 minutes + 5 minutes reading.

Total Marks: 60

Topics: Basic Arithmetic and Algebra, Factorisation, Equations, Plane Geometry and Special Quadrilaterals

Instructions:

- Attempt ALL questions
- There are *4* questions, each worth *15* marks.
- Show all necessary working. Full marks may not be awarded for careless or incomplete working.
- Begin each question on a new page.
- Diagrams are NOT to scale

Name:__________ __________ **Teacher:** __________

Question 1 *(15 marks)* (Start new page)

a) Simplify $\sqrt{18}+\sqrt{112}-\sqrt{28}$ 2

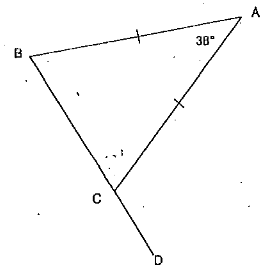
b) Express $0.1\dot{1}\dot{4}$ as simplified fraction. 2

c) Solve $x^2+3x-4=0$ 2

d) Solve $\dfrac{x-5}{3}-\dfrac{x+1}{4}=5$ 3

e) Find integers a and b such that $(5-\sqrt{2})^2=a-b\sqrt{2}$ 3

f) In the diagram ABC is an isosceles triangle with $AB=AC$ and $\angle BAC=38^\circ$. 3
Find the size of $\angle ACD$. Give reasons for your answers.



Question 2 *(15 marks)* (Start new page)

a) Use the "completing the square " method to solve $x^2+2x-5=0$ 3

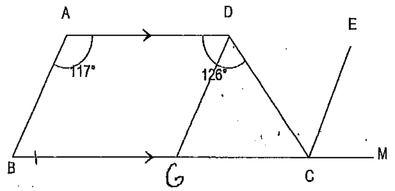
b) Find the values of x for which $|x+1|\le 5$ 2

c) If $a=\frac{3\sqrt{2}}{2}, b=\frac{\sqrt{3}}{3}$ find in simplest form:

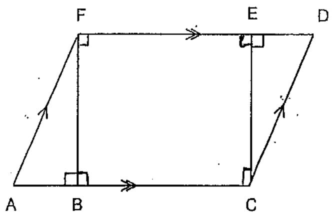
i) $\frac{a}{b}$ 2

ii) $\left(\frac{a}{b}\right)^2-3\left(\frac{b}{a}\right)^2$ 2

d) In the given figure, $AD//BC$, CE bisects $\angle DCM$, $\angle BAD=117^\circ$ and $\angle ADC=126^\circ$. Prove that $AB//CE$. 3



e) Use the given diagram to prove that $\Delta ABF \equiv \Delta DEC$ 3



Question 3 *(15 marks)* (Start new page)

a) Write with a rational denominator $\frac{3\sqrt{3}+5}{3\sqrt{3}-5}$ 2

b) Solve simultaneously $x+y=15$ and $x^2+y^2=125$ 3

c) Given $f(x)=\frac{x^3+1}{x^3-x^2+x}$ and $g(x)=\frac{x^3+x^2+2x+2}{x^3+2x}$

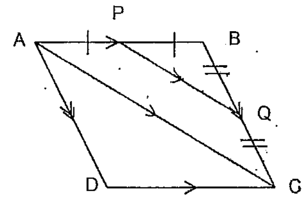
i) Simplify $f(x)$ and $g(x)$. 3

ii) For what value of x does $f(x)=\frac{1}{g(x)}$. 2

d) In the diagram $ABCD$ is a parallelogram. PQ is the line that joins the midpoints of sides AB and BC.

i) Prove using similar triangles that $PQ//AC$ 3

ii) Prove that area $\Delta BPQ=\frac{1}{8}\times$ area of parallelogram $ABCD$ 2



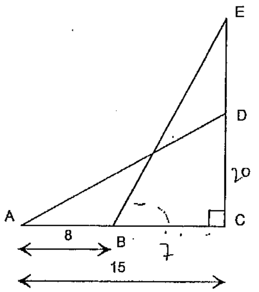
Question 4 *(15 marks)* (Start new page)

a) Find the exact solution (using the quadratic formula) of $\frac{5x}{2}=2+\frac{1}{x}$ 3

b) Solve $|2x+1|=3x-2$ 3

c) In the given diagram $AD=BE=25$, $\angle C$ is a right angle, $AB=8$ and $AC=15$. 3

Find the length of DE.

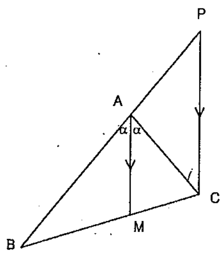


d) Use the given diagram

i) Prove that ΔAPC is isosceles, with $AP=AC$ 2

ii) Prove ΔABM similar to ΔBPC 2

iii) Hence or otherwise show that $\frac{BM}{MC}=\frac{BA}{AC}$ 2



End of the test

2unit

Year 11 Mathematics
2013

Q1 a) $\sqrt{18}+\sqrt{112}-\sqrt{28}$

$= 3\sqrt{2}+4\sqrt{7}-2\sqrt{7}$
$= 3\sqrt{2}+2\sqrt{7}$

b) Let $x = 0.1141414...$
$\therefore 10x = 1.141414...$
$1000x = 114.141414$

$\therefore x = \frac{113}{990}$

c) $x^2+3x-4=0$
$(x+4)(x-1)=0$
$x=-4,\ x=1$

d) $\frac{x-5}{3}-\frac{x+1}{4}=5$

$4(x-5)-3(x+1)=60$
$4x-20-3x-3=60$
$x-23=60$
$x=83$

e) $(5-\sqrt{2})^2 = 25-10\sqrt{2}+2$
$= 27-10\sqrt{2}$
$\therefore a=27 \quad b=10$

f) $\angle ABC = \frac{180-38}{2}$

$= 71^\circ$ (equal $\angle$ of isosceles Δ)
$\angle ACD = 180^\circ - 71^\circ$

Yr Q2 2013

a) $x^2 + 2x + 1 = 5 + 1$ ✓

$(x+1)^2 = 6$ ✓

✓

$x = -1 \pm \sqrt{6}$

b) $-5 \leq x + 1 \leq 5$ ✓

$-6 \leq x \leq 4$ ✓

2

c) i) $\dfrac{\frac{3\sqrt{2}}{2}}{\frac{\sqrt{3}}{3}}$

$\dfrac{9\sqrt{2}}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ ✓

$\dfrac{9\sqrt{6}}{6}$

$\dfrac{3\sqrt{6}}{2}$

ii) $\left(\dfrac{3\sqrt{6}}{2}\right)^2 - 3\left(\dfrac{2}{3\sqrt{6}}\right)^2$

$= \dfrac{9\times6}{4} - 3\left(\dfrac{4}{9\times6}\right)$

$= \dfrac{54}{4} - \dfrac{12}{54}$ ✓

$= \dfrac{27}{2} - \dfrac{2}{9}$ ✓

$= 239 \quad 13\frac{5}{}$

d)

$\angle ABC + 117 = 180°$ (coint. ∠s, AB//

$\angle ABC = 63°$

$\angle DCB + 126 = 180$

$\angle DCB = 54°$

$\angle DCE = \angle ECM = x$ 3

$2x + 54 = 180$

$2x = 126$

$x = 63$

$x = \angle ABC$ corresp. ∠s equal ∴ AB // CE

e) $\angle FBA = \angle CED = 90$ given

$\angle FAB = \angle EDC$ (opp. ∠s of //gram)

$FB = CE$ (From //gram

∴ $\triangle ABF \equiv \triangle CED$ (AAS)

Q3

a. $\dfrac{3\sqrt{3}+5}{3\sqrt{3}-5} \times \dfrac{3\sqrt{3}+5}{3\sqrt{3}+5} = \dfrac{27 + 30\sqrt{3} + 25}{27 - 25}$

$= \dfrac{52 + 30\sqrt{3}}{2}$

$= 26 + 15\sqrt{3}$

b. $x + y = 15$ — ①

$x^2 + y^2 = 125$ — ②

From ①, $x = 15 - y$ sub in ②

$(15-y)^2 + y^2 = 125$

$225 - 30y + y^2 + y^2 = 125$

$2y^2 - 30y + 100 = 0$

$y^2 - 15y + 50 = 0$

$(y-5)(y-10) = 0$

∴ $y = 5, 10$

when $y = 5$, $x = 15 - 5 = 10$

when $y = 10$, $x = 15 - 10 = 5$

$\Rightarrow \left.\begin{array}{l} x = 10, y = 5 \\ x = 5, y = 10 \end{array}\right\}$

c. (i) $f(x) = \dfrac{x^3+1}{x^3 - x^2 + x}$

$= \dfrac{(x+1)(x^2-x+1)}{x(x^2-x+1)}$

$= \dfrac{x+1}{x}$

$$g(x) = \frac{x^3+x^2+2x+2}{x^3+2x}$$
$$= \frac{(x^2+2)(x+1)}{x(x^2+2)}$$
$$= \frac{x+1}{x}$$

(ii) if $f(x) = \frac{1}{g(x)}$

$$\therefore \frac{x+1}{x} = \frac{x}{x+1}$$
$$x^2+2x+1 = x^2$$
$$2x+1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

d, (i) In ΔPBQ, ΔABC

$\frac{PB}{AB} = \frac{1}{2}$ (P is midpoint of AB)

$\angle B$ is common

$\frac{QB}{CB} = \frac{1}{2}$ (Q is midpoint of CB)

$\therefore \Delta PBQ \,|||\, \Delta ABC$ (2 sides in same ratio and included angle =)

$\therefore \angle QPB = \angle CAB$ (corresponding angles of similar Δs)

$\therefore PQ \parallel AC$ (corres. $\angle$s are equal on $\parallel$ lines)

(ii) Let $PB = x$ and $QB = y$

$\therefore AB = 2x$ and $CB = 2y$ (proven above)

$\therefore$ Area of $\Delta PBQ = \frac{1}{2} \times x \times y \times \sin B$

and area of $\Delta ABC = \frac{1}{2} \times 2x \times 2y \times \sin B$

$= 4 \times$ Area of ΔPBQ

Now, area of $ABCD = 2 \times$ area of ΔABC (diagonal of $\parallel$ogram bisect the $\parallel$ogram)

$= 8 \times$ Area of ΔPBQ

$\therefore$ area $\Delta PBQ = \frac{1}{8} \times$ area of $ABCD$

Year 11 - Mathematics - H/Y - 2013.

Question 4: (15 marks)

a) $\frac{5x}{2} = 2 + \frac{1}{x}$ $(\times 2x)$

$$5x^2 = 4x + 2$$
$$5x^2 - 4x - 2 = 0$$
$$a = 5, \quad b = -4, \quad c = -2$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{16-4(5)(-2)}}{2(5)}$$
$$x = \frac{4 \pm \sqrt{16+40}}{10}$$
$$x = \frac{4 \pm \sqrt{56}}{10}$$
$$x = \frac{4 \pm 2\sqrt{14}}{10}$$
$$x = \frac{2 \pm \sqrt{14}}{5}$$

3

b) $|2x+1| = 3x-2$

$2x+1 = 3x-2$ or $2x+1 = -(3x-2)$

$-x = -3$ $\qquad 2x+1 = -3x+2$

$x = 3$ $\qquad 5x = 1$

$x = 1/5$

check solutions

$x=3$, $LHS = |2(3)+1|$

$= 7$

3

$R.H.S = 3(3)-2$

$= 7 = L.H.S.$

$x = 1/5$, $LHS = |2(\frac{1}{5})+1|$

$= 3\frac{1}{5}$

$R.H.S = 3(1/5) - 2 = 1\frac{1}{5} \neq LHS$

$\therefore$ only solution $x=3$

c) In ΔADC,

$(AD)^2 = (DC)^2 + (AC)^2$

$25^2 = (DC)^2 + 15^2$

$DC = \sqrt{25^2 - 15^2}$

$= \sqrt{400}$

$\therefore DC = 20$

In ΔBEC,

$(BE)^2 = (BC)^2 + (CE)^2$

$25^2 = 7^2 + (CE)^2$

$\therefore CE = \sqrt{25^2 - 7^2}$

$= \sqrt{576}$

$CE = 24$

$\therefore DE = EC - DC$

$= 24 - 20$

$= 4$ units

3

d) i) $\angle BAM = \angle APC = \alpha$ (corresponding $\angle$'s $AM \parallel PC$)

$\angle MAC = \angle ACP = \alpha$ (alternate $\angle$'s, $AM \parallel PC$)

$\therefore \Delta APC$ is isosceles since base $\angle$'s are equ[al]

and $AC = AP$.

ii) In ΔBAM and ΔBPC.

$\angle B$ is common

$\angle BAM = \angle BPC = \alpha$ (given)

$\therefore \angle BMA = \angle BCP$ (by subtra[ction])

$\therefore \Delta BAM \,|||\, \Delta BPC$ equiangu[lar]

iii) $\therefore \frac{BM}{MC} = \frac{BA}{AP}$ since $AP = AC$ (from abov[e])

$\therefore \frac{BM}{MC} = \frac{BA}{AC}$

2