11m5 Locus Test Nov 19, 2001

1. Find the equation of the locus of the point which moves so that
   a) the distance from the X axis is twice the distance from the Y axis
   b) the distance from the line x = 2 is equal to the distance from the line y = 1

2. Find the locus of the point which moves so that it is equidistant from A(-2,5) and B(0,3)

3. Find the centre and radius of the circle $x^2 + y^2 - 2x + 4y = 1$

4. Find the locus of the point which moves so that the distance from the point A(4,1) is twice the distance from B(-2,5)

5. Find the locus of the point which moves so that the angle it forms with A(0,6) and B(6,0) is a right angle

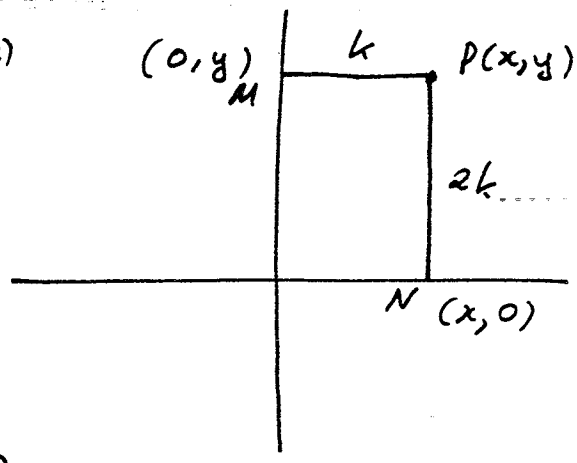
6. Find the locus of the point which moves so that it is equidistant from the point S(3,3) and the line y = -1

7. For the following parabolas, write down
   a) the vertex
   b) the focal length
   c) the focus
   d) equation of the directrix

   i) $x^2 = -12y$ ii) $x^2 - 4x = 4y$ iii) $y^2 = -8(x-1)$

8. A parabola passess through the points (-4,0), (0,8) and (4,0). Given that the vertex is (0,8), determine the equation of the parabola and find the co-ordinates of the focus

Q1 a)



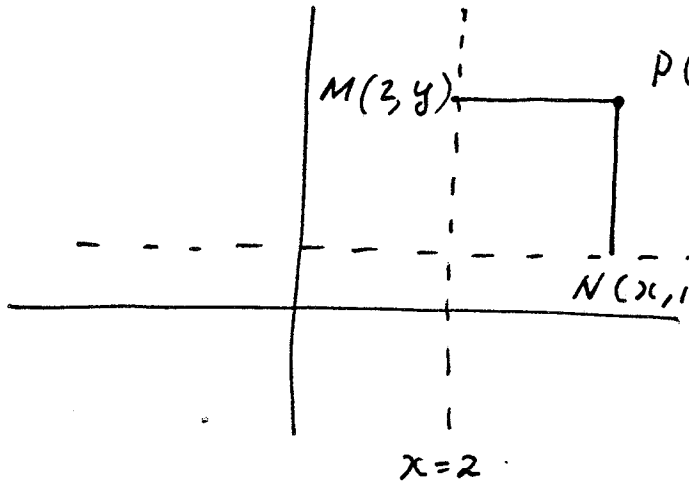
$2PM = PN$

$2\sqrt{(x-0)^2+(y-y)^2} = \sqrt{(x-x)^2+(y-0)^2}$

$2\sqrt{x^2} = \sqrt{y^2}$

$\therefore y = \pm 2x$

b)



$PM = PN$

$\sqrt{(x-2)^2+(y-y)^2} = \sqrt{(x-x)^2+(y-1)^2}$

$y = 1 \quad \therefore \sqrt{(x-2)^2} = \sqrt{(y-1)^2}$

$\therefore y-1 = \pm(x-2)$

$y-1 = x-2$ $\qquad$ $y-1 = -x+2$

$y = x-1$ $\qquad$ $x+y = 3$

Q2 $A(-2,5)$ and $B(0,3)$. $\qquad P(x,y)$

$\therefore PA = PB \Rightarrow PA^2 = PB^2$

$(x+2)^2+(y-5)^2 = (x-0)^2+(y-3)^2$.

$x^2+4x+4+y^2-10y+25 = x^2+y^2-6y+9$.

$4x+20 = 4y$.

$y = x+5$

Q3 $x^2+y^2-2x+4y = 1$

$x^2-2x+1+y^2+4y+4 = 1+1+4$.

$(x-1)^2+(y+2)^2 = 6$.

Centre $(1,-2)$ Radius $\sqrt{6}$

Q4 $A(4,1)$ $B(-2,5)$ $P(x,y)$ $PA = 2PB$

$\therefore PA^2 = 4PB^2$

$(x-4)^2 + (y-1)^2 = 4\{(x+2)^2 + (y-5)^2\}$

$x^2 - 8x + 16 + y^2 - 2y + 1 = 4\{x^2 + 4x + 4 + y^2 - 10y + 25\}$

~~$x^2 - 8x + 16 + y^2 - 2y + 1 = 4x^2 + 16x + 16 + 4y^2 - 40y + 100$~~

$= 3x^2 + 24x + 3y^2 - 38y + 99$

$\therefore x^2 + 8x + y^2 - \frac{38}{3}y + 33 = 0$

$(x^2 + 8x + 16) + (y^2 - \frac{38}{3}y + (\frac{19}{3})^2) + 33 = 16 + (\frac{19}{3})^2$

$(x+4)^2 + (y - \frac{19}{3})^2 = 16 + (\frac{19}{3})^2 - 33$

$(x+4)^2 + (y - \frac{19}{3})^2 = \frac{802}{3}$ $\sqrt{\frac{802}{3}} = 9.4$

Circle: Centre $(-4, \frac{19}{3})$ Radius 9.4

Q5 $A(0,6)$ $B(6,0)$. $P(x,y)$.

Grad of $PA = \frac{y-6}{x}$ Grad of $PB = \frac{y-0}{x-6}$.

Now $PA \perp PB$.

$\therefore \frac{y-6}{x} \times \frac{y}{x-6} = -1$.

$\therefore y(y-6) = -1(x-6)x$.

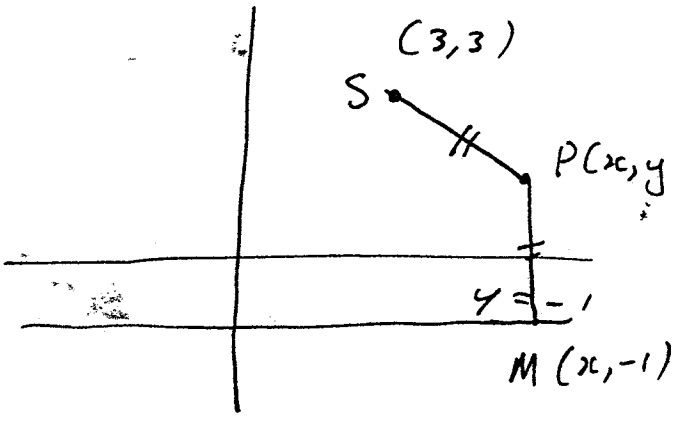
$x(x-6) + y(y-6) = 0$.

$x^2 - 6x + 9 + y^2 - 6y + 9 = 18$

$(x-3)^2 + (y-3)^2 = 18$.

Circle: Centre $(3,3)$ Radius $3\sqrt{2}$.

Q6



$PS = PM$

$(x-3)^2 + (y-3)^2 = 0^2 + (y+1)^2$

$x^2 - 6x + 9 + y^2 - 6y + 9 = y^2 + 2y + 1$

$(x-3)^2 = 8y - 8$

$(x-3)^2 = 8(y-1)$.

Parabola Vertex $(3,1)$

Focal Length 2

Q7 (i) $x^2 = -12y$.

a) Vertex $(0,0)$

b) Focal length $= |-3| = 3$

$a = -3$ $\leftarrow$ -ve direction

c) Focus $(0, -3)$

d) Directrix $y = +3$.

(ii) $x^2 - 4x = 4y$.

$x^2 - 4x + 4 = 4y + 4$

$(x-2)^2 = 4(y+1)$

(a) Vertex $(2, -1)$

(b) Focal length 1

(c) Focus $(2, 0)$

(d) Directrix $y = -2$.

(iii) $y^2 = -8(x-1)$

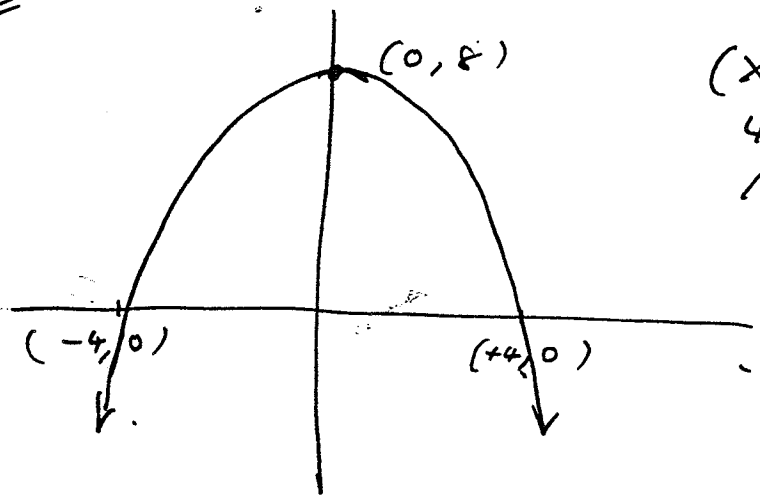
a) Vertex $(1, 0)$.

b) Focal length $a = -2$.

c) Focus $(-1, 0)$

d) Directrix $x = 3$.

Q8



$(X-0)^2 = 4a(Y-8)$

$4^2 = 4a(-8)$

$16 = -32a$

$a = -\frac{1}{2}$.

$\therefore X^2 = -2(Y-8)$