MARCELLIN COLLEGE RANDWICK

YEAR 11
ACCELERATED MATHEMATICS

HSC ASSESSMENT TASK 2
2016

STUDENT NAME: _____   MARK   /33

TEACHER: _____

TIME ALLOWED: 45 minutes
WEIGHTING: 20 %

Directions:
- Answer multiple choice questions on the page provided.
- Use a new sheet for each question.
- Show all necessary working. Where more than one mark is allocated to a question, full marks may not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.
- Calculators may be used

Section I
Start each question on a new sheet of paper

Question 1   (16 marks)

	Marks

a) Differentiate

i) $\sin(\ln x)$ — 2

ii) $x^2 \ln x$ — 2

b) Find $\int \frac{x}{x^2-3} dx$ — 2

c) Find $\int_0^{\frac{\pi}{4}} \sin 2x \, dx$ — 2

d) Solve $2\cos\theta = -\sqrt{3}$ for $0 \le \theta \le 2\pi$ — 2

e) Find the limiting sum of the series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$ — 2

f) A plant has a probability of $\frac{2}{7}$ of producing white flowers. If 3 plants are chosen at random, find the probability that

i) 2 plants will produce white flowers — 1

ii) At least 1 plant will produce white flowers — 1

g) Find the equation of the tangent to the curve $y = 2e^{2x}$ at the point where $x = 0$ — 2

Question 2 (6 marks)
Start a new page

a) John is learning to drive. His first lesson is 30 minutes long. His second lesson is 35 minutes and each lesson thereafter is 5 minutes longer than the lesson before.

    i) How long with John's fifteenth lesson be?   1

    ii) How many hours will John have completed after his fifteenth lesson?   2

b) In the diagram below, the shaded region is bound by the curve $y = 2\sec x$, the x – axis and the lines $x = 0$ and $x = \frac{\pi}{3}$.

The shaded region is rotated about the x – axis
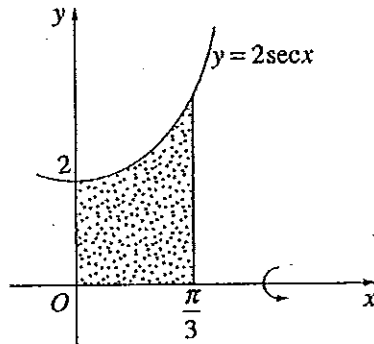
Calculate the exact volume of the solid of revolution formed.   3

Question 3 (7 marks)
Start a new page

a) A bag contains 10 marbles of which only two are black. Each trial involves drawing two marbles from the bag. The marbles are NOT replaced until the end of each trial.   3

How many trials should one make so that the certainty of drawing two black marbles in a trial at least once is greater than 95%?

b) Frances decides to set up a trust fund for her grandson Michael. She invests $100 at the beginning of each month. The money is invested at 3% per annum, compounded monthly.

The trust fund matures at the end of the month of her final investment, 20 years after her first investment. This means that Betty makes 240 monthly investments.

    i) After 20 years, what will be the value of the first $100 invested?   1

    ii) By writing a geometric series for the value of all Frances' investments, calculate the final value of Michael's trust fund.   3

Section II – Multiple Choice (4 marks)
Attempt all questions

Use the Multiple Choice Answer Sheet for Questions 4 to 7

4. Which of the following is the solution to $6^x = 5$?

(A) $x = \frac{\ln 5}{6}$ (B) $x = \frac{5}{\ln 6}$

(C) $x = \ln\left(\frac{5}{6}\right)$ (D) $x = \frac{\ln 5}{\ln 6}$

5. In a school the population of females is 45% and males 55%. If two students are chosen at random, what is the probability that they are both female

(A) 0.2025   (B) 0.2475   (C) 0.3025   (D) 0.5555

6. AOB is a sector of a circle, centre O and radius 5 cm. The sector has an area of 10π
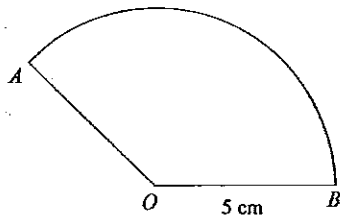
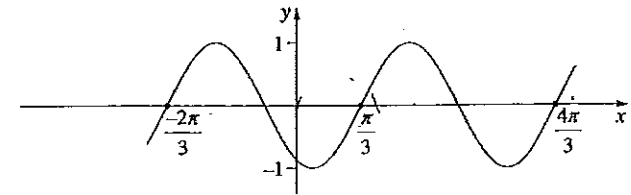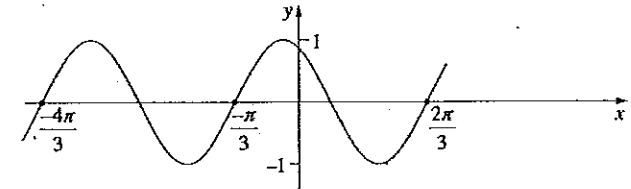
Not to scale

What is the arc length of the sector?

(A) 2π (B) 4π (C) 6π (D) 10π

7. Which diagram shows the graph $y = \sin\left(2x + \frac{\pi}{3}\right)$?
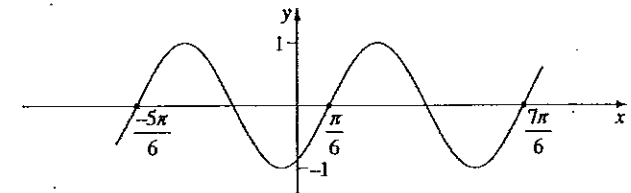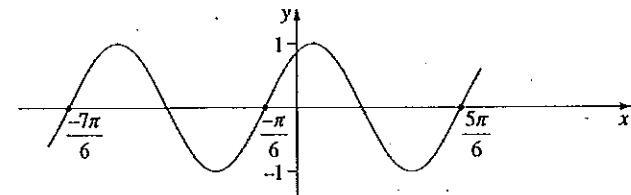
(A)

(B)

(C)

(D)

END OF PAPER

i) $\sin(\ln(x))$

$\Rightarrow$ use chain rule.

$= \frac{d}{dx}(\ln(x)) \times \cos(\ln(x))$

$= \frac{\cos(\ln(x))}{x}$

ii) $x^2 \ln x$

$\Rightarrow$ use product rule

$u = x^2 \qquad v = \ln x$

$u' = 2x \qquad v' = \frac{1}{x}$

$= \frac{x^2}{x} + 2x \ln x$

$= x + 2x \ln x$

$= x(1 + 2\ln x)$

b) $\int \frac{x}{x^2-3} \, dx$
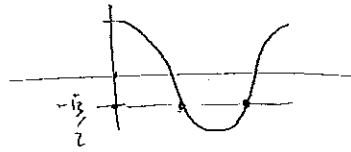
$= \frac{1}{2} \int \frac{2x}{x^2-3} \, dx$

$= \frac{1}{2} \ln(x^2-3) + C$

c) $\int_0^{\pi/4} \sin 2x \, dx$

$= -\frac{1}{2}\cos 2x \Big|_0^{\pi/4}$

$= \frac{1}{2}$.

d) $2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$= 150^\circ, 210^\circ$

$= \frac{8\pi}{6}, \frac{7\pi}{6}$.

e) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27}$

$a = 1, \quad r = -\frac{1}{3}$.

limiting sum $= \frac{a}{1-r}$

$= \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$

f) $\frac{2}{7}$ probability.

$\left(\frac{2}{7} \times \frac{2}{7} \times \frac{5}{7}\right) \times 3$ $\}$ 3 scenarios happening.

$= \frac{60}{343}$.

ii) $1 - P(\text{no white flowers})$

$= 1 - \left(\frac{5}{7}\right)^3 = \frac{218}{343}$.

g) $y = 2e^{2x}$

$\frac{dy}{dx} = 4e^{2x}$

at $x=0$, $M_{\text{tangent}} =$

$4e^{(0)} = 4$.

at $x=0$, $y = 2$.

$(y-2) = 4(x-0)$

$y - 2 = 4x$.

$y - 2 - 4x = 0$

2. Arithmetic series

a)
i) $a = 30$

$d = 5$

$T_{15} = 30 + (14 \times 5)$

$= 100$ minutes

ii) $S_{15} = \frac{15}{2}(60 + (14 \times 5))$

$= 975$ minutes.

$= 16.25$ hours.

b) $y^2 = 4\sec^2 x$

So volume of rotation

$= \pi \int_0^{\pi/3} 4\sec^2 x$

$= 4\pi \left[\tan x\right]_0^{\pi/3}$

$= 4\pi[\sqrt{3}]$

$= 4\sqrt{3}\pi \text{ units}^3$

3. 10 marbles

   2 black

   No REPLACE

   Chance of drawing 2

   black marbles $= \frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$

   So to guarantee, we need

   $\frac{90 \times 0.95}{90}$ tries.

   ie. 86 trials.

   90 tries to statistically
   guarantee that it will happen 100%.
   i.e. the certainty,
   but it might take an
   infinite tries because the
   events are independent of each
   other.

b). $100 \times \left(1 + \frac{0.03}{12}\right)^{240}$

   i) 182.075

   ii) \$32912.28 (2.d.p).

   $T_0 = 100$

   $T_1 = 100 + 100r$

   $T_2 = 100 + 100r + 100r^2$

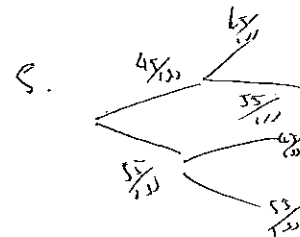 $T_n = 100r + 100r^2 + \dots + 100r^n$.

   where $n = 240$.

   let $P = r = 1 + \frac{0.03}{12} = 1.0025$

   Sum of geometric series

   $a = 100(1.0025)$

   $r = 1.0025$

   $n = 240.$

   $= 100 \cdot \frac{1.0025 \left(1 + 1.0025^{240}\right)}{1.0025 - 1}$

   $= \$32912.28$

4. $6^x = 5$

   $\ln(6^x) = \ln(5)$

   $x \ln 6 = \ln 5$

   $x = \ln 5 \div \ln 6 = D$

5.

   $0.45^2 = 0.2025$

   $= A$

6. Total circumference $= 10\pi$.

   Area of sector $= 10\pi$.

   Area of entire circle $= 25\pi$

   So $\frac{10}{25} = \frac{2}{5} \times 360°$

   $= 144°$.

   Same ratio for the circumference

   So $10\pi \times \frac{2}{5} = 4\pi = B$.

7. Take points

   $\rightarrow$ Sub in $x = 0 \cdot \Rightarrow y = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}$

   So either B or D

   $\rightarrow \sin \theta = \sin\left(2\left(-\frac{\pi}{8}\right) + \frac{\pi}{3}\right)$

   So $= D$.