Student ______________________

BRIGIDINE COLLEGE RANDWICK

PRELIMINARY
EXTENSION 1
MATHEMATICS

Year 11 EXTENSION 1 Task 1
HALF YEARLY

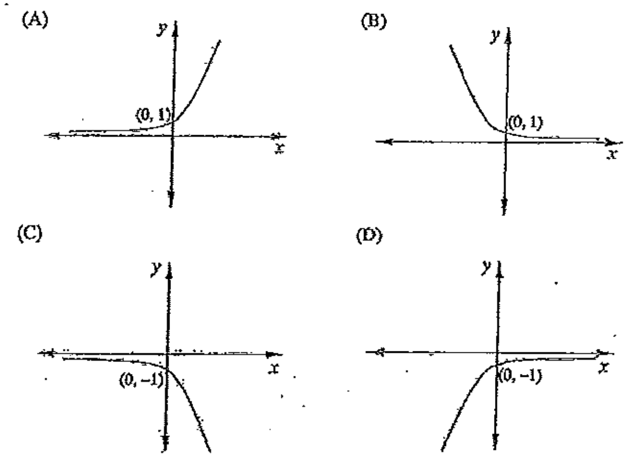
2017

(TIME – 55 Minutes)

Directions to candidates

- *Put your **name at the top** of this paper and on each of the **3 sections** that are to be collected.*
- *All 3 sections are to be attempted.*
- *All questions are to be answered on separate pages and will be collected in separate bundles at the end of this exam.*
- *Use PEN to show all necessary working in every question.*
- *Full marks may not be awarded for careless or badly arranged work.*
- *Diagrams are not to scale unless otherwise stated.*

Q1 Which graph best represents the function $y = -3^{-x}$



Q2 Which expression is the correct simplification of $\frac{25^{2x}}{5^x}$?

(A) 5^2 (B) 5^x (C) 5^{2x} (D) 5^{3x}

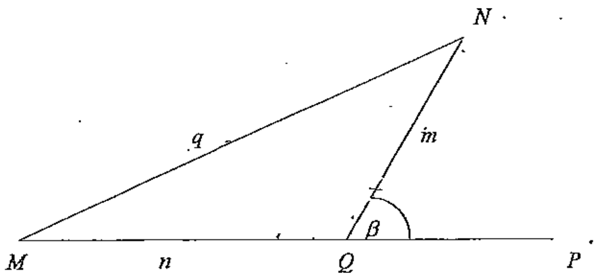
Q3 The solution of $\sqrt{2}\cos x - 1 = 0$, where $0^\circ \le x^\circ \le 360^\circ$

(A) $45^\circ, 225^\circ$ (B) $45^\circ, 315^\circ$ (C) $60^\circ, 120^\circ$ (D) $60^\circ, 300^\circ$

Q4 A coach, manager and six players sit around a circular table to discuss tactics. In how many ways can they sit if the coach and manger are not to sit together?

(A) 3600 (B) 4320 (C) 38880 (D) 39600

Q5 If $\angle NQP = \beta$ in the diagram below, which of the following could be true?



(A) $q^2 = m^2 + n^2 + 2mn\cos\beta$

(B) $q^2 = m^2 + n^2 - 2mn\cos\beta$

(C) $m^2 = q^2 + n^2 - 2qn\cos\beta$

(D) $\dfrac{m}{\sin\angle NMQ} = \dfrac{q}{\cos\beta}$

Question 6 (Start a new page- 20 marks)

a. If $A^m = 3$, find the value of $A^{4m} - 5$. 1

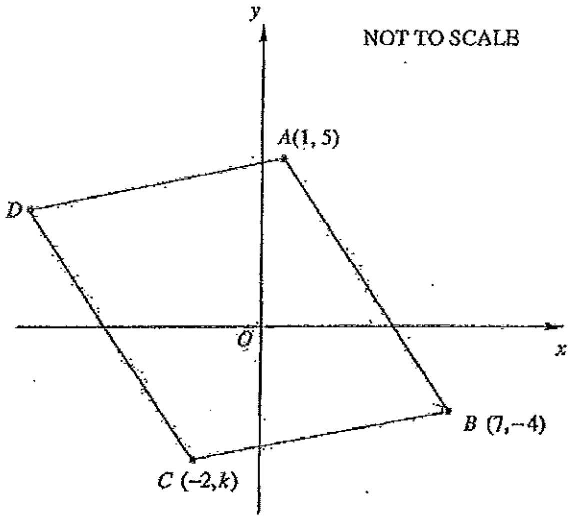
b. A is a point (-2-1) and B is the point (1,5). Find the coordinates of the point Q which divides AB externally in the ratio 5:2. 2

c. Simplify completely $\dfrac{1}{p^2 - pq} - \dfrac{1}{pq - q^2}$ 2

d. Find the number of ways in which the letters of the word EPSILON can be arranged in a straight line so that the three vowels are all next to each other. 2

e. Solve for x: $3\cos x = \sec x + 2$ in the domain $0^0 \le x \le 360^0$ giving your solutions to the nearest minute 3

f. In the diagram A,B and C have coordinates (1,5), (7,-4) and (-2,k) respectively. D is in the 2nd quadrant and C is in the 3rd quadrant. ABCD is a parallelogram.



(i) Find the gradient of AB. 1

(ii) Show that the equation of AB is $3x+2y-13=0$. 1

(iii) Write down an expression, in terms of k, for the perpendicular distance from C to the line AB. 1

(iv) Find the length of interval AB in exact form 1

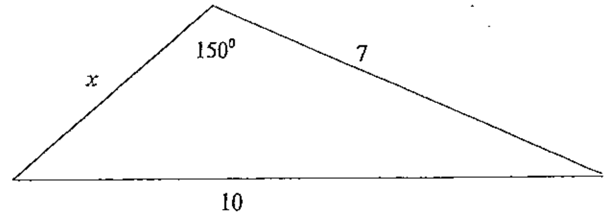
(v) Given that the area of $ABCD$ is 90 square units, find the value of k. 3

(vi) Determine the coordinate of D. 1

g. Sketch the graph of $y=|2x|+x-1$ 2

Question 7 (Start a new page – 18 marks)

a. Find the exact value of x. 3

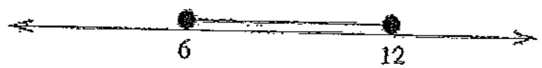


b. From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen. How many different groups of 5 are possible? 2

c. A Mathematics department consists of 5 female and 5 male teachers. How many committees of 3 teachers can be chosen which contain at least one female and at least one male? 2

d. The number line graph represents the solution to the inequality $|x-a|\le b$

NOT TO SCALE



Find the value of a and b. 2

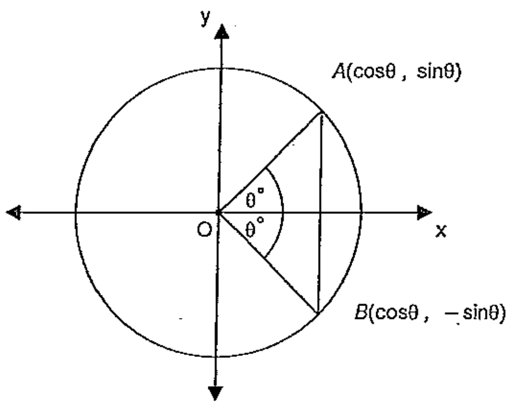
e. i. Find the vertical and horizontal asymptotes of the function $y=\dfrac{x-1}{x+3}$

and hence sketch the graph of $y=\dfrac{x-1}{x+3}$ 3

ii. Hence, or otherwise, find the values of x for which $\dfrac{x-1}{x+3}\geq -2$. 2

iii. Sketch $y=\dfrac{|x-1|}{|x+3|}$ 2

f 2



$A(\cos\theta, \sin\theta)$ and $B(\cos\theta, -\sin\theta)$, $0 < \theta < 90^\circ$, are two points on the circle with centre $O(0,0)$ and radius 1 unit.

Use the cosine rule in ΔAOB to show that:

$$\cos 2\theta = 1-2\sin^2\theta$$

END OF ASSESSMENT TASK

2017 yr 11 EXT 1
HY (TASK 1)

Q1) D

Q2) $\frac{25^{2x}}{5^x}$

$= (5^2)^{2x} \div 5^x$

$= 5^{4x} \div 5^x$

$= 5^{3x}$ ∴ D

Q3) $\sqrt{2}\cos x - 1 = 0$

$\cos x = \frac{1}{\sqrt{2}}$

$x = 45°, 360 - 45°$

$x = 45°, 315°$

∴ B

Q4

Total number of arrangements $(8-1)!$

Number of arrangements if the manager and coach sit together $= 2 \times 6!$

∴ when they don't sit together: $(8-1)! - 2\times 6!$

$= 3600$ ∴ A

Q5

$q^2 = n^2 + m^2 - 2 \times n \times m \times \cos(180 - \beta)$

$\cos(180-\beta) = -\cos\beta$

$q^2 = n^2 + m^2 + 2mn\cos\beta$

∴ A

Q6 a) $A^m = 3$

$A^{4m} - 5$

$(A^m)^4 - 5$

$3^4 - 5 = 76$

1 mark

b) $(-2,-1)$ $(1,5)$

$-5 : 2$

$\frac{2x-2+1x-5}{-5+2}, \frac{2x-1+-5x5}{-5+2}$

$= (3, 9)$

2 marks

1 mark: 1 small error.

(NOTE: 2 marks were awarded for $(3, \frac{23}{3})$ since there was an small error in the test $(-2-1)$ comma missing some students thought it was $(-2,1)$)

c) $\frac{1}{p^2 - pq} - \frac{1}{pq - q^2}$

$\frac{1}{p(p-q)} - \frac{1}{q(p-q)}$

$\frac{q-p}{pq(p-q)}$

$= -\frac{1}{pq}$

2 mark:

1 mark: factorise denominators

Q6d)

(EIO), P, S, L, N can be arranged 5! ways, then (EIO) in 3! ways.

So answer: $5! \times 3!$

$= 720$

2 marks:

1 mark: $3! \times$ ☐ or $5! \times$ ☐

f) i) $m = \frac{5 - -4}{1-7}$

$= -\frac{3}{2}$

1 mark

ii) $A(1,5)$, $m = -\frac{3}{2}$

$y - 5 = -\frac{3}{2}(x-1)$

$2y - 10 = -3x + 3$

$3x + 2y - 13 = 0$

1 mark

iii)

1 mark

Q6f(v)

Area of parallelogram:

A = base × perpendicular height.

$A = \sqrt{117} \times \frac{|2k-19|}{\sqrt{13}}$

$A = \sqrt{9} \times |2k-19|$

$A = 3|2k-19|$

Since $A = 90$

$3|2k-19| = 90$

$|2k-19| = 30$

$2k-19=30 \qquad 2k-19=-30$

$2k = 49 \qquad 2k = -11$

$k = 24\frac{1}{2} \qquad k = -5\frac{1}{2}$

Since C is in 3rd quadrant then

$k = -5\frac{1}{2}$

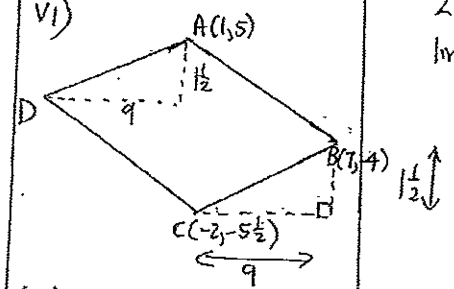
3marks:

2marks: $k = 24\frac{1}{2}$ and $k = -5\frac{1}{2}$

1mark: (part iii) × (part iv) = 90

Finding a "negative k"

vi)



$\therefore D(-8, 3\frac{1}{2})$

2marks: correct from their (v)

1mark: $(-8, \square)$ or $(\square, 3\frac{1}{2})$

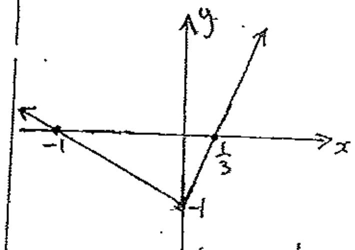
g) $y = |2x| + x - 1$

$y = 2x + x - 1$ if $2x \geq 0$

$y = 3x - 1 \qquad \therefore x \geq 0$

$y = -2x + x - 1$ if $2x < 0$

$y = -x - 1$ if $x < 0$



must show a variation in gradient steepness.

2marks: see solution

1mark: V at -1 on the y-axis.

or an x intercept of either $\frac{1}{3}$ or -1

Q7a)

$10^2 = x^2 + 7^2 - 2 \times x \times 7 \times \cos 150°$

$\cos 150 = -\cos 30$

$= -\frac{\sqrt{3}}{2}$

$100 = x^2 + 49 - 2 \times x \times 7 \times -\frac{\sqrt{3}}{2}$

$100 = x^2 + 49 + 7\sqrt{3}x$

$0 = x^2 + 7\sqrt{3}x - 51$

$x = \frac{-7\sqrt{3} \pm \sqrt{(7\sqrt{3})^2 - 4 \times 1 \times -51}}{2 \times 1}$

3marks.

2marks: $0 = x^2 + 7\sqrt{3}x - 51$

1mark

b) Girls and Boys

${}^7C_3 \times {}^6C_2$

$= 35 \times 15$

$= 525$

2marks:

1mark: ${}^7C_3$ or ${}^6C_2$

c) In any order.

MFF or FMM

2marks:

1mark:

${}^5C \times {}^5C$

Q7e) i)

$y = \frac{x-1}{x+3}$

$x + 3 \neq 0$

$x \neq -3$ vert assymp

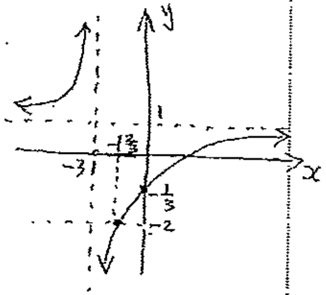
$x \to \infty,\ y \to 1^-$

$x \to -\infty,\ y \to 1^+$

$x \to -3^+,\ y \to -\infty$

$x \to -3^-,\ y \to +\infty$

$y \neq 1$ Horiz assymp



3marks:

2marks:

$x \neq -3$ and $y \neq 1$

or shown on students sketch

1mark:

$x \neq -3$ or $y \neq 1$

or a y intercept of $-\frac{1}{3}$

ii) $\frac{x-1}{x+3} = -2$

$x - 1 = -2x - 6$

$3x = -5$

$x = -1\frac{2}{3}$

from graph:

$x < -3,\ x \geq -1\frac{2}{3}$

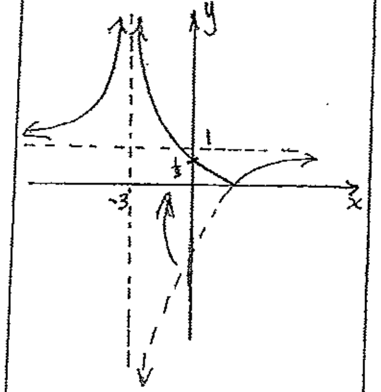
2marks:

1mark:

$x < -3$ or $x \geq -1\frac{2}{3}$

or $3x^2 + 14x + 15 \geq 0$

iii) Abs value is a reflection on the x-axis.

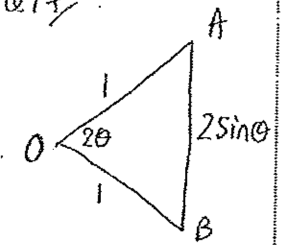


2marks:

1mark:

Some correct part of graph that shows 2 distinct parts.

Q7f



$\cos 2\theta = \frac{1^2 + 1^2 - (2\sin\theta)^2}{2 \times 1 \times 1}$

$\cos 2\theta = \frac{2 - 4\sin^2\theta}{2}$

2marks:

1mark:

$\cos 2\theta = \frac{1^2 + 1^2 - \square}{2 \times 1 \times 1}$

↑ indicates students understood radius = 1

or $AB = \sin\theta + \sin\theta$