**SYDNEY BOYS HIGH SCHOOL**
MOORE PARK, SURRY HILLS

**2011**

**Year 11 Yearly**

# Mathematics Accelerated

**General Instruction**

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Start each **NEW** question in a separate answer booklet.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise instructed.

**Total Marks – 80**

- Attempt questions 1-5.

Examiner: *J. Chen*

**START A NEW ANSWER BOOKLET**

**QUESTION ONE [18 marks]**

(a) [5 marks]

(i) Use the standard integrals to find,

$$\int \sec 2x \tan 2x \, . dx$$

(ii)

$$\int \frac{x-3}{x} . dx$$

(iii)

$$\int \tan x \, . dx$$

(b) Evaluate [5 marks]

(i)

$$\int_{-e}^{e} \sin(e-x) \, . dx$$

(ii)

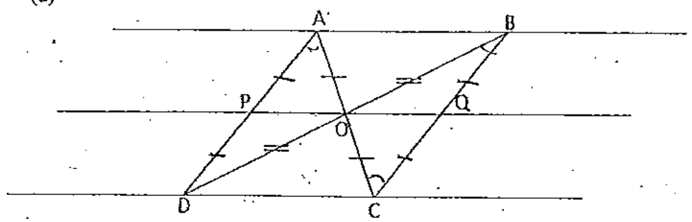
$$\int_0^1 (2+e^x) . dx$$

(c) Differentiate the following with respect to $x$, [3 marks]

(i) $\tan(\sin x)$

(ii) $e^{x+\cos x}$

(d) [5 marks]




(i) Explain why ABCD is a parallelogram.

(ii) If PQ are the midpoints of AD and BC respectively, explain why AB||PQ||CD.

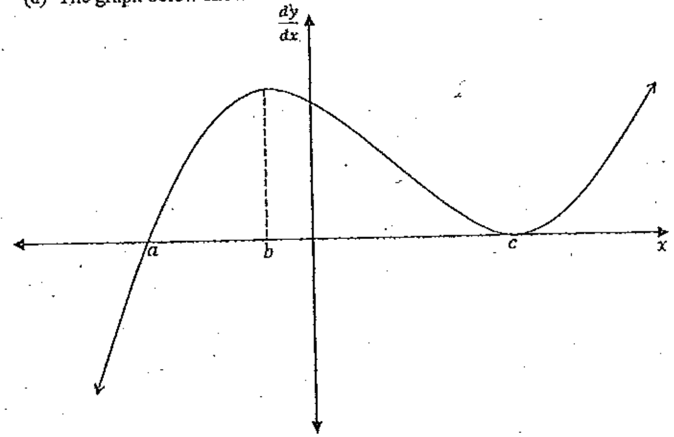
(iii) Prove that OP = OQ.

**End of Question One**

## START A NEW ANSWER BOOKLET

**QUESTION TWO [13 marks]**

(a) The graph below shows the derivative of the curve $y = f(x)$. **[6 marks]**




(i) Explain why the curve $y = f(x)$ has stationary points at $x = a$ and $x = c$.

(ii) What type of stationary point is at $x = a$ and why?

(iii) What type of stationary point is at $x = c$ and why?

(iv) Sketch a possible graph of $y = f(x)$.

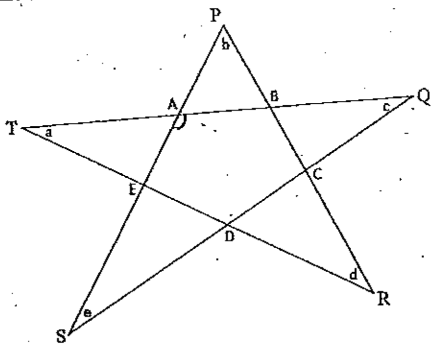
(b) **[4 marks]**

(i) Differentiate $xe^x$.

(ii) Hence, evaluate

$$\int_0^1 xe^x \, .dx$$

(c) Determine the value of a + b + c + d + e, giving reasons. **[3 marks]**



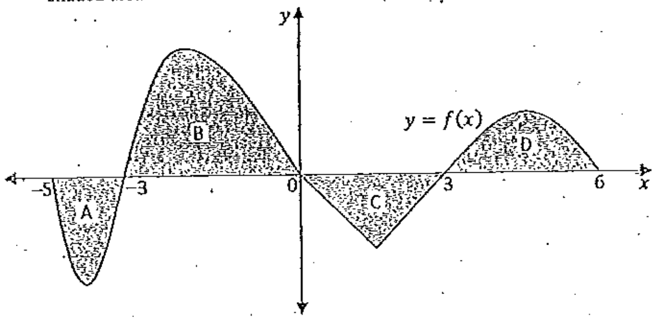

**End of Question Two**



## START A NEW ANSWER BOOKLET

**QUESTION THREE [17 marks]**

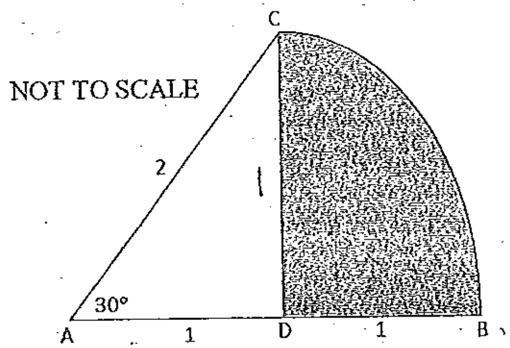
(a) In the diagram, the shaded area A is 5 $cm^2$, the shaded area B is 8 $cm^2$, the shaded area C is 7 $cm^2$ and the shaded area D is 6 $cm^2$. **[1 mark]**




Find

$$\int_{-5}^{6} f(x) \, .dx$$

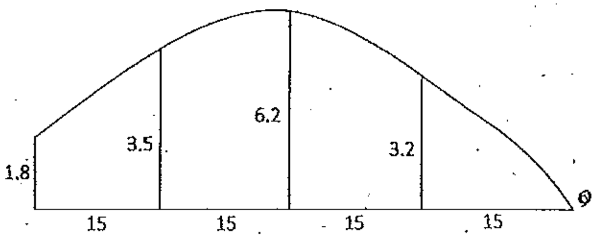
(b) In the diagram below, ABC is the sector of a circle with radius 2 cm, $\angle$CAB is 30° and AD = BD = 1 cm. **[6 marks]**




(i) Find the perimeter of the shaded region BCD correct to the nearest 2 decimal places.

(ii) Find the exact area of the shaded region BCD.

(c) The diagram below shows Mr. Smith's farm. All measurements are in metres. **[2 marks]**




Use Simpson's rule with 5 function values to approximate the area of the farm.

(d) **[4 marks]**

(i) Find the coordinates of the points of intersection of the two curves $y = x^2 - 2x + 1$ and $y = 4x - x^2 - 3$.

(ii) Calculate the area contained by the two curves between the points of intersection.

(e) The temperature of a cup of black coffee is given by $T = 100e^{-t/5}$ where $t$ is the time in minutes. **[4 marks]**
If it is too hot to drink above 55°C and too cold below 25°C. Calculate the length of time during which the coffee is drinkable (to the nearest second).

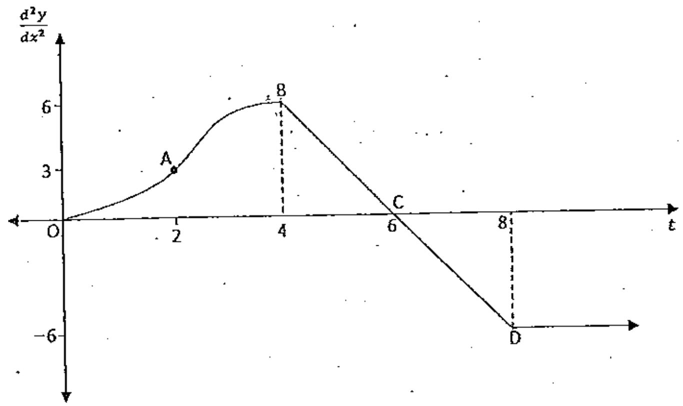
**End of Question Three**



## START A NEW ANSWER BOOKLET

**QUESTION FOUR [16 marks]**

(a) A particle moves along the $x$-axis. Initially it is at rest at the origin. The graph shows the acceleration, $\frac{d^2y}{dx^2}$, of the particle as a function of time $t$. **[6 marks]**




(i) Using Simpson's rule, estimate the velocity of the particle at $t = 4$.

(ii) Write down the time at which the velocity of the particle is a maximum.

(iii) Estimate the time at which the particle is furthest from the origin in the positive direction. Justify your answer.

(b) Consider the function $f(x) = (x^2 - 4)(x^2 - 2)$. **[10 marks]**

(i) Find the $x$ intercepts of the curve.

(ii) Find the coordinates of the stationary points and determine their nature.

(iii) Find any points of inflexion.

(iv) Sketch $y = f(x)$, showing all critical points.

(i) Determine the values of $x$ for which the function concaves up.

**End of Question Four**

## START A NEW ANSWER BOOKLET

**QUESTION FIVE [16 marks]**

(a) A T-shirt company makes 500 shirts per month. At \$30 each, they can sell all the shirts. If the price of each shirt is increased by \$3, then this will result in a 5 shirt reduction in sales for each \$3 increment. Also, the company has fixed costs of \$6500 per month. **[6 marks]**

(i) Let the number of \$3 increments be $x$, prove that the monthly profit P, in dollars, is given by $P = 8500 + 1350x - 15x^2$.

(ii) Find how many shirts would be sold and the price that should be charged per shirt to ensure maximum monthly profit.

(b) Consider the function $f(x) = \frac{x}{\ln x}$, for $x > 1$. **[5 marks]**

(i) Show that the function $y = f(x)$ has a minimum point at $x = e$.

(ii) Hence, use (b) (i) to show that $x^e \leq e^x$ for $x > 1$.

(c) The region bounded by the curve $y = \log_3 x$, the line $y = 2$ and the $x$ and $y$ axes, is rotated about the $y$ axis. **[5 marks]**

(i) Show that the volume of the solid of revolution formed is given by

$$V = \pi \int_0^2 9^y \, . dy$$

(ii) Hence evaluate the volume in exact simplified form.

**End of Exam**

## Sydney Boys' High School

124538

Student No.: [illegible]

Paper: Maths accelerated Yr 11

Section: Q1

Sheet No.: 1 of 1 for this Section.

| Q.No | Tick | Mark |
|---|---|---|
| 1 | | 17 |
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| 10 | | |

a)

i) $\int \sec 2x \tan 2x \, dx$

$= \frac{1}{2} \sec 2x + c$ ✓

ii) $\int \frac{x-3}{x} \, dx = \int \left(\frac{x}{x} - \frac{3}{x}\right) dx$

$= \int \left(1 - \frac{3}{x}\right) dx$ ✓

$= x - 3\ln(x) + c$ ✓

iii) $\int \tan x \, dx$

$= \int \frac{\sin x}{\cos x} \, dx = -\ln(\cos x) + c$ ✓

b)

i) $\int_{-e}^{e} \sin(e-x) \, dx$

$= \left[\cos(e-x)\right]_{-e}^{e}$ ✓

$= \cos(e-e) - \cos(2e)$

$= 1 - \cos(2e)$ ✓

ii) $\int_0^1 (2+e^x) \, dx$

$= \left[2x + e^x\right]_0^1$ ✓

$= (2+e) - (0+1)$ ✓

$= 2+e-1$ ✓

$= 1+e$

c)

(i) $\frac{d}{dx} \tan(\sin x)$ ✓

$= \cos x \sec^2 x (\sin x)$

(ii) $\frac{d}{dx} e^{x+\cos x}$

$= (1-\sin x) e^{x+\cos x}$ ✗ ✓

d)

i) ABCD is a parallelogram because diagonals AC and DB bisect each other at a common point O.

ii) In Parallelogram ABCD,
AD ∥ BC, AB ∥ DC, AD = BC, and AB = DC
but P and Q are midpoints of AD and BC,
∴ AP = BQ
and so AB ∥ PQ as AP = BQ and ABPQ is a parallelogram. ∴ since AB ∥ DC, AB ∥ PQ ∥ DC.

Diagram?

ii) In $\Delta$'s APO and CQO,
AO = CO (given)
AP = CQ (P and Q bisect AD and BC)
$\angle PAO = \angle QCO$ (alt $\angle$'s on AD ∥ BC).

$\therefore \Delta APO \equiv \Delta CQO$ (S.A.S).
$\therefore$ corresp. sides of ~~an~~ cong. $\Delta$'s are equal
$\therefore OP = OQ$

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Sydney Boys' High School

124534

Student No.: [illegible]

Paper: Maths accelerated Yr 11

Section: Q2

Sheet No.: 1 of 1 for this Section.

| Q.No | Tick | Mark |
|---|---|---|
| 1 | | |
| 2 | ✓ | 11 |
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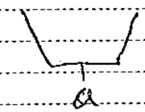
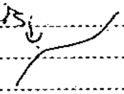
| 10 | | |

a)
i) f(x) has stat pts at ~~x~~ x = a and c as $\frac{dy}{dx}$ of f(x) ~~is~~ = 0 at x = a, c.

1

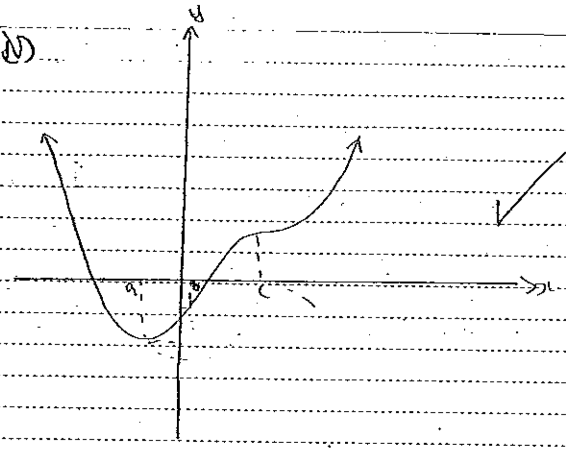
ii) ~~a~~ minimum t.p as for $x<a$, $\frac{dy}{dx}<0$, (negative gradient) and for $x>a$, $\frac{dy}{dx}>0$ (positive gradient)

which looks like this:  so it's a min t.p.

iii) a horizontal point of inflexion as for $x<c$, $\frac{dy}{dx}>0$ (pos. grad.) and for $x>c$, $\frac{dy}{dx}>$ ~~0~~ (pos. grad.) There fore its a h.p.o.i as concavity doesn't change. ~~It~~ It looks like this: 

4

iv)



(1)

b)

i) $u = x$ $\quad v = e^x$

$u' = 1$ $\quad v' = e^x$

$\therefore \frac{d}{dx} = e^x + xe^x$

ii) $\therefore \int_0^1 xe^x \, dx$

$= \int_0^1 (xe^x + e^x) - e^x \, dx$

$= \left[ xe^x - e^x \right]_0^1$

$= (e - e) - (0 - 1)$

$= 1.$

(4)

c) ~~assume~~ ~~Polygon ABCDE is a pentagon, so $\angle$ sum $= 540°$. Assume it is a regular pentagon.~~ ✗ You can't assume this

~~$\therefore$ each $\angle = 108°$.~~

~~$\therefore \angle EAB = 108° = \angle AED = \angle EDC = \angle DCB = \angle CBA$~~

~~$\angle EAT = 72°$ ($\angle$ sum of $\Delta$)~~

~~similarly $\angle AET = \angle DES = \angle EDS = \angle CDR = \angle DCR = \angle BCQ = \angle CBQ = \angle ABP = \angle PAB = 72°$ ($\angle$ sum of $\Delta$)~~

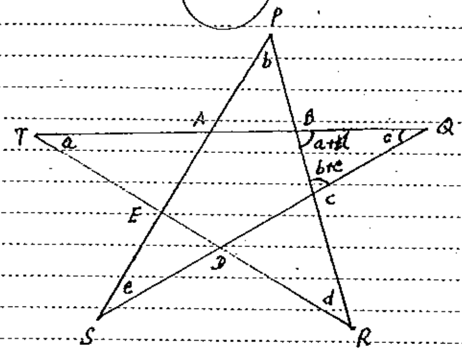
See solution on next page

which means ~~a+b+c~~ $a = b = c = d = e = 36°$ ($\angle$ sum of $\Delta$)

$\therefore a + b + c + d + e = 180°$

(1)

~~Also~~




In $\Delta BCQ$

$\angle QBC = a + d$ (Ext $\angle$ of $\Delta BTR$)

$\angle QCB = b + e$ (Ext $\angle$ of $\Delta PSC$)

$\therefore a + d + c + b + e = 180°$ ($\angle$ sum of $\Delta BQC$)

# Sydney Boys' High School

124535

Student No.: [illegible]

Paper: Maths accelerated Yr 11

Section: Q3

Sheet No.: 1 of 1 for this Section.

| Q.No | Tick | Mark |
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| 10 | | |

a) $\int_{-5}^{8} f(x)\,dx$

$= -5 + 8 - 7 + 6$

$= 2$

b)

i) $\sin 30 = \frac{CD}{2}$

$\therefore CD = 1$

and $CB = 2 \times \frac{\pi}{6}$

$= \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore$ perimiter of $BCD = \frac{\pi}{3} + 2$ cm

$= 3.05$ cm

2

ii) Area of Sector $ABC = \frac{1}{2} \times 4 \times 4 \times \frac{\pi}{6}$

Area of $\Delta\ ADC = \frac{1}{2} \times 2 \times 1 \times \sin 30.$

Area $BCD$ = Area of sector − area of $\Delta ADC$

$= \frac{\pi}{3} - \frac{1}{2}$ cm$^2$.

$= \frac{2\pi - 3}{6}$ cm$^2$.

c) $A = \frac{15}{3}\left(1.8 + 0 + 4(3.5 + 3.2) + 2(6.2)\right)$

$A \doteqdot 205\ m^2$.

6) i) $x^2 - 2x + 1 = 4x - x^2 - 3$

$2x^2 - 6x + 4 = 0.$

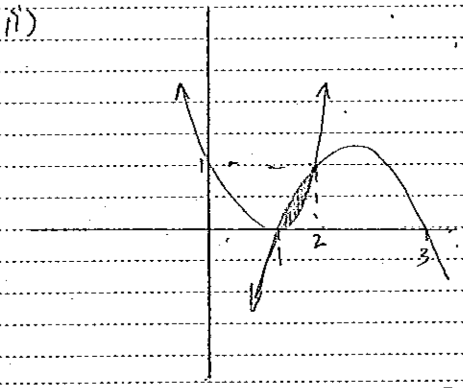
$x^2 - 3x + 2 = 0.$

$(x-2)(x-1) = 0.$

$x = 2, 1$

Points of intersection at $(2,1)$, $(1,0)$

ii)




$A = \int_1^2 4x - x^2 - 3 \;-\; \int_1^2 x^2 - 2x + 1$

$$A = \left[\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right]_1^2 - \left[\frac{x^3}{3} - \frac{2x^2}{2} + x\right]_1^2$$

$$A = \left[\left(8 - \frac{8}{3} - 6\right) - \left(2 - \frac{1}{3} - 3\right)\right] - \left[\left(\frac{8}{3} - 4 + 2\right) - \left(\frac{1}{3} - 1 + 1\right)\right]$$

$$A = \left[\left(-\frac{2}{3}\right) - \left(-\frac{4}{3}\right)\right] - \left[\left(\frac{2}{3}\right) - \frac{1}{3}\right]$$

$$A = \frac{2}{3} - \frac{1}{3}$$

$$A = \frac{1}{3} \text{ units}^2$$

e) $55 = 100e^{-\frac{t}{5}}$ (for $55^\circ C$)

$$\frac{55}{100} = e^{-\frac{t}{5}}$$

$$\therefore \ln\left(\frac{55}{100}\right) = \frac{-t}{5} \ln(e)$$

$$\therefore t = 2^\circ 59'$$

$$25 = 100e^{-\frac{t}{5}}$$

$$\frac{25}{100} = e^{-\frac{t}{5}}$$

$$\ln\left(\frac{25}{100}\right) = \frac{-t}{5} \ln(e)$$

$$\therefore t = 6^\circ \cancel{56}$$

So it is safe to drink between 2 min 54 s to 6 min 56 se.

3

$= 3'57''$

Sydney Boys' High School

124536

Student No.:

Paper: Maths accelerated Yr 11

Section: Q 4

Sheet No.: 1 of 1 for this Section.

| Q.No | Tick | Mark |
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| 2 | | |
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a)

i)

| $t$ | 0 | 2 | 4 |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | 0 | 3 | 6 |

$$A = \frac{2}{3}\left(0 + 6 + 4(3)\right)$$

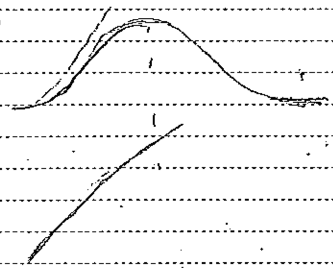
$V = 12$ at 4s

2

ii) at $t = 6$.

1

iii)



at $t = 8$. While acceleration becomes negative, the car is still travelling in a positive direction while it is slowing. The car reaches 0 velocity at $t = 8$ so it is furthest away at $t = 8$.

0

b)

i) ~~[illegible]~~ $(x^2-4)(x^2-2)$

$=(x-2)(x+2)(x-\sqrt{2})(x+\sqrt{2})$

$\therefore$ x int at $f(x)=0$

so x int at $x = 2, -2, \sqrt{2}, -\sqrt{2}$

ii) stat points at $f'(x)=0$.

$f'(x) =$

$U = x^2-4 \qquad V = x^2-2$

$u' = 2x \qquad v' = 2x$

$= 2x(x^2-2)+2x(x^2-4)$

$= 2x(x^2-2+x^2-4)$

$= 2x(2x^2-6)$

$\therefore 2x(2x^2-6)=0.$

$\therefore x=0, \pm\sqrt{3}$

so stat points at $(0,8)$, $(\sqrt{3},-1)$, $(-\sqrt{3},-1)$

nature:

| $x$ | $-2$ | $-\sqrt{3}$ | $-1$ | $0$ | $1$ | $\sqrt{3}$ | $2$ |
|---|---|---|---|---|---|---|---|
| $y'$ | $-8$ | $0$ | $8$ | $0$ | $-8$ | $0$ | $8$ |
| | \ | — | / | — | \ | — | / |

so $(0,8)$ is a max, $(\sqrt{3},-1)$ is a min,
$(-\sqrt{3},-1)$ is a min

(iii) point of inflexion at $f''(x)=0$

$f''(x) =$

$U = 2x \qquad V = 2x^2-6$

$u' = 2 \qquad v' = 4x$

$= 2(2x^2-6)+8x^2$

$= 4x^2-12+8x^2$

$= 12x^2-12$

$\therefore 12x^2-12=0$

$12(x^2-1)=0. \quad x = 1, -1$

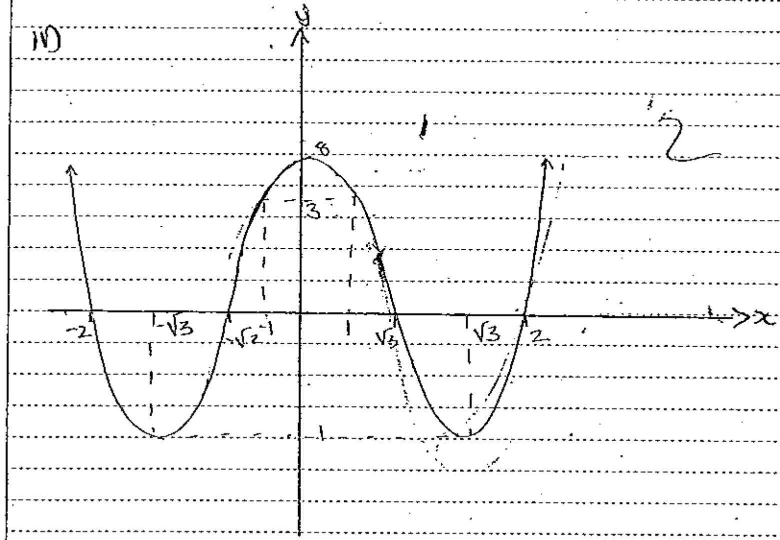
so points of inflexion may be at $(1,3)$, $(-1,3)$

test:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f''(x)$ | $36$ | $0$ | $-12$ | $0$ | $36$ |
| | ∪ | * | ∩ | * | ∪ |

so points of inflexion at $(1,3)$, $(-1,3)$.

iv)




v) concave up for $x<-1$, $x>1$.

Sydney Boys' High School

124537

Student No.:

Paper: Maths accelerated Yr 11

Section: Q5

Sheet No.: 1 of 1 for this Section.

| Q.No | Tick | Mark |
|---|---|---|
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a) i) ~~(500)(30) + ... x(500 - 5x) 150x - 6500~~

~~= 15000 + 1500x - 15x² + 150x - 6500~~ not entered

$P = 500(30) + 3x(500 - 5x) + 5(30) - 6500$

$P = 15000 + 1500x - 15x^2 + 150x - 6500$

$P = 8500 + 1350x - 15x^2$

ii) $P' = 1350 - 30x$ $(P'' = -30)$

max/min at $P' = 0$

$\therefore 1350 - 30x = 0$

$\therefore x = 45$

nature: $x$ 45 80 it has to be a max.

$P''$ -30

subbing 45 shirts into P

$P = 8500 + 1350(45) - 15(45)^2$ $P = \$38675$

no of shirts? (not ?)

b)

i) $f'(x) =$

$u = x$ $v = \ln(x)$

$u' = 1$ $v' = \frac{1}{x}$

$= \dfrac{\ln(x) - 1}{(\ln x)^2}$

$\therefore$ min/max at $f'(x) = 0$.

$\therefore \ln(x) - 1 = 0$

$\therefore \ln(x) = 0$

$\therefore x = e$

nature $x$ 1.5 e 3

$y'$ -3.6 0 0.045

$\therefore$ min point at $x = e$.

ii)

Since $f(e)$ is the min. value from (i)

then $f(e) \leq f(x)$

$\frac{e}{\ln e} \leq \frac{x}{\ln x}$ since $e, x > 0$

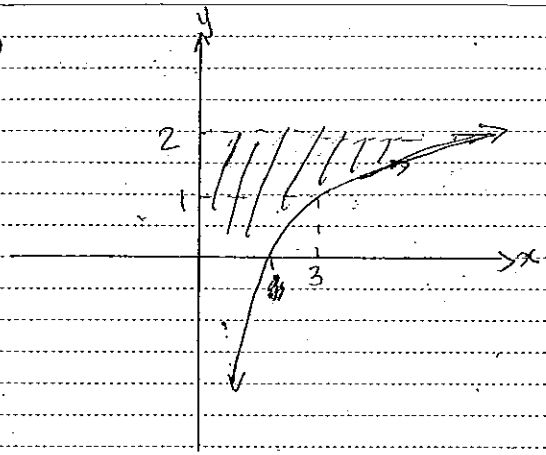
$\therefore e \ln x \leq x \ln e$

$\ln x^e \leq \ln e^x$

$\therefore x^e \leq e^x$ as req$^{d}$.

(c)

i)




We need to express $y = \log_3 x$ in terms of x.

$\therefore 3^y = x$.

$\therefore V = \pi \int_0^2 9^y \, dy$ (since $(3^y)^2 = 9^y$)

$V = \pi \left[ \frac{9^y}{\ln(9)} \right]_0^2$

$\frac{d}{dx} 5^x = \ln(5) \cdot 5^x$

$V = \pi \left( \frac{9^2}{\ln(9)} \right) - \left( \frac{1}{\ln(9)} \right)$

$V = \pi \left( \frac{81}{\ln(9)} - \frac{1}{\ln(9)} \right)$

$V = \pi \left( \frac{80}{\ln(9)} \right)$

$V = \frac{80\pi}{\ln(9)}$

$V = \frac{80\pi}{2\ln(3)} = \frac{40\pi}{\ln(3)}$