Name:

Maths Class:

Sydney Technical High School

Year 11

Mathematics Extension 1

HSC Course

Assessment 2

July, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet has been supplied for your use

Section 1	Multiple Choice Questions 1-5 5 Marks
Section II	Questions 6-11 61 Marks

SECTION 1

Use the multiple choice answer sheet for Questions 1-5

1. What is the value of ab if the lines $ax + 2y = 6$ and $4y = bx - 9$ are perpendicular?

A. $-\frac{1}{2}$

B. $\frac{1}{4}$

C. -8

D. 8

2. Express $x - \frac{1}{x}$ with a rational denominator given that $x = 3 + \sqrt{2}$

A. $\frac{4\sqrt{2}+9}{5}$

B. $\frac{6\sqrt{2}+27}{7}$

C. 6

D. $\frac{8\sqrt{2}+18}{7}$

3. If $\sin\theta = \frac{1}{4}$ and $\cos\theta < 0$, what is the exact value of $\tan 2\theta$?

A. $\frac{-15}{7\sqrt{15}}$

B. $\frac{15}{7\sqrt{15}}$

C. $\frac{-16}{7\sqrt{15}}$

D. $\frac{16}{7\sqrt{15}}$

4. What is the solution to the inequality

$$\frac{3}{x-2} \leq 4$$

A. $x < -2$ and $x \geq \frac{-11}{4}$

B. $x > -2$ and $x \leq \frac{-11}{4}$

C. $x < 2$ and $x \geq \frac{11}{4}$

D. $x > 2$ and $x \leq \frac{11}{4}$

5. Simplify $\frac{x^3-1}{x^2-1} \times \frac{x^2-4x-5}{4x^2+4x+4}$

A. $\frac{x-5}{4}$

B. $\frac{x-1}{4}$

C. $\frac{x+1}{4}$

D. $\frac{x^2+x+1}{4}$

SECTION 2

Total Marks (61)

Attempt Questions 6-11

Answer each question in your writing booklet starting each question on a new page.

QUESTION 6 (10 Marks) **Start on a new page.**

(a) Solve for x : $2.2^x - 5 = \frac{12}{2^x}$ 2

(b) The point $(6, k)$ is 8 units from the straight line $3x + 4y + 2 = 0$. Find k 3

(c) (i) On the same diagram sketch the graphs of

$y = |x - 2|$ and $y = \frac{3}{x}$ 2

(ii) For what values of x is $|x - 2| < \frac{3}{x}$ 1

(d) Determine if $f(x) = x^3 - \sin x$ is odd, even or neither. Show all working. 2

End of Question 6

QUESTION 7 (10 Marks) Start on a new page.

(a) Find the acute angle between $y = 2x - 1$ and $3x + 2y - 4 = 0$ correct to the nearest degree. 2

(b) Find the co-ordinates of the point $P(x, y)$ which divides the interval joining the points $A(-4, 5)$ and $B(5, -1)$ internally in the ratio 2:1. 2

(c) Find the greatest possible domain of

$$y = \frac{2\sqrt{x+3}}{x-7}$$ 2

(d) (i) Express $\sqrt{12}\sin x + 2\cos x$ in the form $R\sin(x+\propto)$ where $R > 0$ and $0 \leq x \leq 90°$ 2

(ii) Hence solve $\sqrt{12}\sin x + 2\cos x = -3$ for $0 \leq x \leq 360°$ 2

End of Question

QUESTION 8 (10 Marks) Start on a new page.

(a) If $\cot(5x - 40)^0 = \tan(3x + 10)^0$, evaluate x. 2

(b) Find in exact form, the value of $\sin 75°$ 2

(c) Use 't' results to prove

$$\frac{1+\sin x-\cos x}{1+\sin x+\cos x} = \tan\frac{x}{2}$$ 2

(d) Prove that $\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$ 2

(e) Solve $\sqrt{3}\tan 2\theta - 1 = 0$ for $0 \leq \theta \leq 360°$ 2

QUESTION 9 (10 Marks) Start on a new page.

(a) Find the equation of the straight line passing through (3, 6) and also passes through the intersection of the lines $x - 2y = 0$ and $3x + y + 7 = 0$ 3

(b) The point (-5, -4) divides the interval joining (1, 5) to (p, q) externally in the ratio 3:2. Find the values of p and q. 2

(c) Using $t = \tan\frac{\theta}{2}$ solve the equation $\sin\theta + 2\cos\theta = 1$ given $0 \leq \theta \leq 360°$ (give your answer in degrees to the nearest minute) 3

(d) Prove the identity $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos 2A$ 2

End of Question 9

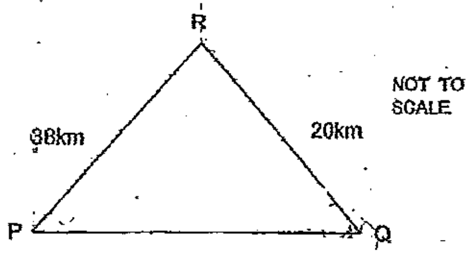
QUESTION 10 (10 Marks) Start on a new page.

(a) The line $3x + ky - 5 = 0$ makes an angle of 135° with the positive x axis.

(i) Show that the gradient of the line is $\frac{-3}{k}$. 1

(ii) Find the value of k. 2

(b)



In the diagram, the point Q is due east of P.

The point R is 38km from P and 20km from Q.

The bearing of R from Q is 325°.

(i) What is the size of angle PQR? 1

(ii) What is the bearing of R from P to the nearest minute? 3

(c) (i) Show that $cosec2\theta - cot2\theta = tan\theta$ 2

(ii) Hence find the exact value of $tan22.5°$ 1

End of Question 10

QUESTION 11 (11 Marks) Start on a new page.

(a) Solve for x: $|x^2 - 5| = 5x + 9$ 2

(b) From three points A, B and C on level ground, the angles of elevation of the top T of a hill are 32°, 30°and 25° respectively. Point A is due west and C is due south of the hill H. Point B lies on the straight line AC.

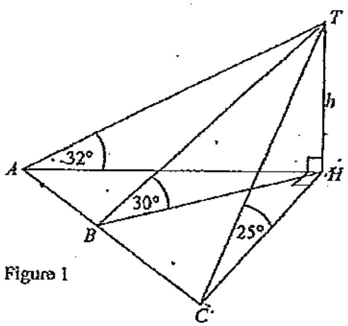


Figure 1

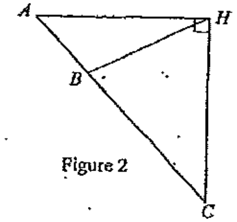


Figure 2

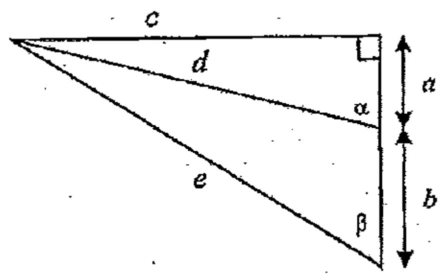
Copy Figure 2 into your writing booklet.

(i) Find an expression for AH in terms of h. 1

(ii) Show that $\angle HCA = 36°44'$ 2

(iii) Find $\angle HBC$ correct to the nearest minute. 2

(iv) Find the bearing of H from B 1



(c) Show that $a = \frac{b\cos\alpha\sin\beta}{\sin(\alpha - \beta)}$ 3

End of Examination

SECTION 1 (Multiple Choice) Yr11 Ext 1

1. $2y = 6 - ax$

$y = \frac{-ax}{2} + 3$

$m_1 = \frac{-a}{2}$

$4y = bx - 9$ (D.)

$y = \frac{bx}{4} - \frac{9}{4}$

$m_2 = \frac{b}{4}$

$\frac{-a}{2} \cdot \frac{b}{4} = -1$

$-ab = -8$

$ab = 8$

2. $\frac{3+\sqrt{2}}{1} - \frac{1}{3+\sqrt{2}}$

$= \frac{(3+\sqrt{2})^2 - 1}{3+\sqrt{2}}$

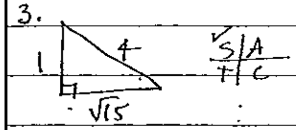
$= \frac{9 + 6\sqrt{2} + 2 - 1}{3+\sqrt{2}}$ (D)

$= \frac{10+6\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{30 + 18\sqrt{2} - 10\sqrt{2} - 12}{9-2}$

$= \frac{8\sqrt{2} + 18}{7}$

3.



$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= \frac{2 \times \frac{-1}{\sqrt{15}}}{1 - \left(\frac{-1}{\sqrt{15}}\right)^2}$ (A.)

$= \frac{-2}{\sqrt{15}} \div \frac{14}{15}$

$= \frac{-15}{7\sqrt{15}}$

4. $\frac{3}{x-2} \le 4$

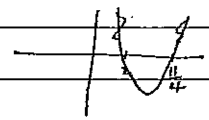
$3(x-2) \le 4(x-2)^2$

$3x - 6 \le 4(x^2 - 4x + 4)$

$3x - 6 \le 4x^2 - 16x + 16$ (C)

$4x^2 - 19x + 22 \ge 0$

$(4x-11)(x-2) \ge 0$



5. $\frac{\cancel{(x-1)}\,\cancel{(x^2+x+1)}}{\cancel{(x-1)}\,\cancel{(x+1)}} \times \frac{(x-5)\cancel{(x+1)}}{4\cancel{(x^2+x+1)}}$ (A)

$= \frac{x-5}{4}$

Student Name: Teacher Name:

QUESTION 6

(a) Let $y = 2^x$

$2y - 5 = \frac{12}{y}$

$2y^2 - 5y = 12$

$2y^2 - 5y - 12 = 0$

$(2y+3)(y-4) = 0$

$y = -\frac{3}{2} \qquad y = 4$

$2^x = -\frac{3}{2} \qquad 2^x = 4$

No solution $\qquad x = 2$

(b) $D = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right| \qquad \overset{A}{3}x + \overset{B}{4}y + \overset{C}{2} = 0$

$D = \left|\frac{3 \times 6 + 4 \times k + 2}{\sqrt{3^2+4^2}}\right|$

$D = \left|\frac{20+4k}{5}\right|$

$\therefore \left|\frac{20+4k}{5}\right| = 8$

$|20+4k| = 40$

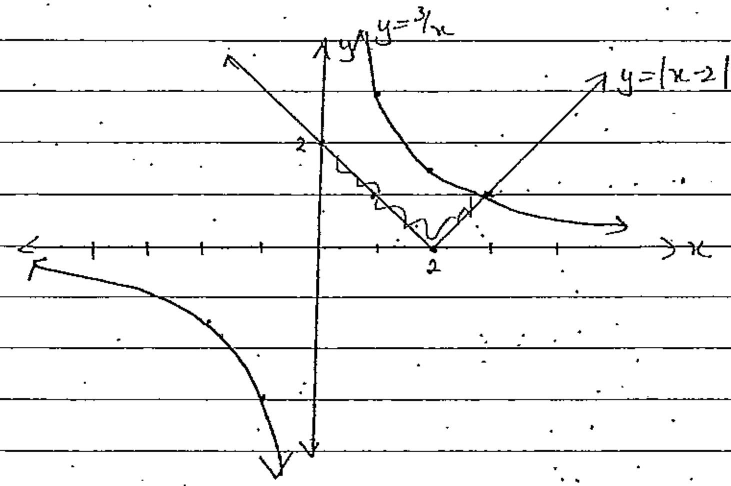
$20 + 4k = 40 \quad$ or $\quad 20 + 4k = -40$

$4k = 20 \qquad 4k = -60$

$k = 5 \qquad k = -15$

Student Name: Teacher Name:

(c) (i)



(ii) Pt of int $\frac{3}{x} = |x-2|$

$3 = x^2 - 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1 \qquad (3, 1)$

$\therefore 0 < x < 3$

(d) $f(x) = x^3 - \sin x$

$f(-x) = (-x)^3 - \sin(-x)$

$= -x^3 + \sin x$

$= -(x^3 - \sin x)$

$= -f(x)$

$\therefore$ as $f(-x) = -f(x)$ function is odd.

Student Name: ______________________ Teacher Name: ______________________

QUESTION 7

(a) $y = 2x - 1$ $\qquad$ $3x + 2y - 4 = 0$

$m_1 = 2$ $\qquad$ $2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

$m_2 = -\frac{3}{2}$

$$\tan\alpha = \left| \frac{2 - -\frac{3}{2}}{1 + 2(-\frac{3}{2})} \right|$$

$$= \left| \frac{4+3}{2-6} \right|$$

$$= \frac{7}{4}$$

$$\alpha = 60^\circ.$$

(b) $A(-4,5)$ $\qquad$ $B(5,-1)$

$2 : 1$

$$= \left(\frac{10-4}{2+1}, \frac{-2+5}{2+1} \right)$$

$$= P(2,1)$$

(c) $x + 3 \geq 0$

$x \geq -3$, $x \neq 7$

(d)(i) $\sqrt{12}\sin x + 2\cos x$

$R = \sqrt{\sqrt{12}^2 + 2^2}$

$R = 4$

$\tan^{-1}\frac{2}{\sqrt{12}} = 30^\circ$.

$\therefore \sqrt{12}\sin x + 2\cos x = 4\sin(x + 30^\circ)$

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(ii) $4\sin(x+30) = -3$ $\qquad$ $0 \leq x \leq 360^\circ$

$\sin(x+30) = -\frac{3}{4}$

$48^\circ 35'$ $\qquad$ $x + 30 = 228^\circ 35', 311^\circ 25'$

S | A
T | C

$x = 281^\circ 25', 198^\circ 35'$

Student Name: ______ Teacher Name: ______

QUESTION 8

(a) $\tan[90-(5x-40)] = \tan(3x+10)$

$90-5x+40 = 3x+10$

$120 = 8x$

$x = 15$

(b) $\sin 75 = \sin(30+45)$

$= \sin 30 \cos 45 + \cos 30 \sin 45$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

(c) $LHS = \frac{1+\sin x - \cos x}{1+\sin x + \cos x}$

$= \dfrac{1 + \frac{2t}{1+t^2} - \left(\frac{1-t^2}{1+t^2}\right)}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}$

$= \dfrac{\frac{1+t^2+2t+t^2-1}{1+t^2}}{\frac{1+t^2+2t+1-t^2}{1+t^2}}$

$= \frac{2t^2+2t}{2t+2}$

$= \frac{2t\cancel{(t+1)}}{2\cancel{(t+1)}}$

$= t$

$= \tan \frac{x}{2}$

$= RHS$

Student Name: ______ Teacher Name: ______

(d) $LHS = \frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$

$= \sec^2\theta - \sin^2\theta$

$= \tan^2\theta + 1 - \sin^2\theta$

$= \tan^2\theta + \cos^2\theta$

$= RHS$

(e) $\sqrt{3}\tan 2\theta - 1 = 0$ S | A / T | C

$\tan 2\theta = \frac{1}{\sqrt{3}}$

$2\theta = 30, 210, 390, 570$

$\theta = 15, 105, 195, 285$

QUESTION 9

(a) $x-2y=0 \quad \times 3$

$3x+y=-7$

$3x-6y=0$

$\underline{3x+y=-7}$

$-7y=7$

$y=-1$

$x+2=0$

$x=-2$

$(-2,-1)\ (3,6)$

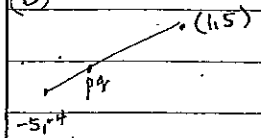
$\dfrac{y-6}{x-3} = \dfrac{6--1}{3--2}$

$5(y-6)=7(x-3)$

$5y-30=7x-21$

$\therefore\ 7x-5y+9=0$

(b)



$-5 = \dfrac{3p-2}{3+-2}$ $\qquad$ $-4 = \dfrac{3q-10}{3+-2}$

$3p-2=-5$ $\qquad$ $3q-10=-4$

$3p=-3$ $\qquad$ $3q=6.$

$p=-1$ $\qquad$ $q=2$

$\therefore\ (p,q)=(-1,2)$

(c) $\sin\theta + 2\cos\theta = 1$

$\dfrac{2t}{1+t^2} + 2\left(\dfrac{1-t^2}{1+t^2}\right) = 1$

$2t + 2(1-t^2) = 1+t^2$

$2t + 2-2t^2-1-t^2 = 0$

$3t^2-2t-1=0$

$(3t+1)(t-1)=0$

$t=-\frac{1}{3},\ \text{or}\ 1$

$\tan\frac{\theta}{2} = -\frac{1}{3}$ $\qquad$ $\tan\frac{\theta}{2}=1$

$\frac{\theta}{2} = 161.565$ $\qquad$ $\frac{\theta}{2}=45$

$\theta = 323°08'$ $\qquad$ $\theta = 90°$

(d) LHS $= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$= \dfrac{1-\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\sin^2 A}{\cos^2 A}}$

$= \dfrac{\cos^2 A-\sin^2 A}{\cos^2 A+\sin^2 A}$

$= \dfrac{\cos 2A}{1}$

$= \cos 2A$

$=$ RHS.

Student Name: Teacher Name:

QUESTION 10.

(a) (i) $3x + ky - 5 = 0$

$ky = -3x + 5$

$y = \frac{-3x}{k} + \frac{5}{k}$

$\therefore m = -3/k$

(ii) $m = \tan\theta$

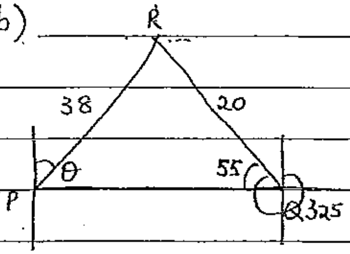
$-3/k = \tan 135$

$\frac{-3}{k} = -1$

$-k = -3$

$k = 3.$

(b)



(i) $\angle PQR = 55°$

(ii) $\frac{\sin\theta}{20} = \frac{\sin 55}{38}$

$\sin\theta = 0.4311326549$

$\theta = 25°32'$

$\therefore$ Bearing of R from P

$= 90 - 25°32'$

$= 64°28'$

(c)(i) $\operatorname{cosec} 2\theta - \cot 2\theta$

$= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$

$= \frac{1 - \cos 2\theta}{\sin 2\theta}$

$= \frac{1 - (\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$

$= \frac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2}\cancel{\sin\theta}\cos\theta}$

(ii) $\tan 22.5°$

$= \operatorname{cosec} 45 - \cot 45$

$= \sqrt{2} - 1$

Student Name: Teacher Name:

QUESTION 11

(a) $|x^2 - 5| = 5x + 9$ NB $5x + 9 \geq 0$

$5x \geq -9$

$x \geq -9/5$

$x^2 - 5 = 5x + 9$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = \cancel{-2}, 7$

$x^2 - 5 = -5x - 9$

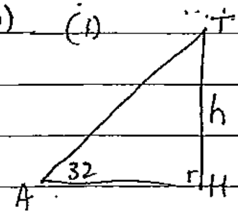
$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = \cancel{-4}, -1$

$\therefore x = 7$ and $x = -1$

(b) (i)

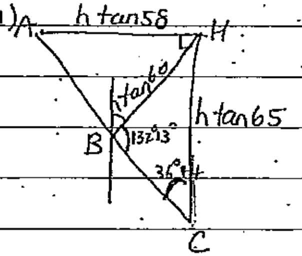


$\tan 32 = \frac{h}{AH}$

$AH = \frac{h}{\tan 32}$

$= h\tan 58$

(ii)



$\tan C = \frac{\cancel{h}\tan 58}{\cancel{h}\tan 65}$

$C = 36°44'$

iii) In ΔHBC

$\frac{\sin B}{h\tan 65} = \frac{\sin C}{h\tan 60}$

$\sin B = \frac{h\tan 65 \sin 36°44}{\tan 60}$

$\therefore B = \cancel{47°46'}$ or $132°13'$

$\therefore \angle B = 132°13'$

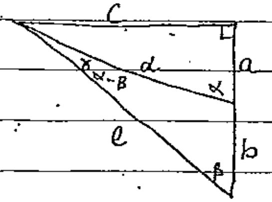
(iv)

Bearing $= 180 - (132°13' + 36°44')$

$= 11°3'$

Student Name: ______________ Teacher Name: ______________

(c)



$$\cos\alpha = \frac{a}{d}$$

$$\alpha = \gamma + \beta$$
$$\gamma = \alpha - \beta.$$

$$d = \frac{a}{\cos\alpha}$$

$$\frac{b}{\sin(\alpha-\beta)} = \frac{d}{\sin\beta}$$

$$\frac{b}{\sin(\alpha-\beta)} = \frac{a}{\cos\alpha\sin\beta}.$$

$$a\sin(\alpha-\beta) = b\cos\alpha\sin\beta$$

$$a = \frac{b\cos\alpha\sin\beta}{\sin(\alpha-\beta)}$$