Sydney Technical High School

Name:

Maths Class:

Year 11

Mathematics

Preliminary Course

Assessment 2

July, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- NESA reference sheet is supplied for your use.

Section I Multiple Choice
Questions 1-5
5 Marks

Section II Questions 6-13
65 Marks

SECTION I

Multiple Choice (5 Marks)
Use the Multiple Choice Answer sheet for Questions 1-5

1. What is the exact value of $\cot 60°$?

   A. $\sqrt{3}$

   B. $\frac{1}{\sqrt{3}}$

   C. $\frac{\sqrt{3}}{2}$

   D. $\frac{2}{\sqrt{3}}$

2. If the lines $3x - 2y + 5 = 0$ and $y = kx - 1$ are perpendicular, what is the value of k?

   A. $-\frac{1}{3}$

   B. $-\frac{2}{3}$

   C. $-\frac{3}{2}$

   D. -3

3. Find all the values of x in the interval $0° \leq x \leq 360°$ for which $\tan x = -1$.

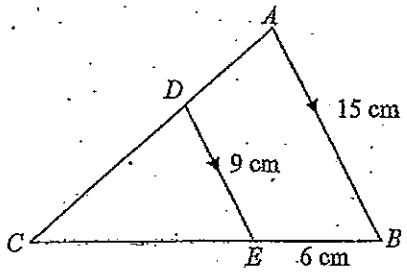
   A. 45°, 225°

   B. 135°, 225°

   C. 135°, 315°

   D. 225°, 315°

4. In the diagram below, ABC is a triangle and $AB \parallel DE$



Given that $AB = 15$ cm, $DE = 9$ cm and $BE = 6$ cm, what is the value of BC?

A. 3.6 cm

B. 6 cm

C. 9 cm

D. 15 cm

5. The midpoint of (a, b) and $(5, -3)$ is $(-1, 4)$. What are the values of a and b?

A. $a = -7, b = 11$

B. $a = 11, b = -10$

C. $a = 2, b = \frac{1}{2}$

D. $a = 3, b = 3\frac{1}{2}$

END OF SECTION I

SECTION II

65 marks
Attempt Questions 6 – 13
Answer each question on a new page in the answer booklet.

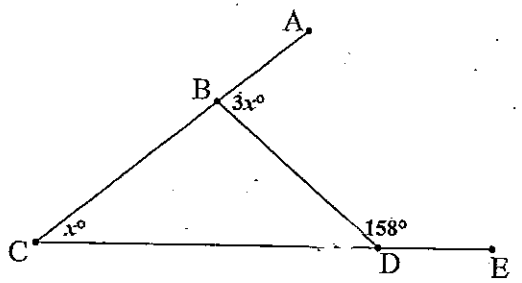
QUESTION 6 (8 marks) Start on a new page.

a) Find the exact value of $\sin 240°$ 2

b) Find the exact value of $\sec 225°$ 2

c) A regular polygon has each interior angle equal to 140°. Find the sum of all its interior angles. 2

d)



In the diagram above, find the value of x, giving reasons. 2

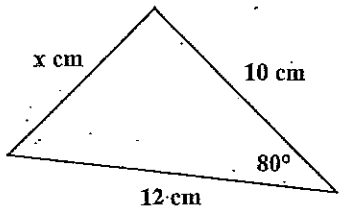
End of Question 6

QUESTION 7 (8 marks) Start on a new page.

a) Evaluate $|-2| - |-6|$ 1

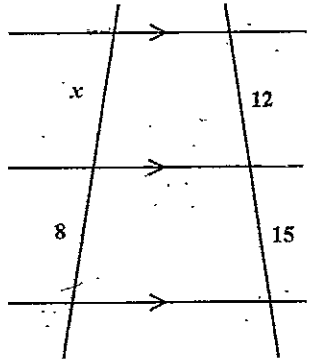
b) Simplify $\frac{\cos(90^\circ - \theta)}{\sin(180^\circ + \theta)}$ 2

c) Find the value of x cm in the diagram below. Leave your answer correct to the nearest cm. 2



d) The line through $P(7,\ p)$ and $Q(4, -5)$ has a gradient of 3. What is the value of p? 1

e) Find the value of x, giving reasons. All measurements are in cm. 2



End of Question 7

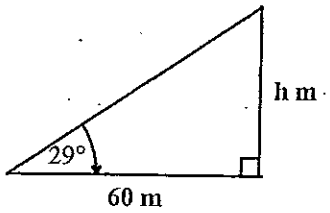
QUESTION 8 (8 marks) Start on a new page.

a) Solve $|2x - 3| \leq 7$ and sketch the solution on a number line. 2

b) For acute angles A and B it is given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{15}{17}$. Find the exact value of $\sec A + \tan B$. 3

c) A man wishes to find out the height of a tower, to the nearest metre. When he is 60 m from the base of the tower, he sees the top of the tower at an angle of elevation of 29°.

Not to scale



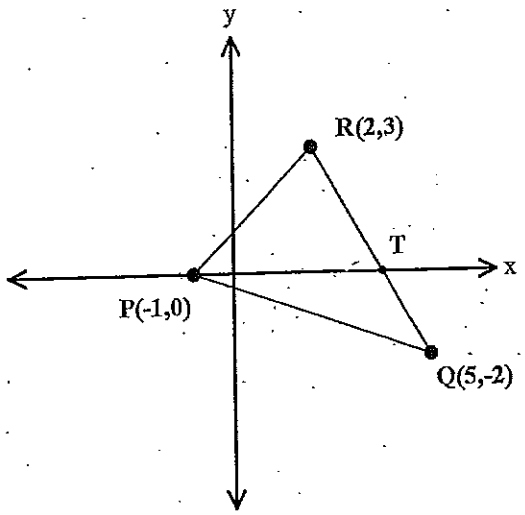
i. Calculate the height of the tower to the nearest metre. 1

ii. If he moves 15 m closer to the tower, what will the angle of elevation to the top now be, to the nearest degree? 2

End of Question 8

QUESTION 9 (8 marks) Start on a new page.

a) The points P $(-1, 0)$, R $(2, 3)$ and Q $(5, -2)$ are shown on the Cartesian Plane below. The interval RQ meets the x-axis at T.
Using the information on the following diagram



Not to scale

i. Show that the equation of the line PQ is $x + 3y + 1 = 0$ — 2

ii. Find the length of PQ — 1

iii. Find the perpendicular distance from R to PQ — 2

iv. Find the area of triangle PRQ — 1

v. Find the size of the angle RTP correct to nearest degree — 2

End of Question 9

QUESTION 10 (8 marks) Start on a new page.

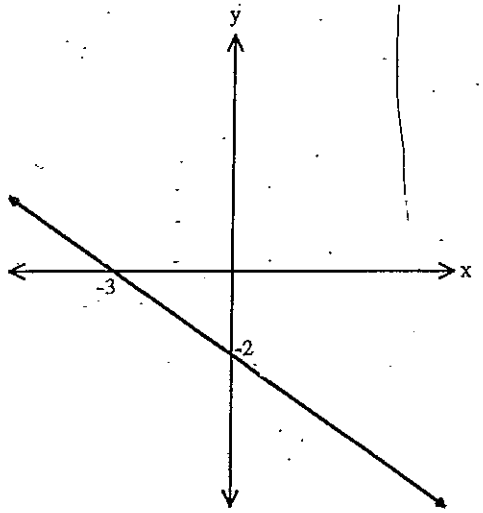
a) Show the exact value of $3\tan 210° + 2\sin 300° = 0$ (Do not use a calculator, show all steps). — 2

b) Simplify $(\sec\theta - 1)(\sec\theta + 1)$ — 2

c) The graph given by $y = |2x + k|$ where k is constant, passes through the point (2, 3). Find the possible values of k. — 2

d) Find a value of x if $\text{cosec}\,(x - 25)° = \sec 65°$ — 1

e) What is the equation of the line below? Leave your answer in general form or gradient-intercept form. — 1



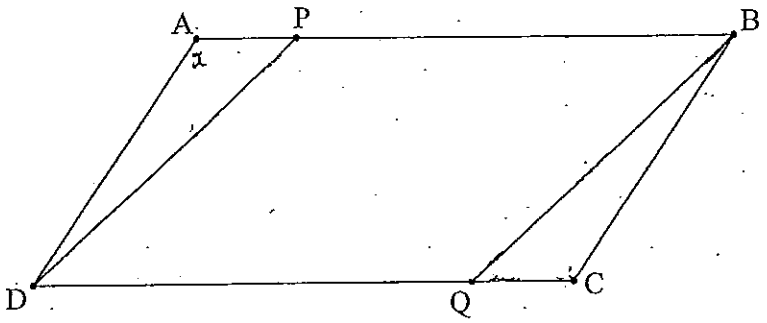
End of Question 10

QUESTION 11 (8 marks) Start on a new page.

a) i. Sketch the graph of $y = |x + 3|$, showing all important features 2

ii. State the domain and range of $y = |x + 3|$ 2

b) $ABCD$ is a parallelogram and $AP = QC$.



i. Prove $\triangle APD \equiv \triangle CQB$. 2

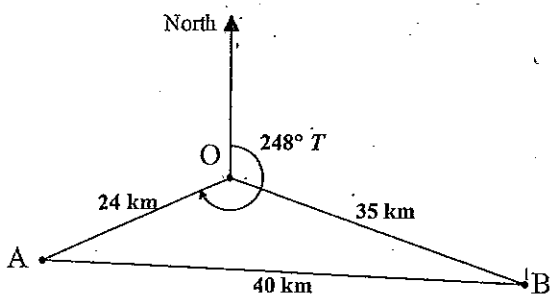
ii. Hence prove that $PD \parallel QB$. 2

End of Question 11

QUESTION 12 (8 marks) Start on a new page.

a) Prove that $\frac{cosx}{1+sinx} + \frac{cosx}{1-sinx} = 2secx$ 3

b) A section of a rainforest is to be scoured in the search for a new species. The shape is shown below.
The bearing of landmark A from landmark O is 248°T and is 24 km in distance.
The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



Not to scale

i. Find the size of $\angle AOB$. 2

ii. Hence or otherwise, calculate the area of this section of the rainforest. 1

iii. What is the bearing of landmark O from landmark B? 2

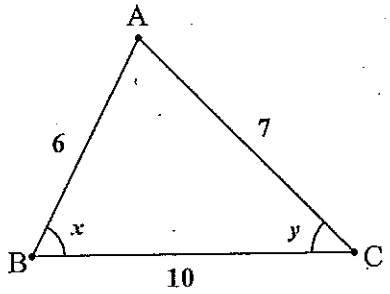
End of Question 12

QUESTION 13 (9 marks) **Start on a new page.**

a) Solve $\cos^2\theta = \frac{3}{4}$ for $0° \leq \theta \leq 360°$ — 3

b) Find the shortest distance between the parallel lines $2x - 3y + 7 = 0$ and $2x - 3y - 3 = 0$ — 2

c) The diagram below shows $\triangle ABC$ with $AC = 7$, $BC = 10$ and $AB = 6$. $\angle ABC = x$ and $\angle ACB = y$.



i. Express the size of $\angle BAC$ in terms of x and y. — 1

ii. Hence, show that $\sin x + \sin y = \frac{13}{10}\sin(x + y)$ — 3

End of Examination ☺

Solutions

Year 11 – Assessment 2 2017

1. $\cot 60° = \frac{1}{\tan 60°}$

$= \frac{1}{\sqrt{3}}$ (B)

2. $3x - 2y + 5 = 0$

$2y = 3x + 5$

$y = \frac{3}{2}x + \frac{5}{2}$

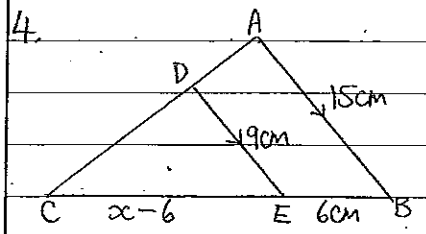
$m_1 = \frac{3}{2}$

$\therefore m_2 = -\frac{2}{3}$ (B)

3. $\tan x = -1$ 2nd, 4th

$x = 135°, 315°$ (C)

4.



Let $BC = x$

$\therefore CE = x - 6$

$\frac{x-6}{x} = \frac{9}{15}$

$15x - 90 = 9x$

$6x = 90$

$x = 15$ (D)

5. $\frac{a+5}{2} = -1$ $\frac{b-3}{2} = 4$

$a + 5 = -2$ $b - 3 = 8$

$a = -7$ $b = 11$ (A)

SECTION II

Question 6

a) $\sin 240° = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

b) $\sec 225° = -\frac{1}{\cos 45°}$

$= -\sqrt{2}$

c) Interior angle $= 140°$

$140n = (n-2) \times 180$

$140n = 180n - 360$

$40n = 360$

$n = 9$

Angle sum $= (9-2) \times 180°$

$= 1260°$

d) $\angle CBD = 180 - 3x$ (angles on a straight line)

$\angle CBD + \angle BCD = 158°$ (exterior angle $\triangle BCD$)

$180 - 3x + x = 158°$

$-2x = -22$

$x = 11°$

Question 7

a) $|-2|-|-1-6| = 2-6$

$= -4$

b) $\frac{\cos(90°-\theta)}{\sin(180°+\theta)} = \frac{\sin\theta}{-\sin\theta}$

$= -1$

c) $x^2 = 10^2+12^2-2(10)(12)\cos 80°$

$x^2 = 202.3244...$

$x = 14.22...$

$= 14\text{cm}$

d) $\frac{-5-p}{4-7} = 3$

$\frac{-5-p}{-3} = 3$

$-5-p = -9$

$p = 4$

e) $\frac{x}{8} = \frac{12}{15}$ (ratio of intercepts between parallel lines)

$15x = 96$

$x = 6.4\text{cm}$

Question 8

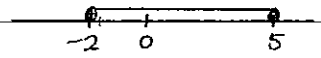
a) $|2x-3| \leq 7$

$2x-3 \leq 7 \qquad -2x+3 \leq 7$

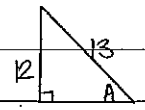
$2x \leq 10 \qquad -2x \leq 4$

$x \leq 5 \qquad x \geq -2$

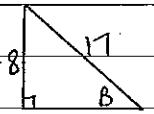
$\therefore -2 \leq x \leq 5$



b) $\sin A = \frac{12}{13}$



$\cos B = \frac{15}{17}$



$\sec A + \tan B = \frac{1}{\cos A} + \tan B$

$= \frac{13}{5} + \frac{8}{15}$

$= \frac{47}{15}$

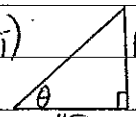
c)(i) $\tan 29° = \frac{h}{60}$

$h = 60 \times \tan 29°$

$= 33.258...$

$= 33\text{m}$

(ii)



$\tan\theta = \frac{h}{45}$ [Used full h value]

$\theta = 36.467°...$

$= 36°$

Question 9

a)(i) $P(-1,0) \quad Q(5,-2)$

$m_{PQ} = \frac{-2-0}{5--1}$

$= -\frac{1}{3}$

Equation: $y-0 = -\frac{1}{3}(x+1)$

$y = -\frac{1}{3}x - \frac{1}{3}$

$3y = -x-1$

$\therefore x+3y+1 = 0$

(ii) $d_{PQ} = \sqrt{(-1-5)^2+(0--2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$

(iii) $R(2,3) \qquad x+3y+1=0 \quad a=1, b=3, c=1$

$d_{\perp} = \frac{|1(2)+3(3)+1|}{\sqrt{1^2+3^2}}$

$= \frac{12}{\sqrt{10}}$ OR $\frac{12\sqrt{10}}{10} = \frac{6\sqrt{10}}{5}$

(iv) Area $\triangle PRQ = \frac{1}{2} \times \frac{12}{\sqrt{10}} \times 2\sqrt{10}$

$= 12 \text{ units}^2$

(v) $m_{RQ} = \frac{-2-3}{5-2}$

$= -\frac{5}{3}$

$\tan(\angle RTP) = \left|-\frac{5}{3}\right|$

$\angle RTP = 59°$

Question 10

a) $LHS = 3\tan 210° + 2\sin 300°$

$= 3\times(+\tan 30°) + 2\times(-\sin 60°)$

$= 3\times\frac{1}{\sqrt{3}} + 2\times-\frac{\sqrt{3}}{2}$

$= \frac{3}{\sqrt{3}} - \sqrt{3}$

$= \frac{3\sqrt{3}}{3} - \sqrt{3}$

$= \sqrt{3} - \sqrt{3}$

$= 0$

$= RHS$

b) $(\sec\theta-1)(\sec\theta+1)$

$= \sec^2\theta - 1 \qquad (1+\tan^2\theta = \sec^2\theta)$

$= \tan^2\theta$

c) $y = |2x+k| \qquad (2,3)$

$3 = |2(2)+k|$

$3 = 4+k \qquad 3 = -4-k$

$k = -1 \qquad k = -7$

d) $\text{cosec}(x-25)^\circ = \sec 65^\circ$

$x - 25 = 90 - 65$

$x = 50^\circ$

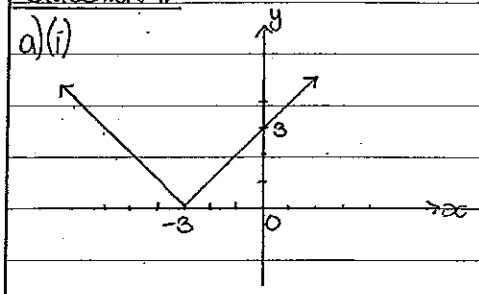
e) $m = -\frac{2}{3}$ y-intercept: -2

$\therefore y = -\frac{2}{3}x - 2$

OR $2x + 3y + 6 = 0$

Question 11

a)(i)



(ii) Domain: all real x

Range: $y \geq 0$

b)(i) In $\triangle APD$ and $\triangle CQB$,

$AP = QC$ (given)

$\angle DAP = \angle BCQ$ (opposite angles in a parallelogram are equal)

$AD = BC$ (opposite sides of parallelogram ABCD are equal)

$\therefore \triangle APD \equiv \triangle CQB$ (SAS)

(ii) As $AB = DC$ (opposite sides of parallelogram ABCD are equal) and $AP = QC$ (given),

$PB = QD$

Now $PD = BQ$ (corresponding sides in congruent triangles)

$\therefore PBQD$ is a parallelogram (2 pairs of opposite sides equal)

$\therefore PD \parallel QB$ (opposite sides of a parallelogram are parallel)

Question 12

a) $\text{LHS} = \frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}$

$= \frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{1-\sin^2 x}$

$= \frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{\cos^2 x}$

$= \frac{2\cos x}{\cos^2 x}$

$= \frac{2}{\cos x}$

$= 2\sec x$

$= \text{RHS}$

b)(i) Let $\angle AOB = \theta$

$\cos\theta = \frac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}$

$\theta = 83^\circ 8'$

$\therefore \angle AOB = 83^\circ 8'$

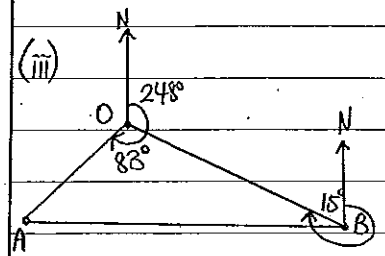
(ii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ 8'$

$= 416.987\ldots$

$= 417\ \text{km}^2$ (nearest kilometre)

(iii)



$\angle NOB = 248^\circ - 83^\circ$

$= 165^\circ$

$\angle NBO = 180^\circ - 165^\circ$ (co-interior angles)

$= 15^\circ$

$\therefore$ Bearing of O from B $= 360^\circ - 15^\circ$

$= 345^\circ$

Question 13

a) $\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

b) $2x - 3y + 7 = 0$

$(0, \frac{7}{3})$ lies on this line.

$2x - 3y - 3 = 0$ $a = 2, b = -3, c = -3$

$\therefore d_\perp = \frac{|2(0) - 3(\frac{7}{3}) - 3|}{\sqrt{2^2 + (-3)^2}}$

$= \frac{10}{\sqrt{13}}$ OR $\frac{10\sqrt{13}}{13}$ units

c)(i) $\angle BAC = 180 - (x+y)$

(ii) $\sin(180 - (x+y)) = \sin(x+y)$

$\therefore$ In $\triangle ABC$,

$\frac{\sin x}{7} = \frac{\sin y}{6} = \frac{\sin(x+y)}{10}$

Now, $\frac{\sin x}{7} = \frac{\sin(x+y)}{10}$

$\sin x = \frac{7\sin(x+y)}{10}$

$$\frac{\sin y}{6} = \frac{\sin(x+y)}{10}$$

$$\sin y = \frac{6\sin(x+y)}{10}$$

$$\text{LHS} = \sin x + \sin y$$

$$= \frac{7\sin(x+y)}{10} + \frac{6\sin(x+y)}{10}$$

$$= \frac{13}{10}\sin(x+y)$$

$$= \text{RHS}$$