Alexandria Park Community School

2016 HIGHER SCHOOL CERTIFICATE
TASK 3

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
  Black pen is preferred
- Board-approved calculators may be used
- A Board-provided reference sheet is allowed
- In Questions 4-5, show relevant mathematical reasoning and/or calculations

Total Marks – 33

Section I

3 marks

- Attempt questions 1-3
- Allow about 5 minutes for this section

Section II

30 marks

- Attempt questions 4-5
- Allow about 50 minutes for this section

Section I

3 marks
Attempt Questions 1-3
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1-3.

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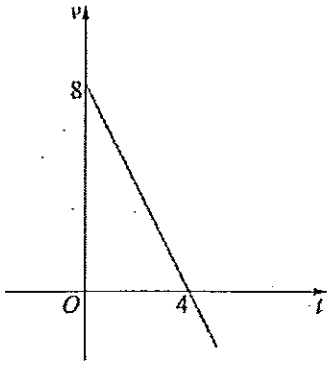
1 The first three terms of an arithmetic series are 3, 7 and 11.
What is the 16^{th} term of this series?

(A) 59 (B) 63 (C) 465 (D) 495

2 A particle is moving along the x-axis. The graph shows its velocity v metres per second at time t seconds.



When $t = 0$ the displacement x is equal to 2 metres. What is the maximum value of the displacement x?

(A) 8 m (B) 14 m (C) 16 m (D) 18 m

3 A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time, $\dot{x} = -3\ ms^{-1}$ and $\ddot{x} = 2\ ms^{-2}$.

Which statement describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed.

(B) The particle is moving to the left with increasing speed.

(C) The particle is moving to the right with decreasing speed.

(D) The particle is moving to the left with decreasing speed.

End of Section I

Section II

30 marks
Attempt Questions 4-5
Allow about 55 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 4-5, your responses should include relevant mathematical reasoning and/or calculations.

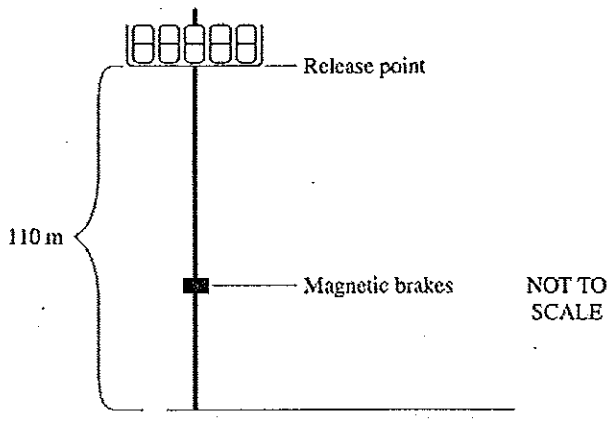
Question 4 (15 marks) Use the Writing Booklet.

(a) Evaluate 2

$$\sum_{r=2}^{4} \frac{1}{r}$$

(b) Find the limiting sum of the geometric series $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots$ 2

(c) In a theme park ride, a chair is release from a height of 110 metres and falls vertically. Magnetic brakes are applies when the velocity of the chair reaches -37 metres per second.



The height of the chair at time t seconds is x metres. The acceleration of the chair is given by $\ddot{x} = -10$. At the release point, $t = 0$, $x = 110$, and $\dot{x} = 0$.

(i) Using calculus, show that $x = -5t^2 + 110$ 2

(ii) How far has the chair fallen when the magnetic brakes are applied? 2

Question 4 continues on the following page

(d) Sam borrows \$100 000 to be repaid at a reducible interest rate of 0.6% per month. Let $\$A_n$ be the amount owing at the end of n months and $\$M$ be the monthly repayment.

(i) Show that $A_2 = 100\,000(1.006)^2 - M(1 + 1.006)$ **1**

(ii) Show that $A_n = 100\,000(1.006)^n - M\left(\frac{(1.006)^n - 1}{0.006}\right)$ **2**

(iii) Same makes monthly repayments of \$780. **1**

Show that after making 120 monthly repayments the amount owing is \$68 500 to the nearest \$100.

(iv) Immediately after making the 120th repayment, Sam makes a one-off payment, reducing the amount owing to \$48 500. The interest rate and monthly repayment remain unchanged. **3**

After how many more months will the amount owing be completely repaid?

End of Question 4

Question 5 (15 marks) Use the Writing Booklet.

(a) The amount of caffeine, C, in the human body decreases according to the equation

$$\frac{dC}{dt} = -0.14C$$

where C is measured in mg and t is the time in hours.

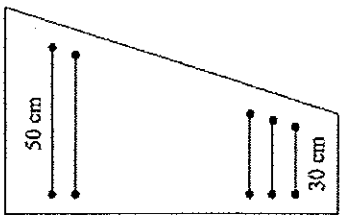
(i) Show that $C = Ae^{0.14t}$ is a solution to $\frac{dC}{dt} = -0.14C$, where A is a constant. 1

(ii) When $t = 0$, there are 130 mg of caffeine in Lee's body. Find the value of A. 1

(iii) What is the amount of caffeine in Lee's body after 7 hours? 1

(iv) What is the time taken for the amount of caffeine in Lee's body to halve? 2

(b)



NOT TO SCALE

A simple instrument has many strings, attached as shown in the diagram. The difference between the lengths of adjacent strings is a constant, so that the lengths of the strings are the terms of an arithmetics series.

The shortest string is 30 cm long and the longest string is 50 cm. The sum of the lengths of all the strings is 1240 cm.

(i) Find the number of strings. 2

(ii) Find the difference in length between adjacent strings. 2

Question 5 continues on the following page

(c) The displacement of a particle moving along the x-axis is given by

$$x = t - \frac{1}{1+t}$$

where x is the displacement from the origin in metres, t is the time in seconds, and $t \geq 0$.

(i) Show that the acceleration of the particle is always negative. **2**

(ii) What value does the velocity approach as t increases indefinitely? **1**

(d) At the beginning of every 8-hour period, a patient is given 10 mL of a particular drug.

During each of these 8-hour periods, the patient's body partially breaks down the drug. Only $\frac{1}{3}$ of the total amount of the drug present in the patient's body at the beginning of each 8-hour period remains at the end of that period.

(i) How much of the drug is in the patient's body immediately after the second dose is given? **1**

(ii) Show that the total amout of the drug in the patient's body never exceeds 15 mL. **2**

End of paper

Student Name _____ Teacher _____

2016 HSC 2U Mathematics Task 3
Multiple Choice Answer Sheet
Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○ ✓
2. A ○ B ○ C ○ D ● ✓
3. A ○ B ○ C ○ D ● ✓

Question 4

a) $\sum_{r=2}^{4} \frac{1}{r}$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$= \frac{13}{12}$

$= 1\frac{1}{12}$ ✓✓

b) $\frac{1}{1-(-\frac{1}{4})}$

$\frac{1}{1+\frac{1}{4}} = \frac{4}{5}$ ✓✓

c) $\ddot{x} = -10$

$\dot{x} = -10t + C$ ✓

when $\dot{x}=0$, $t=0$

$C=0$

$x = \frac{-10t^2}{2} + C$

when $x = 110$, $t=0$

$x = -5t^2 + 110$ ✓

ii) $-37 = -10t$

$t = 3.7$

$110 - 41.55 =$

$= 68.45$ m ✓✓

when $t = -3.7$

$x = -5(3.7)^2 + 110$

$= 41.55$

d $P = 100\,000$

$r = 0.6\%$

(i) $A_1 = 100\,000(1.006) - M$

$A_2 = (A_1)(1.006) - M$

$= 100\,000(1.006)^2 - M(1 + 1.006)$ ✓

(ii) ~~Since $A_2 = 100\,000(1.$~~

$A_3 = 100\,000(1.006)^3 - M(1 + 1.006 + 1.006^2 \ldots)$

$\therefore A_n = 100\,000(1.006)^n - M(1 + 1.006 + 1.006 \ldots 1.006^{n-1})$

$a = 1, r = 1.006, n = n$

$\Rightarrow A_n = 100\,000(1.006)^n - M\left(\frac{(1.006)^n - 1}{.006}\right)$ ✓ ✓

(iii) $M = 780, n = 120$

$\therefore A_n = 100\,000(1.006)^{120} - 780\left(\frac{(1.006)^{120} - 1}{.006}\right)$

$= 68499.45$

$\approx \$68\,500$ as required ✓

(IV) ~~48500~~ $= 48\,500(1.006)^n - 780\left(\frac{1.006^n - 1}{.006}\right) \geq 0$

~~48000~~ $(.006)(48500)(1.006)^n - 780(1.006^n) + 780 \geq 0$

$291(1.006)^n - 780(1.006^n) + 780 = 0$

$(1.006)^n(291 - 780) + 780 = 0$

$(1.006)^n(-489) + 780 = 0$

$(1.006)^n = \frac{-780}{-489}$

$= \frac{780}{489}$

$n \ln(1.006) = \ln \frac{780}{489}$

$n = \frac{\ln \frac{780}{489}}{\ln 1.006}$

$n = 78.055$

or 79 months ✓✓✓

Question 5

a) $\frac{dC}{dt} = -0.14C$ to prove

$C = Ae^{-0.14t}$

$\frac{dC}{dt} = -0.14Ae^{-0.14t}$

$= -0.14(C)$ ✓

$= -0.14C$ as required

ii) when $t = 0$

$Ae^{-0.14t} = 130$

$Ae^{0} = 130$

$A = 130$ ✓

iii) $130e^{-0.14(7)}$

$= 48.790\,44285$

or 48.8 (1 dp) mg ✓

iv) $130 = A$

$\frac{A}{2} = Ae^{-0.14(t)}$

$\frac{1}{2} = e^{-0.14(t)}$

$\frac{\ln \frac{1}{2}}{-0.14} = t$

$t = 4.95$ hours ✓

or 4 hours 57 ✓ minutes

b)

$1240 = \frac{n}{2}(30+50)$

$2480 = 80n$

$n = 31$ strings //

ii) $a = 30$, $n = 31$

~~$1240 = 31[\frac{1240}{31} = 30 + (n-1)d$~~

$1240 = 30 + 30d$

$1240 = \frac{31}{2}(60 + (30)d)$

$2480 = 31(60 + 30d)$

$80 = 60 + 30d$

$20 = 30d$

$d = \frac{2}{3}$ //

c) $x = t - \frac{1}{1+t}$

$v = 1 - (1+t)^{-1}$

$= 1 + (1+t)^{-2}$

$= 1 + \frac{1}{(1+t)^2}$

$\frac{dx}{dt}$

$\frac{dv}{dt}$

$a = (1+t)^{-2}$

$a = -2(1+t)^{-3}$

$= \frac{-2}{(1+t)^3}$

Since it is -2, for $t \geq 0$ acceleration will always be negative //

(y)

ii) $v = 1 + \frac{1}{(1+t)^2}$

as $\frac{1}{(1+t)^2} \to \infty$ $\quad = 0$

$\therefore$ ~~$v = 1$~~ 1

d) $a = 10$

~~10~~

i) $10 + 10\frac{1}{3}$

$= 13\frac{1}{3}$ mL ✓

ii) $10 + 10\frac{1}{3} + (10\frac{1}{3})^2$

$\frac{10}{1 - \frac{1}{3}} = 15$ ✓✓

Since the limiting sum = 15

it can never exceed