Student ______________________

BRIGIDINE COLLEGE RANDWICK

HALF YEARLY

MATHEMATICS EXTENSION 2

- 2013 -

(TIME - 2 HOURS)

DIRECTIONS TO CANDIDATES

- *Put your **name** at the top of this paper and on **each of the 5 sections** to be collected.*
- *All **questions** are to be attempted.*
- *All **questions** are to be answered on **separate pages** and will be collected in **5 separate bundles** at the end of this exam.*
- *All necessary working should be shown in every question in PEN.*
- *Full marks may not be awarded for careless or badly arranged work*

TABLE OF STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}; \quad n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0 \qquad \text{Note: } \ln x = \log_e x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

MULTIPLE CHOICE (PLACE ANSWERS ON THE ANSWER SHEET PROVIDED)

1. Which one of the following is an expression for $\int \frac{1}{\sqrt{x^2-6x+10}}\,dx$?

(A) $\ln\left(x-3-\sqrt{x^2-6x+10}\right)+c$

(B) $\ln\left(x+3-\sqrt{x^2-6x+10}\right)+c$

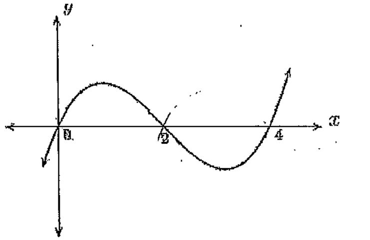
(C) $\ln\left(x-3+\sqrt{x^2-6x+10}\right)+c$

(D) $\ln\left(x+3+\sqrt{x^2-6x+10}\right)+c$

2. Find the value of $\int_0^{\pi} 5\sin x\cos^4 x\,dx$

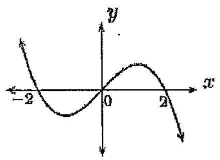
(A) 0 B 2 (C) -2 (D) 20

3.

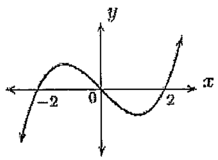


The graph of $y = f(x)$ is shown above. The graph of $y = f(2-x)$ is:

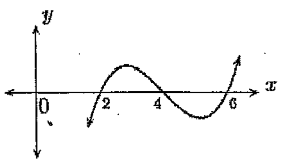
(A)



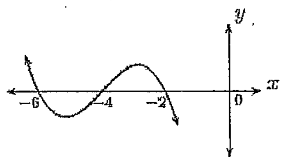
(B)



(C)

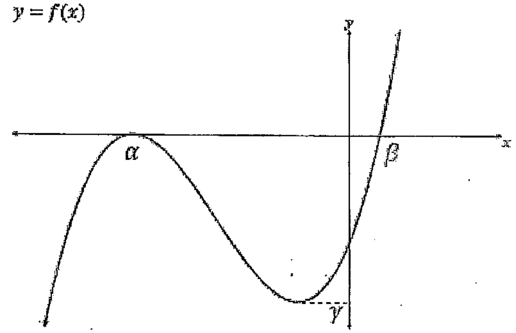


(D)



Multiple Choice Continued

4. The graph of $y = f(x)$ is given below



For what value(s) of k will the graph of $y = f(x)+k$ have exactly one root ?

(A) $k \le 0 \; and \; k \ge \gamma$

(B) $\alpha < k < \beta$

(C) $0 \le k \le \gamma$

(D) $k < 0 \; and \; k > \gamma$

5. The real factors of the polynomial expression $x^4 + x^2 - 12$ are :

(A) $(x-2)(x+2)(x^2+3)$

(B) $(x^2+4)(x^2-3)$

(C) $(x^2+4)(x+\sqrt{3})(x-\sqrt{3})$

(D) $(x^2+4)(x^2+3)$

Multiple Choice Continued

6. Let $z=1+2i$ and $w=-2+i$. What is the value of $\frac{5}{iw}$?

   (A) $-1-2i$

   (B) $-1+2i$

   (C) $1-2i$

   (D) $1+2i$

7. What is the value of $\arg\bar{z}$ given the complex number $z=1-i\sqrt{3}$?

   (A) $-\frac{\pi}{3}$

   (B) $-\frac{2\pi}{3}$

   (C) $-\frac{\pi}{3}$

   (D) $\frac{\pi}{3}$

8. It is given that $3+i$ is a root of $P(z)=z^3+az^2+bz+10$ where a and b are real numbers.

   Which expression factorises $P(z)$ over the real numbers?

   (A) $(z-1)(z^2+6z-10)$

   (B) $(z-1)(z^2-6z-10)$

   (C) $(z+1)(z^2+6z+10)$

   (D) $(z+1)(z^2-6z+10)$

Multiple Choice Continued

9. Which of the following complex numbers equals $(\sqrt{3}+i)^4$?

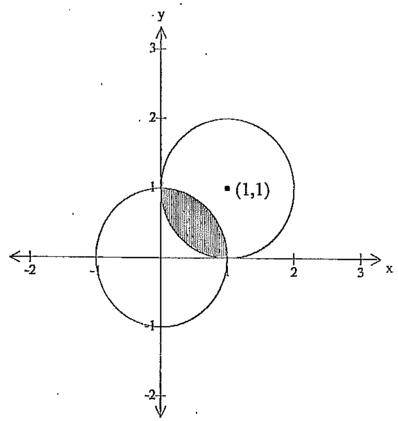
   (A) $-2+\frac{2}{\sqrt{3}}i$

   (B) $-8+\frac{8}{\sqrt{3}}i$

   (C) $-2+2\sqrt{3}i$

   (D) $-8+8\sqrt{3}i$

10. Consider the Argand diagram below.



Which inequality could define the shaded area?

   (A) $|z|\leq 1$ and $|z-(1-i)|\geq 1$

   (B) $|z|\leq 1$ and $|z-(1+i)|\geq 1$

   (C) $|z|\leq 1$ and $|z-(1-i)|\leq 1$

   (D) $|z|\leq 1$ and $|z-(1+i)|\leq 1$

Question 11 (15 Marks) (Start a New Page) **Marks**

(a) Find $\int \frac{dx}{\sqrt{16-9x^2}}$ 2

(b) Find $\int \left(\frac{1-\sin x}{\cos^2 x}\right) dx$ 2

(c) Evaluate $\int_1^e x \ln x \, dx$ 3

(d) Evaluate $\int_2^3 \frac{dx}{x^2-1}$ 4

(e) Using the substitution $t = \tan\frac{\theta}{2}$ find

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$$ 4

Question 12 (15 marks) (Start a New Page) **Marks**

(a) (i) Express $\sqrt{-5+12i}$ in the form $a+ib$ where a, b are real. 3

(ii) Hence solve $2z^2-(6+i)z+5=0$ for z. 2
Express your answer in the form $a+ib$.

(b) (i) Express each of the complex numbers $z_1=\sqrt{2}+\sqrt{2}i$ and $z_2=-1+\sqrt{3}i$ in modulus-argument form. 2

Represent the vectors z_1, z_2 and z_1+z_2 on an Argand diagram.

(ii) Find the exact values of $\arg\left(\frac{z_2}{z_1}\right)$ and $\arg(z_1+z_2)$. 2

(c) Let ω be a complex root of $z^3=1$

(i) Solve $z^3=1$, giving the value/s of ω in modulus-argument form 2

(ii) Given $\omega \neq 1$, show that $\omega^2+\omega+1=0$ 2

(iii) Hence simplify $(1+\omega)^8$ 2

Question 13 (15 marks) (Start a New Page) **Marks**

(a) (i) If $P(x)=x^4+x^3-3x^2-5x-2$, show that $P(x)=0$ has a multiple root, find this root and its multiplicity. 3

(ii) Hence factorise $P(x)=x^4+x^3-3x^2-5x-2$ into its linear factors. 1

(b) Consider the curve given implicitly by the equation $4y^3-3y=x^3+x-1$

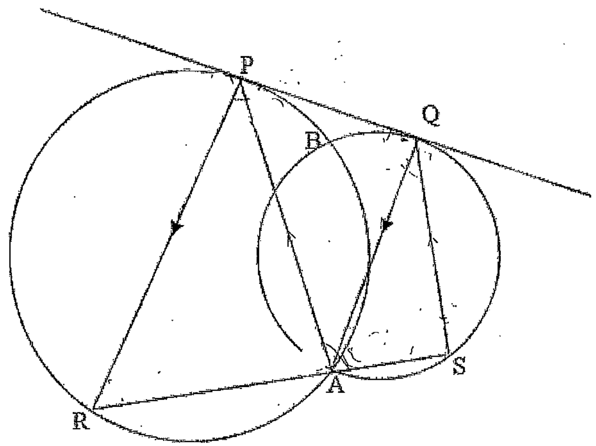
(i) Show that $\frac{dy}{dx}=\frac{3x^2+1}{12y^2-3}$ 2

(ii) Find the equation of the tangent to $4y^3-3y=x^3+x-1$ at the point $(0,-1)$ 2

(c) The polynomial equation $z^3-7z^2+25z-39=0$ has one zero equal to $(2+3i)$. Find the other two zeros. 2

Question 13 Continued Over Page

(d) Two circles intersect at A and B and a common tangent touches them at P and Q as shown.



A chord PR is drawn parallel to QA.
RA is produced to meet the other circle at S.

(i) Explain why $\angle PQA=\angle QSA$. 1

(ii) Prove that $PQSR$ is a cyclic quadrilateral. 2

(iii) Hence prove that PA is parallel to QS. 2

Question 14 (15 marks) (Start a New Page) **Marks**

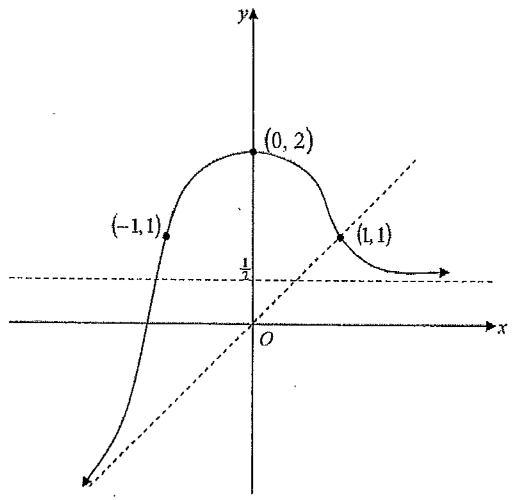
(a) The equation $x^3+2x-1=0$ has roots α,β,γ. Find the monic equation with roots

(i) $\alpha^2,\beta^2,\gamma^2$ 2

(ii) $\alpha\beta,\ \beta\gamma,\ \alpha\gamma$ 2

(iii) Evaluate $\alpha^3+\beta^3+\gamma^3$ 2

(b)



The diagram shows the graph of $y=f(x)$. The lines $y=x$ and $y=\frac{1}{2}$ are both asymptotes.
Draw separate graphs of the following clearly indicating all important features.

(i) $y=\left(f(x)\right)^2$ 2

(ii) $y=f(x)-x$ 2

(c) (i) Show that the recurrence (reduction) formula for

$$I_n=\int \tan^n x\,dx \text{ is } I_n=\frac{1}{n-1}\tan^{n-1}x-I_{n-2}$$ 3

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}}\tan^3 x\,dx$ 2

End of Exam Paper

Brigidine College Randwick
2013 Extension 2 Half Yearly Exam

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Multiple Choice Answer Sheet

Name : ______________

Select Your Answers

1.	A	B	(C)	D
2.	A	(B)	C	D
3.	(A)	B	C	D
4.	A	B	(C) ✓	D
5.	A	B	(C)	D
6.	A	(B)	C	D
7.	A	B	C	(D)
8.	A	B	C	(D) ✓
9.	A	B	C	(D) ✓
10.	A	B	C	(D)

7/10

11

13

a) $\int \frac{dx}{\sqrt{16-9x^2}}$

$= \int \frac{dx}{\sqrt{9\left(\frac{16}{9}-x^2\right)}}$

$= \int \frac{dx}{3\sqrt{\frac{16}{9}-x^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right) + c$

$= \frac{1}{3}\sin^{-1}\frac{3x}{4} + c$

(b) $\int \frac{1-\sin x}{\cos^2 x}\,dx$

$= \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\,dx$

$= \int \sec^2 x - \sec x \tan x\,dx$

$= \tan x - \sec x + c$

c) $\int_1^e x\ln x$

$u = \ln x \qquad v' = x$

$u' = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$\int_1^e x\ln x = \frac{x^2}{2}\ln x\Big]_1^e - \int_1^e \frac{x}{2}\,dx$

$= \left(\frac{e^2}{2}\ln(e) - \frac{1}{2}\ln 1\right) - \frac{1}{2}\int_1^e x\,dx$

$= \frac{e^2}{2} - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e$

$= \frac{e^2}{2} - \frac{1}{2}\left[\frac{e^2}{2} - \frac{1}{2}\right]$

$= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4}$

$= \frac{e^2}{4} + \frac{1}{4}$ ✓✓✓

d) $\int_2^3 \frac{dx}{x^2-1} = \int_2^3 \frac{dx}{(x-1)(x+1)}$

$= \int_2^3 \frac{A}{(x-1)} + \frac{B}{(x+1)}\,dx$

$\therefore \frac{1}{x^2-1} \equiv \frac{A}{(x-1)} + \frac{B}{(x+1)}$

$1 \equiv A(x+1) + B(x-1)$

when $x = 1 \qquad x = -1$

$1 = 2A \qquad 1 = -2B$

$A = \frac{1}{2} \qquad -\frac{1}{2} = B$

$\therefore \int_2^3 \frac{\frac{1}{2}}{(x-1)} + \frac{-\frac{1}{2}}{(x+1)}\,dx$

$\frac{1}{2}\int_2^3 \frac{1}{x-1} \quad - \frac{1}{2}\int_2^3 \frac{1}{x+1}$

$= \frac{1}{2}\left[\ln(x-1)\right]_2^3 - \frac{1}{2}\left[\ln(x+1)\right]_2^3$

$= \frac{1}{2}\left[\ln 2 - \ln 1\right] - \frac{1}{2}\left[\ln 4 - \ln 3\right]$

$= \frac{1}{2}\ln 2 - \frac{1}{2}\ln\frac{4}{3}$

$= \frac{1}{2}\ln\frac{3}{2}$ // ✓✓✓✓

e) $t = \tan\frac{\theta}{2}$

$\int_0^{\pi/2} \frac{d\theta}{2+\cos\theta}$.

when $\theta = \frac{\pi}{2} \qquad \theta = 0$

$t = 1 \qquad t = 0$

$\frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2}$

$\frac{dt}{d\theta} = \frac{1}{2}\left(1+\tan^2\frac{\theta}{2}\right)$

$\frac{dt}{d\theta} = \frac{1}{2}(1+t^2)$

$d\theta = \frac{2dt}{1+t^2}$

$\therefore \int_0^1 \frac{\frac{2dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}}$

$= \int_0^1 \frac{\frac{2dt}{1+t^2}}{\frac{2(1+t^2)+1-t^2}{1+t^2}}$

$= \int_0^1 \frac{2dt}{2+2t^2+1-t^2}$

$$\int_0^1 \frac{2dt}{t^2+3}$$

$$= 2\int_0^1 \frac{1}{t^2+3}\,dt$$

$$= 2\int_0^1 \frac{1}{3+t^2}\,dt$$

$$= 2\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}\right]_0^1$$

$$= 2\left[\frac{1}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{0}{\sqrt{3}}\right]$$

$$= 2\left[\frac{1}{\sqrt{3}}\cdot\frac{\pi}{6}\right]$$

$$= 2\left[\frac{\pi}{6\sqrt{3}}\right]$$

$$= \frac{\pi}{3\sqrt{3}}$$ ✓✓✓✓

14/15 excellent

12

a) i $\sqrt{-5+12i}$ in the form $a+ib$

$$\left(\sqrt{-5+12i}\right)^2 = (a+ib)^2$$

$$-5+12i = a^2-b^2+2aib$$

i ... $-5 = a^2-b^2$

ii ... $12 = 2ab$

$$\frac{6}{b} = a$$

$\therefore$ sub $\frac{6}{b} = a$ into (i)

$$-5 = \left(\frac{6}{b}\right)^2 - b^2$$

$$-5 = \frac{36}{b^2} - b^2$$

$$-5b^2 = 36 - b^4$$

$$b^4 - 5b^2 - 36 = 0$$

$$(b^2+4)(b^2-9) = 0$$

$\therefore b^2+4=0$ $b^2-9=0$

$b^2=-4$ $b^2=9$

$b^2 \neq -4$ $\therefore b=\pm 3$

when $b=3$ $b=-3$

$a=2$ $a=-2$

$\therefore a=2,\ b=3$

$a=-2,\ b=-3$

$\therefore \pm(2+3i)$ ✓✓✓

a ii $2z^2 - (6+i)z + 5 = 0$

$z = \dfrac{(6+i) \pm \sqrt{(6+i)^2 - 4(2)(5)}}{4}$

$z = \dfrac{(6+i) \pm \sqrt{36+12i-1-40}}{4}$

$z = \dfrac{(6+i) \pm \sqrt{-5+12i}}{4}$

from part (i)
$\sqrt{-5+12i} = \pm(2+3i)$

$\therefore z = \dfrac{6+i \pm (2+3i)}{4}$

$z = \dfrac{6+i+2+3i}{4}$ or $z = \dfrac{6+i-2-3i}{4}$

$z = \dfrac{8+4i}{4}$ $\quad z = \dfrac{4-2i}{4}$

$z = 2+i$, $\quad z = 1 - \dfrac{1}{2}i$, $z = 1 - \dfrac{i}{2}$

b i $z_1 = \sqrt{2} + \sqrt{2}i$
$|z| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$
$= 2$
$\arg z = \tan^{-1}\left(\dfrac{\sqrt{2}}{\sqrt{2}}\right)$
$= \dfrac{\pi}{4}$

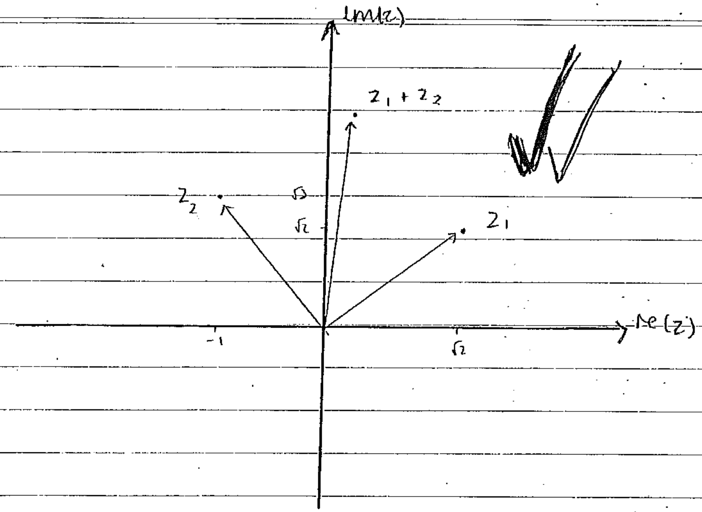
$\therefore 2(\cos(\pi/4) + i\sin(\pi/4))$
$= 2\text{cis}(\pi/4)$

$z_2 = -1 + \sqrt{3}i$
$|z| = \sqrt{(-1)^2 + (\sqrt{3})^2}$
$= 2$
$\arg z = \tan^{-1}\left(\dfrac{\sqrt{3}}{-1}\right)$
$= \dfrac{2\pi}{3}$

$\therefore 2\text{cis}(2\pi/3)$

good

PTO



b ii $\arg\left(\dfrac{z_2}{z_1}\right)$

$\arg z_2 = \dfrac{2\pi}{3}$ $\quad \arg z_1 = \dfrac{\pi}{4}$

$\therefore \arg\left(\dfrac{z_2}{z_1}\right) = \arg z_2 - \arg z_1$

$= \dfrac{2\pi}{3} - \dfrac{\pi}{4}$

$= \dfrac{5\pi}{12}$

$\arg(z_1 + z_2)$
$= \dfrac{\pi}{4} + \dfrac{2\pi}{3}$

$= \dfrac{11\pi}{12}$

c) $\omega \rightarrow z^3 = 1$ / $w^3 = 1$

i) $z^3 = 1$

$\rightarrow z^3 - 1 = 0$

$(z-1)(z^2+z+1) = 0$

$\therefore (w-1)(w^2+w+1) = 0$

$\therefore W = 1$ or $W = \frac{-1 \pm \sqrt{1-4}}{2}$

$\therefore W = 1$ $\quad W = \frac{-1 \pm \sqrt{3}i}{2}$

$W = 1$ $\quad W = \frac{-1}{2} + \frac{\sqrt{3}i}{2}$ $\quad W = \frac{-1}{2} - \frac{\sqrt{3}i}{2}$

$W = cis 0$ $\quad W = cis\left(\frac{-\pi}{3}\right)$

$W = cis(-2\pi/3)$

$W = cis(2\pi/3)$

good

ii $W \neq 1$

From part (i) $(w-1)(w^2+w+1) = 0$

$w - 1 = 0$ or $w^2 + w + 1 = 0$

therefore since $W \neq 1$

$w^2 + w + 1 = 0$

iii $(1+w)^8 = (-\omega^2)^8 = \omega^{16} = (\omega^3)^5 \cdot \omega = \omega$

$W^2 + w + 1 = 0$

$W^2 = -1 - W$

$W^2 = -(1+W)$ $\quad$ since $\quad w - 1 = 0$

$-W^2 = 1 + w$ $\quad W = 1, \; W^2 = 1$

$-(1)^2 = -1$ ~~$-1 = 1+w$~~

~~$\therefore (-1)^8 = 1$~~

13 $\quad$ Excellent $\quad$ 15/15

a) i $P(x) = x^4 + x^3 - 3x^2 - 5x - 2 = 0$

$P'(x) = 4x^3 + 3x^2 - 6x - 5 = 0$

$P''(x) = 12x^2 + 6x - 6 = 0$ $\quad 2x^2 + x - 1 = 0$

$P^3(x) = 24x + 6 = 0$ $\quad (2x-1)(x+1) = 0$

~~$\therefore 24x = -6$~~ $\quad x = \frac{1}{2} \quad x = -1$

~~$\therefore x = \frac{-1}{4}$~~

~~Test $x = \frac{-1}{4}$~~ $\quad$ Test $x = \frac{1}{2}$ and $x = -1$

$P(1/2) \neq 0$

~~$P(-1/4) =$~~

$P(-1) = 0$

$P'(-1) = 0$

$P''(-1) = 0$

$P^3(-1) \neq 0$

$\therefore P(x)$ has a multiple root of $x = -1$ with multiplicity of 3

$\therefore (x+1)^3$

ii $P(x) = x^4 + x^3 - 3x^2 - 5x - 2$

$(x+1)^3(x+k)$

$\therefore$ matching coefficients

$k = -2$

$\therefore (x+1)^3(x-2)$

b) i $4y^3 - 3y = x^3 + x - 1$

Show $\frac{dy}{dx} = \frac{3x^2+1}{12y^2-3}$

$\frac{dy}{dx}\left(\frac{d}{dy}4y^3\right) - \frac{dy}{dx}\left(\frac{d}{dy}3y\right) = \frac{d}{dx}x^3 + \frac{d}{dx}x - \frac{d}{dx}1$

$\frac{dy}{dx}(12y^2) - \frac{dy}{dx}(3) = 3x^2 + 1$

$\frac{dy}{dx}(12y^2 - 3) = 3x^2 + 1$

$\frac{dy}{dx} = \frac{3x^2+1}{12y^2-3}$

ii $4y^3 - 3y = x^3 + x - 1$

$\frac{dy}{dx} = \frac{3x^2+1}{12y^2-3}$

when $x=0$ and $y=-1$

$\frac{dy}{dx} = \frac{3(0)^2+1}{12(-1)^2-3}$

$\frac{dy}{dx} = \frac{1}{9}$

$\therefore y+1 = \frac{1}{9}(x-0)$

$9y+9 = x$

$0 = x-9y-9$

$\therefore$ eqn of tangent $\Rightarrow$ $x-9y-9=0$

c) $z^3 - 7z^2 + 25z - 39 = 0$

$(2+3i)$ = ~~roots~~ zeros

by conjugate theorem $(2-3i)$ is also a root

$\therefore (2+3i), (2-3i)$ are ~~roots~~ zeros

$\therefore$ quadratic eqn

$z^2 - (\alpha+\beta)z + (\alpha\beta) = 0$

$\alpha+\beta = 2+3i + 2-3i$

$= 4$

$\alpha\beta = (2+3i)(2-3i)$

$= 4-9i^2$

$= 4+9$

$= 13$

$\therefore z^2 - 4z + 13 = 0$

Girls need to know (i) (ii)

Must be clearly shown

$\therefore$ ~~$(z^2-4z+13)(z+k)$~~

$(z^2-4z+13)(z-k)=0$

matching coefficents.

$-39 = -13k$

$k=3$

$\therefore (z^2-4z+13)(z-3) = 0$

$\therefore$ zeroes are $2+3i$, $2-3i$, 3

(d) i $\angle PQA = \angle QSA$ since the $\angle$ between a tangent and a chord is = to alternate segment.

ii PQSR is a cyclic quad:

~~$\angle QPR = 180$~~

Let $\angle PQA = \theta$, then $\angle QSA = \theta$ (part i)

Then $\angle QPR = 180 - \theta$ (cointerior $\angle$s are supplementary in parallel lines $PR \parallel QA$)

$\therefore \angle QPR + \angle QSR = 180°$ (opp $\angle$ in cyclic quad are supplementary)

$180 - \theta + \theta = 180°$

$180° = 180°$

$\therefore$ PQSR is a cyclic quadrilateral

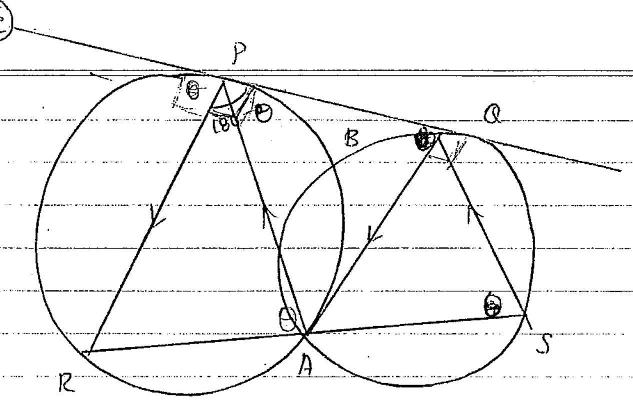
iii Prove ~~$\triangle PRA \parallel\!\parallel \triangle QAS$~~

~~$\angle QSA = \angle PQA$ (part i)~~

next page

(c)

d) iii



Prove $\triangle PRA \ ||| \ \triangle QAS$.

From part (ii). $\angle PQA = \theta = \angle QSA$

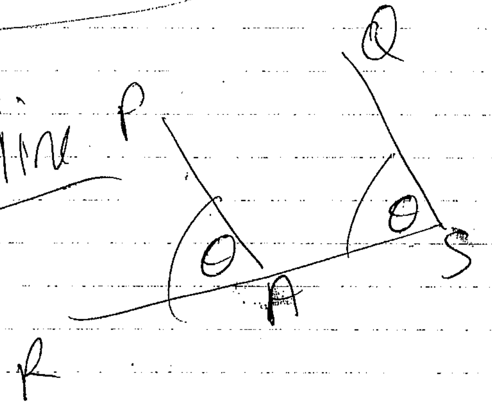
$\therefore \angle CPR = \theta$ (corresponding $\angle$ are $=$ in perpendicular lines $PR \parallel QA$)

Then $\angle PAR = \theta$ ($\angle$ between tangent and chord $=$ to alternate $\angle$ segment)

~~since PQRS is a cyclic quad~~

$\angle QPR = 90°$ (Tangent on a circle is perpendicular) ?

One more line



(14) $x^3 + 0x^2 + 2x - 1 = 0$

9

a) i $x^3 + 2x - 1 = 0$

$\alpha^2, \beta^2, \gamma^2$

$\therefore x = \alpha^2 \quad x = \beta^2 \quad x = \gamma^2$

$\therefore \sqrt{x} = \alpha \quad \beta = \sqrt{x} \quad \gamma = \sqrt{x}$

$\therefore (\sqrt{x})^3 + 2\sqrt{x} = 1$

$\sqrt{x}\,(x+2) = 1$

Squaring both sides

$x\,(x+2)^2 = 1$

$x\,(x^2+4x+4) - 1 = 0$

$x^3 + 4x^2 + 4x - 1 = 0$

(ii) Roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.

Since $\alpha\beta\gamma = 1$

$\therefore \alpha\beta = \frac{1}{\gamma}$

$\therefore \left(\frac{1}{x}\right)^3 + 2\left(\frac{1}{x}\right) - 1 = 0$ is the reqd. equation

$\frac{1}{x^3} + \frac{2}{x} - 1 = 0$

$\therefore 1 + 2x^2 - x^3 = 0$

$\Rightarrow x^3 - 2x^2 - 1 = 0$

iii $\alpha^3 + \beta^3 + \gamma^3$

$P(x) = x^3 + 2x - 1$

$P(\alpha) = \alpha^3 + 2\alpha - 1 = 0$ ①

$P(\beta) = \beta^3 + 2\beta - 1 = 0$ ②

$P(\gamma) = \gamma^3 + 2\gamma - 1 = 0$ ③

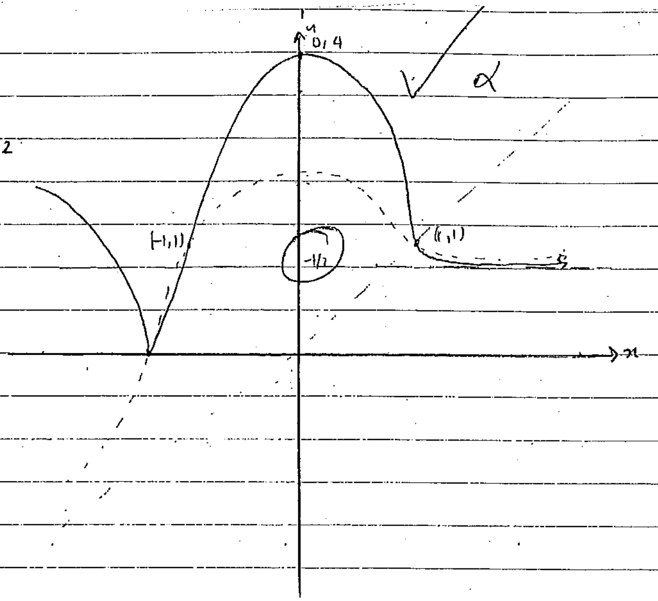
1+2+3 $\therefore (\alpha^3 + \beta^3 + \gamma^3) + 2(\alpha + \beta + \gamma) - 3 = 0$

$(\alpha^3 + \beta^3 + \gamma^3) + 2(0) - 3 = 0$

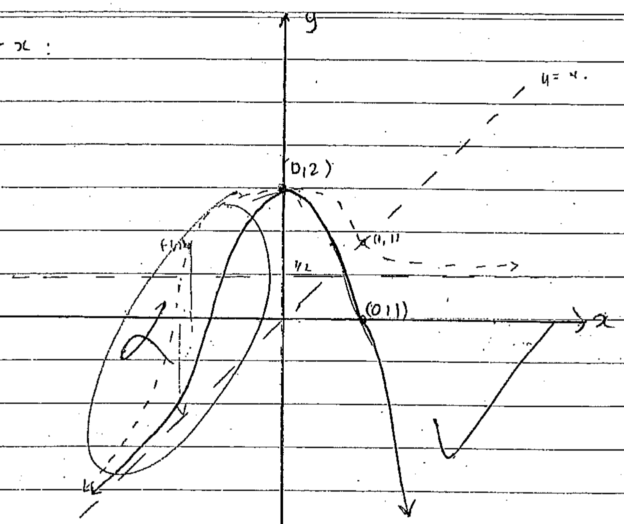
$\therefore \alpha^3 + \beta^3 + \gamma^3 = 3$

(14)(6)

→ i $y = (f(x))^2$



ii $y = f(x) - x$:



c) (i) $I_n = \int \tan^n x \, dx \Rightarrow I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$

$I_n = \int \tan^{n-2} x \cdot \tan^2 x \, dx$

$I_n = \int \tan^{n-2} x \, (\sec^2 x - 1) \, dx$

$I_n = \int \tan^{n-2} x \cdot \sec^2 x - \tan^{n-2} x \, dx$

$I_n = \int \tan^{n-2} x \sec^2 x \, dx - \int \tan^{n-2} x \, dx$

$I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

$I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$

$\therefore I_n = \int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$

ii $\int_0^{\pi/4} \tan^3 x \, dx$

n = 3

$I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$

$I_3 = \frac{1}{2} \tan^2 x \Big]_0^{\pi/4} - I_1 = \left(\frac{1}{2} \tan^2(\pi/4) - \frac{1}{2}\tan^2 0\right) - I_1$

$= \frac{1}{2} - I_1 = \frac{1}{2} - \ln \frac{1}{\sqrt{2}}$.

$I_1 = \int_0^{\pi/4} \tan x \, dx$

$= -\ln(\cos x)\Big]_0^{\pi/4}$

$= -\ln \frac{1}{\sqrt{2}} - \ln 1 = -\ln 1/\sqrt{2}$