Centre Number | | | |

Student Number | | | | | | | | |

CRANBROOK SCHOOL

2016

HSC Assessment Task 2

Mathematics (2 Unit)

Reading time	5 minutes
Writing time	70 minutes
Total Marks	50
Task weighting	10%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- A formula Reference Sheet will be provided
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Reference Sheet
- Multiple Choice Answer Sheet
- 3 writing booklets

Structure & Suggested Time Spent

Section I
Multiple Choice Questions

- Answer Q1 – 5 on the multiple choice answer sheet
- Allow 10 minutes for this section

Section II
Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 60 minutes for this section

This paper must not be removed from the examination room

Disclaimer

The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

5 Marks

Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1 - 5.

1 The locus of a point $P(x,y)$ that moves so that it is always 5 units above the x-axis is:

(A) $x = -5$

(B) $x = 5$

(C) $y = -5$

(D) $y = 5$

2 The minimum value of $x^2 - 2x + 6$ is;

(A) -1

(B) 1

(C) 5

(D) 6

3 Which definite integral represents the area bounded by the curve $y = 4 - x^2$ and the x-axis?

(A) $\int_0^2 (4 - x^2)dx$

(B) $\int_{-2}^0 (4 - x^2)dx$

(C) $\int_{-2}^2 (4 - x^2)dx$

(D) $\int_{-\sqrt{2}}^{\sqrt{2}} (4 - x^2)dx$

4 What is the approximation of the integral $\int_1^5 \ln x \, dx$, using the trapezoidal rule and 4 sub-intervals?

(A) $\frac{1}{3}[\ln 1 + \ln 5 + 4(\ln 3) + 2(\ln 2 + \ln 4)]$

(B) $\frac{1}{3}[\ln 1 + \ln 5 + 2(\ln 2 + \ln 3 + \ln 4)]$

(C) $\frac{1}{2}[\ln 5 + 2(\ln 2 + \ln 3 + \ln 4)]$

(D) $\frac{1}{2}[\ln 1 + \ln 5 + 4(\ln 3) + 2(\ln 2 + \ln 4)]$

5 If $\frac{dy}{dx} = \frac{1}{x}$ and $y = 0$ when $x = 2$, then the correct expressions for y in terms of x is:

(A) $y = \log_e\left(\frac{x}{2}\right)$

(B) $y = \frac{1}{2}\log_e x$

(C) $y = \log_e x - 2$

(D) $y = 2\log_e x$

END OF SECTION I

Section II

45 Marks

Allow about 60 minutes for this section

Answer question 6 - 8 in separate booklets.

Question 6 **Begin a new booklet** **15 Marks**

(a) A point $P(x,y)$ moves so that it is equidistant from point (2, 3) and the line $y=4$. What is the equation of the locus point? **2**

(b) The focus of a parabola is (2,0) and the equation of the directrix is $x=6$.

Find the equation of this parabola. **3**

(c) Given the equation of the parabola $x^2-6x+25=8y$, find the;

(i) focal length **1**

(ii) vertex **1**

(iii) equation of the directrix **1**

(d) Find the value of k in the equation $2x^2-(k+3)x+4k=0$ if;

(i) the sum of the roots is 6. **1**

(ii) one root is the reciprocal of the other. **1**

(e) Find the discriminant of $2x^2+3x-4=0$ and describe the nature of the roots (real or unreal, rational or irrational, equal or unequal). **2**

(f) If α and β are the roots of $3x^2+5x+1=0$, find;

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{2}{\alpha}+\frac{2}{\beta}$ **1**

Question 7 **Begin a new booklet** **15 Marks**

(a) Find the primitive function of $\frac{2x^2}{3}+x$ **2**

(b) Evaluate $\int_2^3 (2x-3)^4\,dx$ **2**

(c) Find the area bounded by $y=x(x-3)$ and the x-axis between $x=0$ and $x=4$. **3**

(d) Consider the functions $y=x$ and $y=x^3$;

(i) Find their point(s) of intersection. **2**

(ii) Sketch both curves on the same number plane. **1**

(iii) Find the area of the region bound by the functions. **2**

(e) Using Simpson's rule and 5 function values, approximate $\int_0^4 \frac{1}{x+1}\,dx$. **3**

Question 8 **Begin a new booklet** **15 Marks**

(a) Differentiate the following:

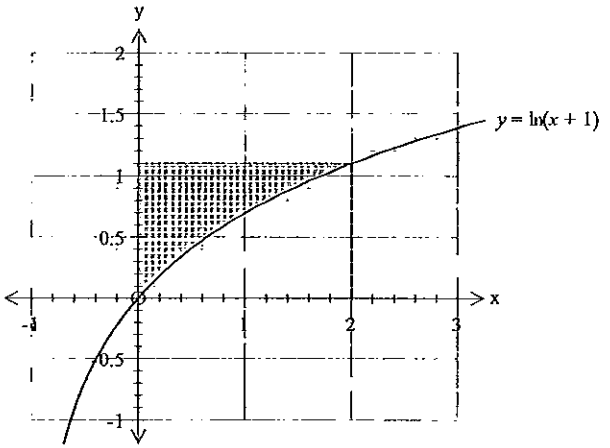
(i) $y=(3x-1)e^{2x}$ **2**

(ii) $y=2\ln(x^3+1)$ **1**

(b) Evaluate $\int_2^3 \frac{3x}{x^2+1}\,dx$, leaving your answer in exact form. **2**

(c) Solve $\frac{1}{2}\ln(6x-8)=\ln x$ **2**

(d) Consider the graph below.

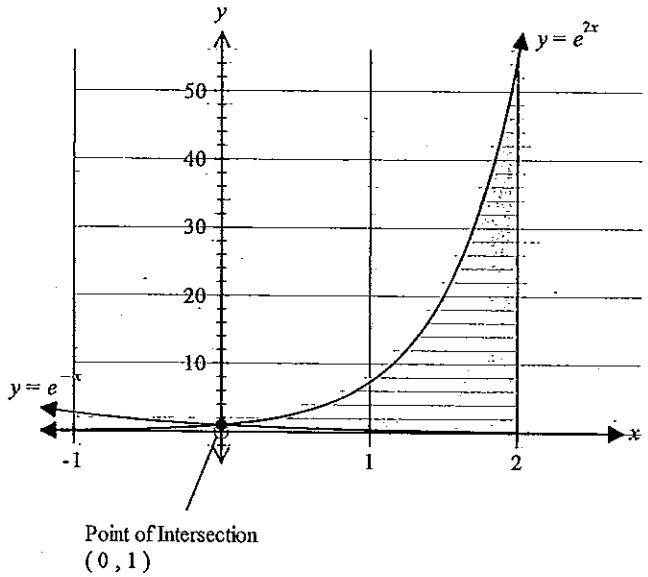


(i) Show that the volume of the solid formed when the shaded area is rotated about the y-axis is given by: **2**

$$V=\pi\int_0^{\ln 3}\left(e^{2y}-2e^{y}+1\right)dy$$

(ii) Hence find the volume of the solid of revolution. **2**

(e) The curves $y = e^{2x}$ and $y = e^{-x}$ intersect at the point (0,1). Find the volume generated when the region between the curves and the line $x = 2$ is rotated around the x-axis. Give your answer to the nearest integer. 4



END OF SECTION II

END OF ASSESSMENT

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QUESTION 6 SOLNS

a. $P(x,y)$ $A(2,3)$ $B(x,4)$ ... HOW TO REPRESENT $y=4$.

$PA = PB$ ... EQUIDISTANT

$(PA)^2 = (PB)^2$

$(x-2)^2 + (y-3)^2 = (x-x)^2 + (y-4)^2$ ✓

$x^2 - 4x + 4 + y^2 - 6y + 9 = 0 + y^2 - 8y + 16$

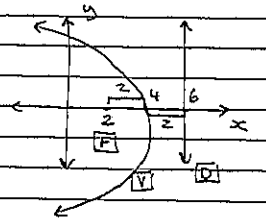
$\therefore x^2 - 4x + 2y - 3 = 0$ ✓

OR

$(x-2)^2 = -2(y - 7/2)$

- MIXED RESPONSES... MANY HAD FORGOTTEN THIS CONCEPT.
- ALMOST TO A PERSON, THE EXT. 1 CANDIDATES USED THE DEFINITION OF A PARABOLA TO SOLVE & DID NOT CONSIDER $(PA)^2 = (PB)^2$. MANY STRUGGLED WITH THIS.

b. SKETCH TO HELP:



$\therefore$ VERTEX = $(4,0)$ ✓

FOCAL LENGTH = 2 ✓

$\therefore y^2 = -8(x-4)$ ✓

CONCAVE LEFT $\therefore$ -VE.

- ANSWERED WELL.
- SOME ANSWERED WITH NO OBVIOUS WORKING ... YOU CANNOT DO THIS IN THE HSC!
- MANY MIXED UP FOCUS & VERTEX
- REM: SIDEWAYS PARABOLA IS $y^2 = \pm 4ax$!

c. $x^2 - 6x + 25 = 8y$ ... COMPLETE THE SQUARE!

$x^2 - 6x + 9 = 8y - 25 + 9$

$(x-3)^2 = 8y - 16$

$\therefore (x-3)^2 = 8(y-2)$

(i) FOCAL LENGTH = 2 ✓

(ii) VERTEX = $(3,2)$ ✓

(iii) DIRECTRIX = $y = 0$ ✓ ... CONCAVE UP PARAB. VERTEX = $(3,2)$ ∴ 2 UNITS BELOW IS $y=0$

- MIXING UP FOCUS & VERTEX WAS COMMON
- A FEW SUBTRACTED FOCAL LENGTH FROM x VAL, NOT THE y VAL

d. $2x^2 - (k+3)x + 4k = 0$

(i.) $\alpha + \beta = 6$

$\frac{-b}{a} = 6$

$\frac{k+3}{2} = 6$

$k + 3 = 12$

$k = 9$ ✓

(ii) PRODUCT = $\alpha \times \frac{1}{\alpha} = 1$

$\therefore \frac{c}{a} = 1$

$\frac{4k}{2} = 1$

$4k = 2$

$k = 1/2$ ✓

Section I

1. D
2. C
3. C
4. C
5. A

QUESTION 7

(a) $\int \frac{2x^2}{3} + x \, dx$

$= \frac{2x^3}{9} + \frac{x^2}{2} + c$ ✓✓

(b) $\int_2^3 (2x-3)^4 \, dx$

$= \left[\frac{(2x-3)^5}{(5)(2)}\right]_2^3$ ✓

$= \left[\frac{(2x-3)^5}{10}\right]_2^3$

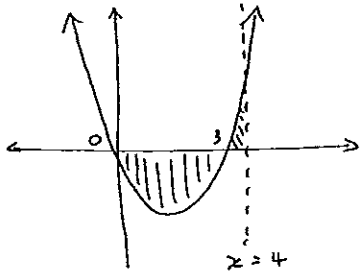
$= \left(\frac{(2(3)-3)^5}{10}\right) - \left(\frac{(2(2)-3)^5}{10}\right)$

$= \frac{243}{10} - \frac{1}{10}$

$= \frac{242}{10}$

$= \frac{121}{5}$ ✓

(c) $y = x(x-3)$



$A = \left|\int_0^3 x(x-3)\,dx\right| + \int_3^4 x(x-3)\,dx$ ✓

$= \left|\int_0^3 x^2 - 3x\,dx\right| + \int_3^4 x^2 - 3x\,dx$

$= \left|\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3\right| + \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]$ ✓

$= \left|\left(\frac{(3)^3}{3} - \frac{3(3)^2}{2}\right) - \left(0-0\right)\right|$

$+ \left[\left(\frac{(4)^3}{3} - \frac{3(4)^2}{2}\right) - \left(\frac{(3)^3}{3} - \right.\right.$

$= \left|9 - \frac{27}{2}\right| + \left(-\frac{8}{3} + \frac{9}{2}\right)$

$= \frac{9}{2} + \frac{11}{6}$

$= \frac{19}{3}$ units^2 ✓

(d) $y = x^4$ and $y = x^3$

(i) $x = x^3$

$x^3 - x = 0$ ✓

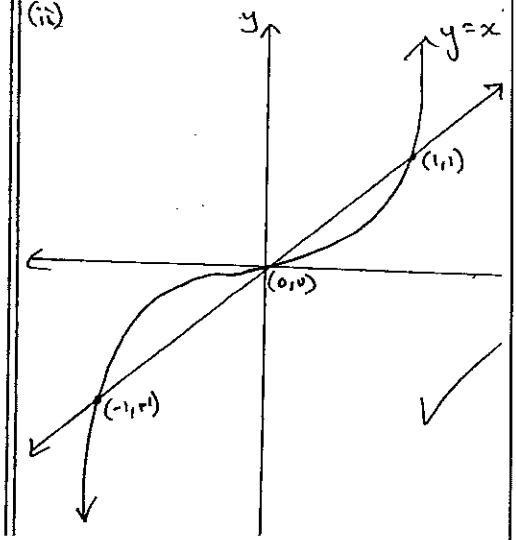
$x(x^2-1) = 0$

$x(x-1)(x+1) = 0$

$\therefore x = -1, 0, 1$ ✓

$\therefore$ P.O.I's @ $(-1,-1)$, $(0,0)$, $(1$

(ii)



✓

e.g. $\Delta = b^2 - 4ac$

$= 9 - 4.2.-4$

$= 9 + 32$

$\Delta = 41$ ✓✓

$\Delta > 0 \therefore$ 2 REAL, IRRATIONAL, UNEQUAL ROOTS

(-1 FOR EACH ERROR)

- ANSWERED WELL.
- MANY DID NOT RECOGNISE 41 IS NOT A PERFECT SQUARE SO ROOTS WOULD BE IRRATIONAL

i. (i) $\alpha + \beta = \frac{-b}{a} = \frac{-5}{3}$ ✓

(ii) $\alpha\beta = \frac{c}{a} = \frac{1}{3}$ ✓

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta}{\alpha\beta} + \frac{2\alpha}{\alpha\beta}$ ... COMMON DENOM

$= \frac{2(\alpha+\beta)}{\alpha\beta}$

$= 2 \times (-5/3)$

$= -10$ ✓

- ANSWERED WELL. NO PROBS.

QUESTION 8 MARKED BY HRK ✓ = 1 mark

a:

(i) $y = (3x-1)e^{2x}$

$u = (3x-1) \quad v = e^{2x}$

$u' = 3 \quad v' = 2e^{2x}$

$y' = 2(3x-1)e^{2x} + 3e^{2x}$ ✓

$= e^{2x}\left(2(3x-1)+3\right)$ *

$= e^{2x}(6x+1)$ ✓

[2]

(ii) $y = 2\ln(x^3+1)$

$y' = 2 \cdot \frac{3x^2}{x^3+1}$

$= \frac{6x^2}{x^3+1}$ ✓

Absolutely do not put the 2 up front. It makes it much longer & is wasting time. [1]

(i) * 2 marks given for this. BUT it is always wise to simplify fully, otherwise you risk losing a mark or make it difficult to use if needed in a later step.

b. $\int_2^3 \frac{3x}{x^2+1}\,dx$

$= \frac{3}{2}\int_2^3 \frac{2x}{x^2+1}\,dx$ ✓

$= 3/2\left[\ln(x^2+1)\right]_2^3$

$= 3/2\left(\ln 10 - \ln 5\right)$ ✓

$= 3/2 \ln 2$

RTQ !! Question says leave EXACT ie [2] DO NOT pick up a calculator !!!

c. $\frac{1}{2}\ln(6x-8) = \ln x$

$\ln\sqrt{6x-8} = \ln x$

$\sqrt{6x-8} = x$ ✓

$6x-8 = x^2$

$0 = x^2 - 6x + 8$

$0 = (x-2)(x-4)$

$\therefore x = 2, 4.$ ✓

Learn your log laws !!

[2]

(i) $A = 2\int_0^1 x - x^3\,dx$ ✓

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 2\left[\left(\frac{1^2}{2} - \frac{1^4}{4}\right) - \left(\frac{0^2}{2} - \frac{0^4}{4}\right)\right]$

$= 1/2$ units^2 ✓

(ii) $A \doteq \frac{1}{3}\left[y_0 + 4y_1 + 2y_2 + \cdots + y_5\right]$

x	0	1	2	3	4
f(x)	1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{5}$

$h = 1.$ ✓

$A \doteq \frac{1}{3}\left[\left(1+\frac{1}{5}\right) + 4\left(\frac{1}{2}+\frac{1}{4}\right) + 2\left(\frac{1}{3}\right)\right]$

$\doteq \frac{73}{45}$ ✓

e) $y = e^{2x}$ $\quad y = e^{-x}$

$y^2 = e^{4x}$ $\quad y^2 = e^{-2x}$ ✓

$V = \pi \int_0^2 (e^{4x} - e^{-2x})\, dx$ ✓

$= \pi \left[\frac{e^{4x}}{4} + \frac{e^{-2x}}{2} \right]_0^2$ ✓

$= \pi \left[\left(\frac{e^8}{4} + \frac{e^{-4}}{2} \right) - \left(\frac{1}{4} + \frac{1}{2} \right) \right] = \pi \left(\frac{e^8}{4} + \frac{1}{2e^4} - \frac{3}{4} \right)$

≈ 2339 units^3 ✓

RTQ Question says to nearest integer

NB approximate only once otherwise you may introduce an error.

d) (i) $x = 2 \quad \therefore y = \ln(x+1)$

$= \ln(2+1)$

$= \ln 3$ ✓

$y = \ln(x+1)$

$e^y = x + 1$

$x = e^y - 1$

$x^2 = (e^y - 1)^2$

$= (e^y - 1)(e^y - 1)$

$= e^{2y} - e^y - e^y + 1$

$= e^{2y} - 2e^y + 1$ ✓

$\therefore V = \int_0^{\ln 3} (e^{2y} - 2e^y + 1)\, dy$

(ii) $V = \pi \int_0^{\ln 3} (e^{2y} - 2e^y + 1)\, dy$

$= \pi \left[\frac{e^{2y}}{2} - 2e^y + y \right]_0^{\ln 3}$ ✓

$= \pi \left[\left(\frac{e^{2\ln 3}}{2} - 2e^{\ln 3} + \ln 3 \right) - \left(\frac{e^0}{2} - 2e^0 + 0 \right) \right]$

$= \pi \left[\left(\frac{9}{2} - 6 + \ln 3 \right) - \left(\frac{1}{2} - 2 \right) \right]$

$= \pi \ln 3$ units^3 ✓