



KAMBALA

Student Number: _____

Preliminary Task 1
March 2016

Mathematics Extension 1

General Instructions

- Working time – 50 minutes
- Write using black or blue pen
Black pen is preferred
- Board-approved calculators may be used
- Answer questions 1 – 4 on the multiple choice answer sheet provided.
- Answer questions 5 – 6 on the paper provided.
Start each question on a new page.
- Show all necessary working in Questions 5 – 6

Total marks – 30

Section I

4 marks

- Attempt Questions 1-4
- Allow about 7 minutes for this section

Section II

26 marks

- Attempt Questions 5-6
- Allow about 43 minutes for this section

Section I

4 Marks

Attempt Questions 1 – 4

Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 4.

1 Simplify $\frac{(x-3)}{x} + 1$.

(A) $3x$

(B) $-\frac{3}{x}$

(C) $\frac{2x-3}{x}$

(D) $\frac{x-2}{x}$

2 The curve $y = \frac{2x^2 - 6x}{x^3 - 1}$ has asymptote(s)

(A) $x=1, y=0$

(B) $x=-1$

(C) $x=1, y=2$

(D) $y=0$

3 The point $(1, \sqrt{8})$ lies on the graph of $y = 2^{kx}$. Find the value of k .

(A) $k = 3$

(B) $k = -3$

(C) $k = \frac{1}{3}$

(D) $k = \frac{3}{2}$

4 Solve for c : $\frac{2-c}{6} + 8 = 1 - c$

(A) $c = 2$

(B) $c = -\frac{4}{5}$

(C) $c = -\frac{44}{5}$

(D) $c = -8$

Section II

26 Marks

Attempt Questions 5 – 6

Allow about 43 minutes for this section

Answer each question on the writing paper provided. Start each question on a new page.

In Questions 5 – 6 your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (13 marks) Start a new page.

(a) Factorise

(i) $x^2 - 4xy + 4y^2 - 64$ 2

(ii) $x^{2a} - 3x^a - 4$ 1

(b) Simplify $\frac{a}{2b} + \frac{6}{3a}$ 2

(c) Factorise $a^3 - \frac{8}{27}b^6$ 2

(d) Solve $\frac{x+1}{x-2} \leq 2$ 3

(e) Find possible values of a , b and c if $3x^2 - 7 = a(x+3)^2 + bx + c$ 3

Question 6 (13 marks) Start a new page.

- (a) (i) Sketch the graph of the function $f(x) = |2x - 6|$. 1
- (ii) Hence, or otherwise, solve $|2x - 6| > x + 3$. 1
- (b) (i) Given $f(x) = 2x + 6$ and $g(x) = 2x^2$ find the composite function $f[g(x)]$. 2
- (ii) Show that $f(6) - g(3) = 0$. 1
- (iii) Sketch the graphs of $f(x)$ and $g(x)$ on the same axes and hence determine the value(s) of x for which $f(x) > g(x)$. 2
- (c) Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$. 1
- (d) Consider the function $f(x) = \frac{ax + 3}{x - b}$.
- (i) Find a and b given that $y = f(x)$ has asymptotes with equations $x = -1$ and $y = 2$. 2
- (ii) Write down the domain and range of $f(x)$. 1
- (e) If the graph of the function $f(x) = 4x^2 - 20x - 4$ intersects the graph of the function $g(x) = 1 + 4x - x^2$ at the point (m, k) , show that the other point of intersection occurs where $x = -\frac{1}{m}$. 2

End of Assessment

Student Number: _

Mathematics Extension 1

Preliminary Task 1 March 2016

Section I

Multiple-Choice Answer Sheet
Circle the correct response

- | | | | | |
|----|------------------------------------|---|------------------------------------|------------------------------------|
| 1. | A | B | <input checked="" type="radio"/> C | D |
| 2. | <input checked="" type="radio"/> A | B | C | D |
| 3. | A | B | <input checked="" type="radio"/> C | <input checked="" type="radio"/> D |
| 4. | A | B | <input checked="" type="radio"/> C | D |

✓
 4



(1)

(13) well done.

Question 5

$$\begin{aligned}
 \text{a) } & x^2 - 4xy + 4y^2 - 64 \\
 &= (x - 2y)^2 - 64 \\
 &= (x - 2y)^2 - 8^2 \\
 &= (x - 2y + 8)(x - 2y - 8) \quad \checkmark\checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } & x^{2a} - 3x^a - 4 \\
 & \text{let } u \text{ be } x^a \\
 & u^2 - 3u - 4 \\
 & = (u - 4)(u + 1) \quad \text{sub in } u = x^a \\
 & = (x^a - 4)(x^a + 1) \quad \checkmark\checkmark
 \end{aligned}$$

~~$x^2 = 4$
 $x = \pm 2$
 $x = \pm 2$
 $x = \pm 2$~~

$$\begin{aligned}
 \text{b) } & \frac{a}{2b} + \frac{b}{3a} \\
 & = \frac{3a(a)}{6ab} + \frac{2b(b)}{6ab} \\
 & = \frac{3a^2 + 2b^2}{6ab} \\
 & = \frac{3(a^2 + 4b)}{6(2ab)} = \frac{a^2 + 4b}{2ab} \quad \checkmark\checkmark
 \end{aligned}$$



(2)

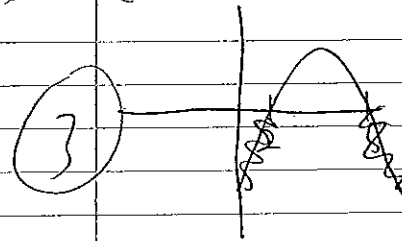
over question 5 cont.

$$\begin{aligned}
 \text{c) } & a^3 - \frac{8}{27} b^3 \\
 & \sqrt[3]{a^3 - \frac{8}{27} b^3} \\
 & \text{let } u \text{ be } \sqrt[3]{a^3 - \frac{8}{27} b^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & a^3 - \frac{8}{27} b^3 \\
 \text{(f) } & \left(a - \frac{2}{3} b\right) \left(a^2 + \frac{2}{3} ab + \frac{4}{9} b^2\right) \quad \checkmark\checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{x+1}{x-2} \leq 2 \quad (x \neq 2) \quad \checkmark \\
 & (x+1)(x-2) \leq 2(x-2)^2 \\
 & (x+1)(x-2) - 2(x-2)^2 \leq 0 \\
 & (x-2)[(x+1) - 2(x-2)] \leq 0 \\
 & (x-2)(5-x) \leq 0
 \end{aligned}$$

Roots:
 $x = 2, x = 5$



$x < 2$ or $x > 5$



Q5 (3)

Question 5 continued

a) $3x^2 - 7 = a(x+3)^2 + bx + c$

LHS:

RHS:

$a(x^2 + 6x + 9) + bx + c$
 $ax^2 + 6ax + 9a + bx + c$

LHS:

$3x^2 - 7$

Equate:

$3x^2 - 7 = ax^2 + 6ax + 9a + bx + c$

$3x^2 = ax^2$

$a = 3$

Sub in $a = 3$

$3x^2 + 18x + 27 + bx + c$
 $= 3x^2 + 27 + 18x + bx + c$

~~$3x^2 - 7 = 3x^2 + 27 + 18x + bx + c$~~

$7 = 27 + c$

$c = -20$

$0x = 18x + bx$

$b = -18$



Q6 (4)

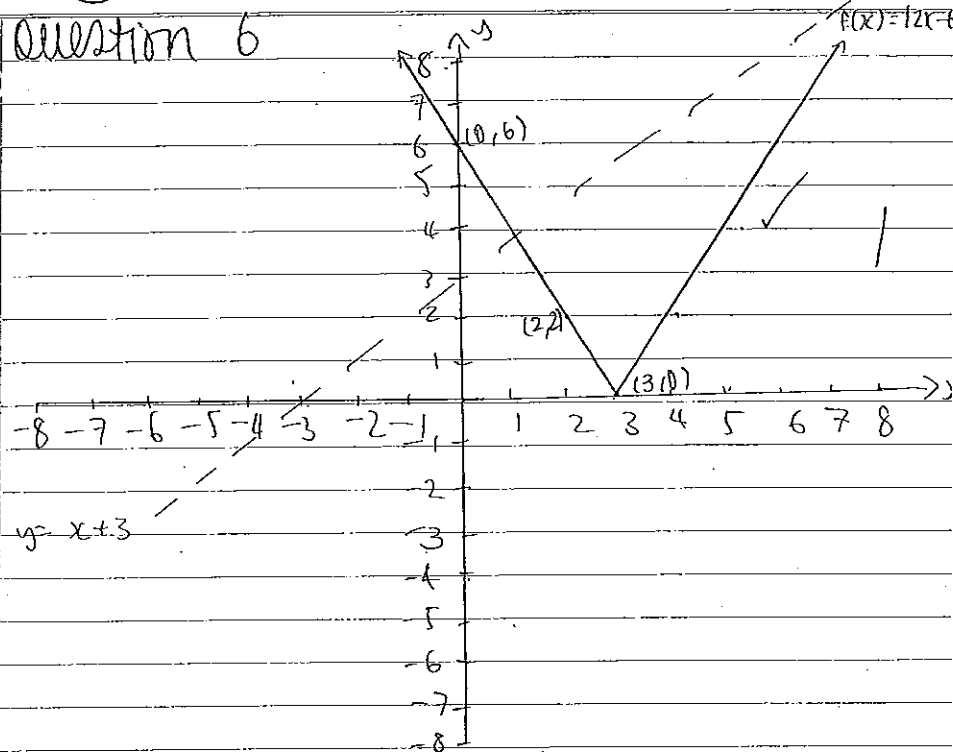
(13)

Well Done

😊

Question 6

a)



ii) $12x - 6 > x + 3$

If $2x - 6 > 0$, $2x - 6 > x + 3$

If $2x - 6 < 0$, $2x - 6 > -x - 3$
 $3x > 3$
 $x > 1$

TEST

$x > 9$ or $x = 10$

$12(10) - 6 > 10 + 3$

$120 - 6 > 10 + 3$

$114 > 13$

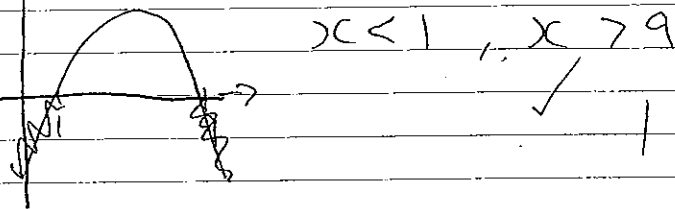
$x > 1$



QUESTION 6 CONT.

5

$$\begin{aligned} \text{ii) } |2x-6| &> x+3 \\ (2x-6)^2 &> (x+3)^2 \\ 0 &> (x+3)^2 - (2x-6)^2 \\ 0 &> (x+3+2x-6)(x+3-2x+6) \\ 0 &> (3x-3)(9-x) \end{aligned}$$



$$\begin{aligned} \text{b) } f(x) &= 2x+6 \\ g(x) &= 2x^2 \end{aligned}$$

$$\begin{aligned} f[g(x)] &= 2(2x^2) + 6 \\ &= 4x^2 + 6 \end{aligned}$$

$$f[g(x)] = 4x^2 + 6$$

$$\begin{aligned} \text{ii) } f(6) &= 2(6) + 6 \\ &= 12 + 6 \\ &= 18 \end{aligned}$$

$$\begin{aligned} g(3) &= 2(3)^2 \\ &= 2 \times 9 \\ &= 18 \end{aligned}$$

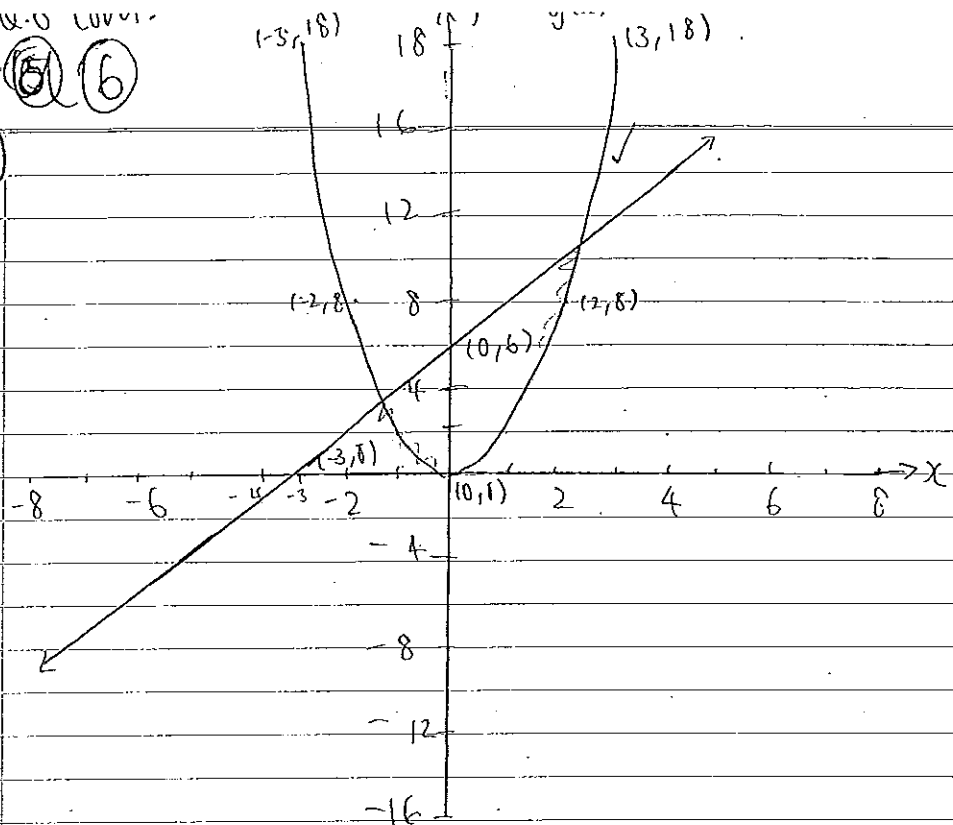
$$\begin{aligned} f(6) - g(3) &= 18 - 18 \\ &= 0 \quad (\text{as required}) \end{aligned}$$



QUESTION 6 CONT.

6

iii)



$$\text{ii) } 2x+6 = 2x^2 \quad \textcircled{1}$$

REARRANGE $\textcircled{1}$:

$$2x = 2x^2 - 6$$

$$x = x^2 - 3 \quad \textcircled{2}$$

$$\begin{aligned} \text{Sub } \textcircled{2} \text{ into } \textcircled{1} \\ y = 2 \left(\frac{y}{2} - 3 \right) \quad \textcircled{3} \end{aligned}$$

Q7

Equating $f(x) = g(x)$ to find points of intersection

$$\begin{aligned}
 \text{ii) } 2x^2 &= 2x + 6 \\
 2x^2 - 2x - 6 &= 0 \\
 2(x^2 - x - 3) &= 0 \\
 x^2 - x - 3 &= 0 \\
 -b \pm \sqrt{b^2 - 4ac} \\
 &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1 + 12}}{2} \\
 &= \frac{1 \pm \sqrt{13}}{2}
 \end{aligned}$$

$f(x) > g(x)$ ✓

$$\frac{1 - \sqrt{13}}{2} < x < \frac{1 + \sqrt{13}}{2}$$

c) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

$$\frac{(x+2)(x-2)}{x-2}$$

Sub in $x = 2$:

$$\frac{2+2}{1} = 4 \quad \checkmark$$

Q6 cont

d) $f(x) = \frac{ax+3}{x-b}$

f(x)

$$\begin{aligned}
 x-b &\neq 0 \\
 -b &\neq 0
 \end{aligned}$$

$$\begin{aligned}
 x - b &= 0 \\
 -1 - b &= 0
 \end{aligned}$$

$$\boxed{b = -1} \quad \checkmark$$

$$\frac{ax+3}{x-b} = a = 2 \quad \text{asymptote at } y=2$$

$$\boxed{a = 2} \quad \checkmark$$

~~ii) Domain: $\{x : x \neq -1, x \in \mathbb{R}\}$
 Range: $\{y : y \neq 2, y \in \mathbb{R}\}$~~

~~$f(x) = 4x^2 - 2x - 4$ ①
 $g(x) = 1 + 4x - x^2 - 4$ ②~~

~~② $\times 4$
 $4 + 16x - 4x^2$~~

~~① + ②~~

~~$4x^2 - 2x - 4$
 $+ 16x - 4x^2 + 4$
 $-16x + 0$~~

ii) Domain: $\{x : x \neq 1, x \in \mathbb{R}\}$ ✓
 Range: $\{y : y \neq 2, y \in \mathbb{R}\}$

Q6 cont.

$$e) f(x) = 4x^2 - 20x - 4$$

$$g(x) = 1 + 4x - x^2$$

$$4x^2 - 20x - 4 = 1 + 4x - x^2$$

~~$$5x^2 + 24x - 5 = 0$$~~

$$5x^2 + 24x - 85 = 0$$

~~$$5x^2 - 25x + x$$~~

$$5x^2 + x - 25x - 5 = 0$$

$$x(5x + 1) - 5(5x + 1) = 0$$

$$(5x + 1)(x - 5) = 0$$

$$x = 5 \text{ or } x = -\frac{1}{5} \quad \checkmark$$

~~$m = 5$~~

$$\text{At } x = 5, y = 4(5)^2 - 20(5) - 4$$

$$= 4 \cdot 25 - 100 - 4$$

$$= 100 - 100 - 4$$

$$= -4$$

$$m = 5, k = -4$$

Points of intersection:

$$(5, -4), (-\frac{1}{5}, y)$$

As $m = 5$ the other point of intersection is $-\frac{1}{5} = -\frac{1}{m}$

\checkmark (as requir)