Student Number: _____

YEAR 12 HSC

Assessment 1

FEBRUARY 2016

Mathematics Extension 1

Time Allowed: 45 minutes

Total Marks: 30

Instructions to Candidates

- Do not open this paper until instructed to do so.
- Board approved calculators may be used.
- To obtain full marks, well set out logical reasoning or explanations must accompany your answers.
- Please answer Multiple Choice questions on this paper and Section II questions on the coloured paper provided.
- Include your student number/name at the top of each coloured sheet.

Teacher Use Only

	Section I	Section II	Total
Combinations and Permutations	1	3(2) 6(2)	5
Polynomials	2	4(3)	4
Circle Geometry	3	7(4)	5
Integration	4	1(3) 2(4)	8
Geom. Apps of Calculus	5	5(7)	8
Total	5	25	

Section I
5 marks

Circle the correct answer for each of these multiple choice questions:

1. How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order?

(A) 5! (B) 6! (C) $2 \times 5!$ (D) $2 \times 6!$

2. A polynomial equation has roots α, β and γ where

$\alpha + \beta + \gamma = -2$, $\alpha\beta + \alpha\gamma + \beta\gamma = 3$ and $\alpha\beta\gamma = 1$.

Which polynomial equation has the roots α, β and γ?

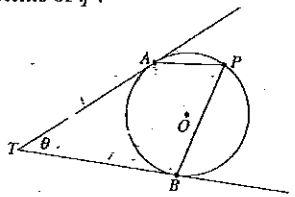
(A) $x^3 + 2x^2 + 3x + 1 = 0$

(B) $x^3 + 2x^2 + 3x - 1 = 0$

(C) $x^3 - 2x^2 + 3x + 1 = 0$

(D) $x^3 - 2x^2 + 3x - 1 = 0$

3. The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB = q$. What is $\angle APB$ in terms of q?



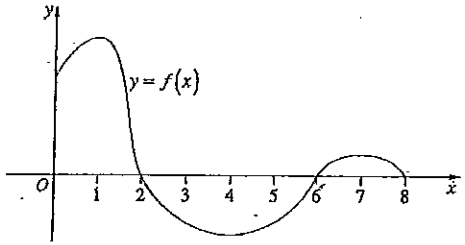
(A) $\frac{\theta}{2}$

(B) $90° - \frac{\theta}{2}$

(C) θ

(D) $90° - \theta$

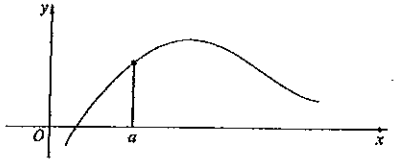
4. The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$.
Which of the following integrals has the greatest value?



(A) $\int_0^1 f(x)\,dx$ (B) $\int_0^2 f(x)\,dx$

(C) $\int_0^7 f(x)\,dx$ (D) $\int_0^8 f(x)\,dx$

5. The diagram shows the graph of $y = f(x)$.
Which of the following statements is true?



(A) $f'(a) > 0$ and $f''(a) < 0$

(B) $f'(a) > 0$ and $f''(a) > 0$

(C) $f'(a) < 0$ and $f''(a) < 0$

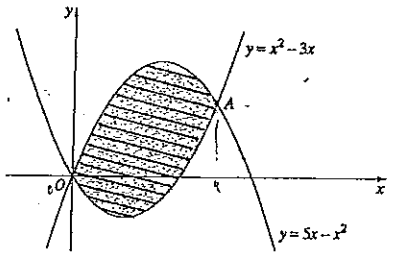
(D) $f'(a) < 0$ and $f''(a) > 0$

Section II
25 marks

Answer each question on the coloured paper provided.

1. Use the substitution $u = 1 - x$ to evaluate $\int_0^1 x\sqrt{1-x}\,dx$ (3)

2. The diagram shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$.
The parabolas intersect at the origin O and the point A.
The region between the two parabolas is shaded.



(i) Find the x-coordinate of the point A. (1)
(ii) Find the area of the shaded region. (3)

3. Alex's playlist consists of 40 different songs that can be arranged in any order.

I. How many arrangements are there for the 40 songs? (1)
II. Alex decides that she wants to play her three favourite songs first, in any order. How many arrangements of the 40 songs are now possible? (1)

4. Let $P(x) = x^3 - ax^2 + x$ be a polynomial, where a is a real number. (3)
When $P(x)$ is divided by $x - 3$ the remainder is 12.
Find the remainder when $P(x)$ is divided by $x + 1$.

5. Consider the function $f(x) = \dfrac{x^4 + 3x^2}{x^4 + 3}$ (7)

(i) Show that $f(x)$ is an even function.

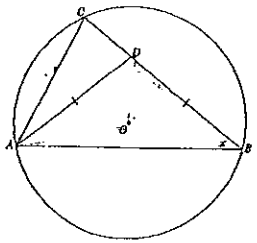
(ii) What is the equation of the horizontal asymptote to the graph $y = f(x)$?

(iii) Find the x-coordinates of all stationary points for the graph $y = f(x)$.

(iv) Sketch the graph $y = f(x)$. You are not required to find any points of inflexion.

6. Find the number of ways in which a group of 3 people can be chosen from 4 men and 5 women so that the group contains a majority of women. (2)

7. In the diagram, the vertices of $\triangle ABC$ lie on the circle with centre O. The point D lies on BC such that $\triangle ABD$ is isosceles and $\angle ABC = x$.



Copy or trace the diagram into your writing booklet.

(i) Explain why $\angle AOC = 2x$. (1)

(ii) Prove that $ACDO$ is a cyclic quadrilateral. (2)

(iii) Let M be the midpoint of AC and P the centre of the circle through A, C, D and O. Show that P, M and O are collinear. (1)

END OF ASSESSMENT

Student Number: Solutions

YEAR 12 HSC

Assessment 1

FEBRUARY 2016

Mathematics Extension 1

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Total	5	25	

Section I
5 marks

Circle the correct answer for each of these multiple choice questions:

1. How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order?

(A) 5! (B) 6! (C) 2 × 5! (D) 2 × 6!

Regard C and L as a single unit arranged 2! ways
Six units can be arranged 6! ways
∴ No ways = 2!6! or 2×6!

2. A polynomial equation has roots α, β and γ where

$\alpha+\beta+\gamma=-2$, $\alpha\beta+\alpha\gamma+\beta\gamma=3$ and $\alpha\beta\gamma=1$.
$-b/a=-2$ $c/a=3$ $-d/a=1$

Which polynomial equation has the roots α, β and γ?

a b c d

(A) $x^3+2x^2+3x+1=0$ $-\frac{2}{1}=-2$ ✓ $3/1=3$ ✓ $-1/1$ ✓

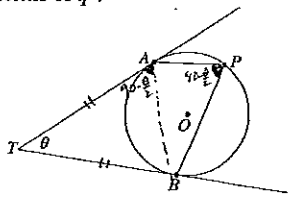
(B) $x^3+2x^2+3x-1=0$ ✓ ✓ ✓

(C) $x^3-2x^2+3x+1=0$ ×

(D) $x^3-2x^2+3x-1=0$ ×

3. The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB=q$. What is $\angle APB$ in terms of q?

0.66 State mean



$\triangle ATB$ is isos $\triangle$ (tangents from external pt.)
$\angle TAB = 90-\theta/2$ (isos $\triangle$, $\angle$ sum $\triangle$)
$\angle APB = 90-\theta/2$ ($\angle$ in altern. segment)

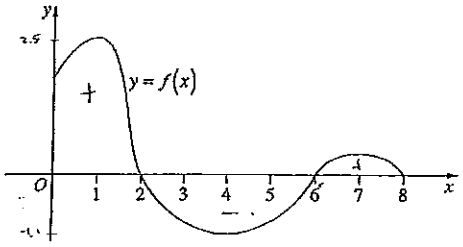
(A) $\frac{\theta}{2}$

(B) $90°-\frac{\theta}{2}$

(C) θ

(D) $90°-\theta$

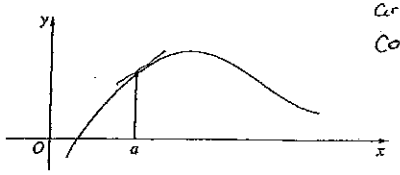
4. The graph of $y=f(x)$ has been drawn to scale for $0\le x\le 8$.
Which of the following integrals has the greatest value?



(A) $\int_0^1 f(x)\,dx$ (B) $\int_0^2 f(x)\,dx$ (all positive) greater than A

(C) $\int_0^7 f(x)\,dx$ (D) $\int_0^8 f(x)\,dx$ (greater than C with extra positive area) but not greater than B because adding an overall negative area.

5. The diagram shows the graph of $y=f(x)$.
Which of the following statements is true?



Grad positive at a $f'(a)>0$
Concave down at a $f''(a)<0$

Grad at a Concavity at a

(A) $f'(a)>0$ and $f''(a)<0$

(B) $f'(a)>0$ and $f''(a)\overset{>}{\lessdot}0$

(C) $f'(a)\overset{<}{\gtrdot}0$ and $f''(a)<0$

(D) $f'(a)\overset{<}{\gtrdot}0$ and $f''(a)\overset{>}{\lessdot}0$

Section II
25 marks

Answer each question on the coloured paper provided.

1. Use the substitution $u = 1 - x$ to evaluate $\int_0^1 x\sqrt{1-x}\,dx$ (3)

$\int = -\int_1^0 (1-u)\,u^{1/2}\,du$ ✓

$= -\int_1^0 u^{1/2} - u^{3/2}\,du$

$= -\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_1^0$

$= -\left[(0-0) - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$ ✓

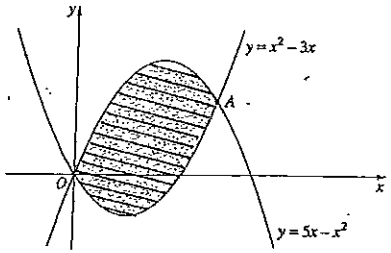
$= +4/15$

$u = 1 - x$ $\therefore x = 1 - u$

$\frac{du}{dx} = -1$ If $x=0$, $u=1$

$du = -dx$ If $x=1$, $u=0$

2. The diagram shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$.
The parabolas intersect at the origin O and the point A.
The region between the two parabolas is shaded.



(i) Find the x-coordinate of the point A. (1)
(ii) Find the area of the shaded region. (3)

i) Solve simult. $5x - x^2 = x^2 - 3x$ or by Trial & Error

$0 = 2x^2 - 8x$

$0 = 2x(x-4)$

$x = 0, 4$

At A: $x = 4$ ✓

ii) Area = $\int_{\text{top curve}} - \int_{\text{bottom curve}}$

$= \int_0^4 (5x - x^2) - (x^2 - 3x)\,dx$ ✓

$= \int_0^4 8x - 2x^2\,dx$

$= \left[4x^2 - \frac{2x^3}{3}\right]_0^4$ ✓

$= \left(64 - \frac{128}{3}\right) - (0-0)$

$= 21\frac{1}{3}$ units^2 $\frac{64}{3}$ u^2 ✓

3. Alex's playlist consists of 40 different songs that can be arranged in any order.

40!

I. How many arrangements are there for the 40 songs? (1)

$8.15915\ldots \times 10^{47}$ (Note if rounding eg. 3 sig figs)

3!37!

II. Alex decides that she wants to play her three favourite songs first, in any order. How many arrangements of the 40 songs are now possible? (1)

$8.25825\ldots \times 10^{43}$

4. Let $P(x) = x^3 - ax^2 + x$ be a polynomial, where a is a real number. (3)
When $P(x)$ is divided by $x - 3$ the remainder is 12.
Find the remainder when $P(x)$ is divided by $x + 1$.

$P(3) = 27 - 9a + 3 = 12$

$18 = 9a$ ✓

$\therefore a = 2$

$P(x) = x^3 - 2x^2 + x$ ✓

$P(-1) = -1 - 2 - 1$

$= -4$ ✓

$\therefore$ Remainder = -4

5. Consider the function $f(x) = \frac{x^4 + 3x^2}{x^4 + 3}$ (7)

(i) Show that $f(x)$ is an even function. (1)
(ii) What is the equation of the horizontal asymptote to the graph $y = f(x)$? (1)
(iii) Find the x-coordinates of all stationary points for the graph $y = f(x)$. (3)
(iv) Sketch the graph $y = f(x)$. You are not required to find any points of inflexion. (2)

6. Find the number of ways in which a group of 3 people can be chosen from 4 men and 5 women so that the group contains a majority of women. (2)

SOLUTIONS

5. $f(x) = \frac{x^4 + 3x^2}{x^4 + 3}$

i) $f(-x) = \frac{(-x)^4 + 3(-x)^2}{(-x)^4 + 3}$ ½

$= \frac{x^4 + 3x^2}{x^4 + 3}$

$= f(x)$ ∴ Even function ½
(symmetrical about y axis)

ii) For horiz. asymptote

$y = \lim_{x \to \pm\infty} f(x)$

$= \lim_{x \to \pm\infty} \frac{x^4/x^4 + 3x^2/x^4}{x^4/x^4 + 3/x^4}$

$= \lim_{x \to \pm\infty} \frac{1 + 3/x^2}{1 + 3/x^4}$

÷ top & bottom by highest power of x in denominator

$y = 1$ ✓ Must be stated as an Equation.

iii) $f'(x) = \frac{(4x^3 + 6x)(x^4 + 3) - 4x^3(x^4 + 3x^2)}{(x^4 + 3)^2}$ ✓

$= \frac{4x^7 + 12x^3 + 6x^5 + 18x - 4x^7 - 12x^5}{(x^4 + 3)^2}$

$= \frac{-6x^5 + 12x^3 + 18x}{(x^4 + 3)^2}$

For stat. pts $f'(x) = 0$

$0 = -6x^5 + 12x^3 + 18x$ ✓ mult. denom. both sides

$0 = -6x(x^4 - 2x^2 - 3)$

$0 = -6x(x^2 - 3)(x^2 + 1)$

$0 = -6x(x + \sqrt{3})(x - \sqrt{3})(x^2 + 1)$

$\therefore x = 0,\ x = \sqrt{3}$ or $x = -\sqrt{3}$ ✓

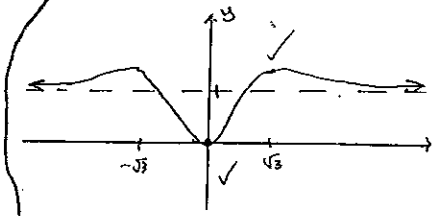
iv) Finding $f''(x)$ complicated ∴ use

x	-2	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	2
$f'(x)$	$-$	0	$+$	0	$+$	0	$-$

even ∴ only need →

$x = \pm\sqrt{3}$ Max
$x = 0$ Min

Intercept $(0,0)$



(should be ⅓ to ½ page + use a ruler!)

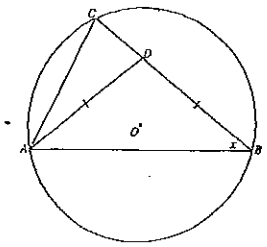
No. ways = 2 women, 1 man + 3 women, 0 men

$= {}^5C_2\,{}^4C_1 + {}^5C_3\,{}^4C_0$ ✓

$= 10 \times 4 + 10 \times 1$

$= 50$ ways ✓

7. In the diagram, the vertices of $\triangle ABC$ lie on the circle with centre O. The point D lies on BC such that $\triangle ABD$ is isosceles and $\angle ABC = x$.



Copy or trace the diagram into your writing booklet.

(i) Explain why $\angle AOC = 2x$. (1)

(ii) Prove that $ACDO$ is a cyclic quadrilateral. (2)

(iii) Let M be the midpoint of AC and P the centre of the circle through A, C, D and O. Show that P, M and O are collinear. (1)

7. i) $\angle AOC = 2x$ ($\angle$ at centre = double $\angle$ circumf.) ✓

ii) $\angle BAD = x$ (base $\angle$'s of isos $\triangle ABD$)
$\angle CDA = 2x$ (exterior $\angle$ of $\triangle ABD$) ✓
$\angle CDA = \angle DOA$ (both $2x$)
$\therefore$ ACDO is a cyclic quad ($\angle$'s in same segment are equal). ✓

iii) OM $\perp$ AC (perp. from centre bisects chord in circle centre O)
PM $\perp$ AC (perp. from centre bisects chord in circle centre P) ✓
$\therefore$ P is on line OM
$\therefore$ P, M and O are collinear.