SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2017

HSC Task #2

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes.
- Working time – 90 minutes.
- Write using black pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Leave your answers in the simplest exact form, unless otherwise stated.
- Start each **NEW** question in a separate answer booklet.
- A reference sheet is provided for this paper.

Total Marks – 55

Section I Pages 3–5

6 Marks

- Attempt Questions 1–6
- Allow about 10 minutes for this section.

Section II Pages 6–11

49 marks

- Attempt Questions 7–9
- Allow about 1 hour and 20 minutes for this section

Examiner: *DY*

Section I – Multiple Choice

6 Marks
Attempt question 1–6
Allow about 10 minutes for this section
Use the multiple-choice answer sheet for Questions 1–6

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1 A point $P(x, y)$ is moving on the curve defined by $x = 2 + 3t$ and $y = 2 - 3t$, where t is a variable.

What is the Cartesian equation of the curve?

(A) $x + y + 4 = 0$

(B) $x + y - 4 = 0$

(C) $x - y + 4 = 0$

(D) $x - y - 4 = 0$

2 Dennis, Eric and five friends arrange themselves in a circle.
In how many ways can they be arranged so that Dennis and Eric are NOT together?

(A) $6! - 4! \times 2!$

(B) $7! - 5! \times 2!$

(C) $6! - 5! \times 2!$

(D) $7! - 4! \times 2!$

3 Consider the function $f(x) = x^3 + 3x - 1$. It has one root α, $0 < \alpha < 0.6$.
Take $x = 1$ as a first approximation for this root.

Using ONE application of Newton's method, which of the following is the second approximation for the root?

(A) $x = \frac{1}{2}$

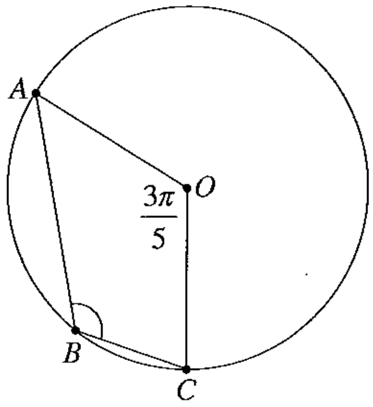
(B) $x = \frac{1}{3}$

(C) $x = \frac{1}{4}$

(D) $x = \frac{1}{5}$

4 The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{3\pi}{5}$ radians.



NOT TO SCALE

What is the size of $\angle ABC$ in radians?

(A) $\frac{3\pi}{10}$

(B) $\frac{2\pi}{5}$

(C) $\frac{7\pi}{10}$

(D) $\frac{4\pi}{5}$

5 There are m boys and m girls at a particular school and a committee of 4 students is to chosen from the students.

How many different committees can be formed having 2 boys and 2 girls on it?

(A) $m(m-1)$

(B) $m^2(m-1)^2$

(C) $\frac{m(m-1)}{2}$

(D) $\frac{m^2(m-1)^2}{4}$

6 Find $\int \cos x \sin^3 x \, dx$.

(A) $4\sin^4 x + C$

(B) $-4\sin^4 x + C$

(C) $\frac{1}{4}\sin^4 x + C$

(D) $-\frac{1}{4}\sin^4 x + C$

Section II

49 marks
Attempt Questions 7–9
Allow about 1 hour and 20 minutes for this section

Answer each question in a NEW writing booklet. Extra pages are available

In Questions 7-9, your responses **should include** relevant mathematical reasoning and/or calculations.

Question 7 (17 Marks) Start a NEW Writing Booklet

(a) Solve the inequality $\frac{x}{2x+1} < 2$. **3**

(b) Find the value of k if $\lim_{x \to 0} \frac{\sin kx}{2x} = 5$. **1**

(c) The point $P(2p, p^2)$ lies on the parabola $x^2 = 4y$.

(i) Write down the equation of the normal at P. **1**

(ii) Find the co-ordinates of the points L and M where the normal at P cuts the x-axis and y-axis respectively. **2**

(iii) Find the area of $\Delta\ OLM$ where O is the origin. **1**

(d) Find $\int \cos^2 4x \, dx$. **2**

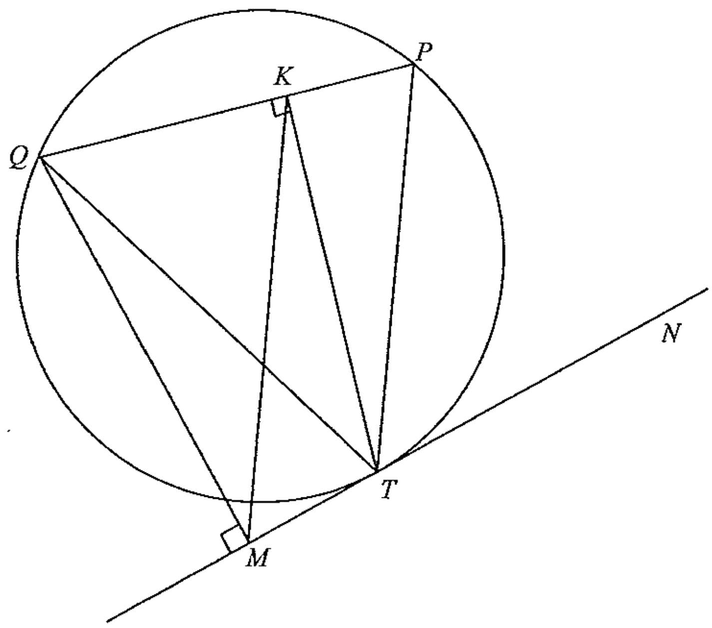
(e) Prove by mathematical induction that $6^{2n} - 5^{2n}$ is divisible by 11 for integers $n \geq 1$. **3**

(f) (i) Sketch the graph of $y = \frac{x+2}{|x|}$. **2**

(ii) Hence, solve $-1 < \frac{x+2}{|x|} \leq 1$. **2**

Question 8 (17 marks) Start a NEW Writing Booklet

(a) In the diagram below, the points P, Q and T lie on a circle.
The line MN is tangent to the circle at T with M chosen so that QM is perpendicular to MN.
The point K on PQ is chosen so that TK is perpendicular to PQ.



(i) Copy the diagram to your Writing Booklet.

(ii) Show that $QKTM$ is a cyclic quadrilateral. **1**

(iii) Show that $\angle KMT = \angle KQT$. **1**

(iv) Hence, or otherwise, show that MK is parallel to TP. **2**

(b) (i) Show that $\dfrac{d}{dx}(\tan^3 x) = 3\sec^2 x(\sec^2 x - 1)$. **2**

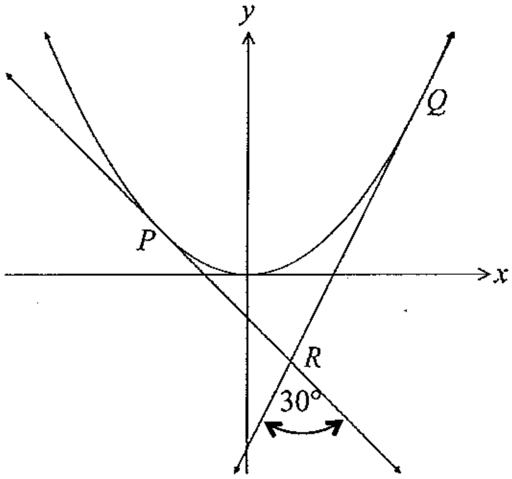
(ii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$. **2**

Question 8 continues on page 9

Question 8 (continued)

(c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

The tangents to the parabola at P and Q intersect at R and at an angle of 30°.



(i) Show that the co-ordinates of R are $(ap + aq, apq)$ **2**

(ii) Show that $\sqrt{3}\,|\,p-q\,| = |\,1+pq\,|$ **1**

(iii) Find the equation of the locus of R as P and Q move on the parabola. **3**

(d) Consider the number 77…77 which has $n + 1$ digits, all sevens. **3**

$$\underbrace{777...7}_{n+1\text{ digits}} = 7 + 7\times 10 + 7\times 10^2 + ... + 7\times 10^n \text{ (Do NOT prove)}$$

Prove by mathematical induction that

$$7 + 77 + 777 + ... + \underbrace{77...77}_{n\text{ digits}} = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$$

for integers $n \geq 1$.

End of Question 8

Question 9 (15 marks) Start a NEW Writing Booklet

(a) Let α be an approximation to a root of $x^2 = k$.

(i) Using Newton's method, show that a further approximation is given by $\dfrac{\alpha^2 + k}{2\alpha}$. **2**

(ii) Hence, find a rational approximation to $2\sqrt{2}$ using $\alpha = 3$. **1**

(b) In a set of 40 cards there are 10 red, 10 green, 10 black and 10 blue cards. Cards of the same color are numbered separately 1, 2, 3, …, 10.

(i) Find the number of different ways three cards may be selected at random from the entire set. **1**

(ii) Show that the number of selections in part (i) for which the cards are not all of different colors is 5880. **2**

(iii) What is the probability that the three cards selected have different colours and are not consecutive numbers? **3**

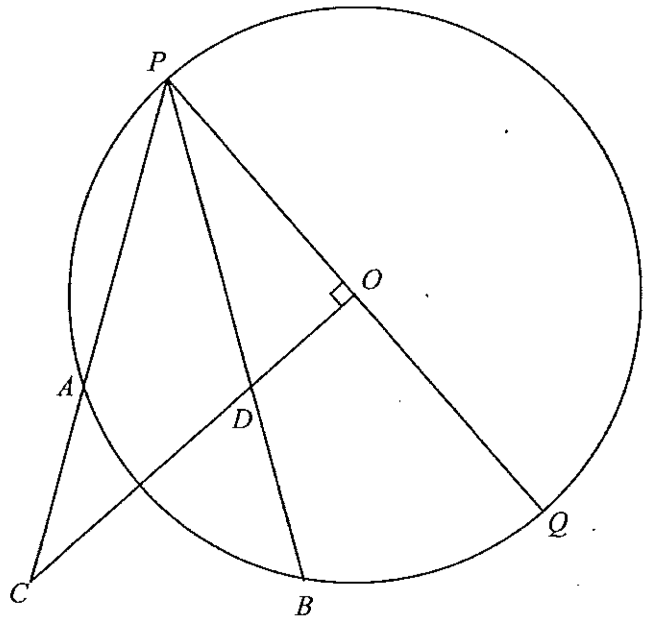
Question 9 continues on page 10

Question 9 (continued)

(c) In the diagram below, PA and PB are two chords on the same side of a diameter POQ, of a circle with centre O. **3**

Chord PA is produced to C, so that OC is perpendicular to PQ.
OC cuts PB at D.

Prove $ACBD$ is a cyclic quadrilateral.



(d) The straight lines $y = mx$ and $y = kx$ meet the line $y = c$ at points P and Q respectively.

The perpendiculars to the two given lines at P and Q intersect at R, which has coordinates $R\left(\dfrac{cm+ck}{mk}, \dfrac{c(mk-1)}{mk}\right)$ (Do NOT prove).

(i) Draw a diagram to represent the above information.

(ii) If the length of the interval PQ is $3c$, find the equation of the curve on which R lies. **3**

End of paper

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HSC Task #2

Mathematics Extension 1

Suggested Solutions

MC Answers

1.	B
2.	C
3.	A
4.	C
5.	D
6.	-

MARKERS

MC	BD
7.	AYW
8.	BK
9.	PSP

X1 Y12 Assessment Task 2 2017 Multiple choice solutions

Mean (out of 5): 4.40

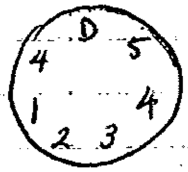
Q1. $x = 2 + 3t,\ y = 2 - 3t$

$\therefore x + y = 4$

$\therefore x + y - 4 = 0$ (B)

A	2
B	163
C	1
D	3

Q2.



$5! \times 4 = 480$

$6! = 720$

$\therefore 6! - 5! \times 2!$

$= 720 - 240$

$= 480$ (C)

A	13
B	8
C	148
D	0

Q3. $f(x) = x^3 + 3x - 1$

$f'(x) = 3x^2 + 3$

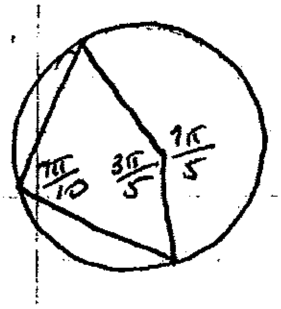
$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1 - \frac{3}{6}$

$= \frac{1}{2}$ (A)

A	162
B	5
C	2
D	0

Q4



(C)

A	11
B	12
C	145
D	1

Q5. ${}^mC_2 \cdot {}^mC_2$

$= \frac{m(m-1)}{2} \cdot \frac{m(m-1)}{2}$

$= \frac{m^2(m-1)^2}{4}$ (D)

A	6
B	27
C	10
D	126

Q6. $\int \sin x \cos^3 x\,dx$ let $u = \cos x$, $du = -\sin x\,dx$

$= -\int u^3\,du$

$= -\frac{1}{4}u^4 + c$

$= \frac{-\cos^4 x}{4} + c$

No answer

A	0
B	5
C	63
D	100

7. a) $(2x+1)^2 \times \frac{x}{2x+1} < 2 \times (2x+1)^2$ (1 mark.

Marker's comments:
Poor results for this question by many candidates due to one (or more) of the following:
- Not taking account of the critical values or not multiplying both sides by $(2x+1)^2$.
- Cant factorise the resulting quadratic equation properly.
- Using the wrong inequality and/or inequality sign for their final solution.
Marks were deducted for the errors above.

$x(2x+1) < 2(2x+1)^2$

$0 < 2(2x+1)^2 - x(2x+1)$

$2(2x+1)^2 - x(2x+1) > 0$

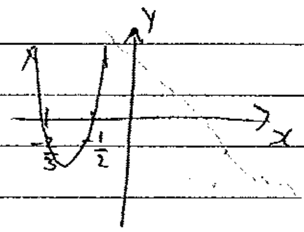
$(2x+1)[2(2x+1) - x] > 0$

$(2x+1)(3x+2) > 0$ (1 mark When $(2x+1)(3x+2) = 0$.

$x = -\frac{1}{2}, -\frac{2}{3}$

$\therefore x < -\frac{2}{3}$ and $x > -\frac{1}{2}$

(1 mark



b) $\lim_{x \to 0} \frac{\sin kx}{2x} = 5$

Marker's Comments:
Majority of the candidates did well in this question.

$LHS = \frac{1}{2} \lim_{x \to 0} \frac{\sin kx}{x}$

$= \frac{k}{2} \lim_{x \to 0} \frac{\sin kx}{kx}$

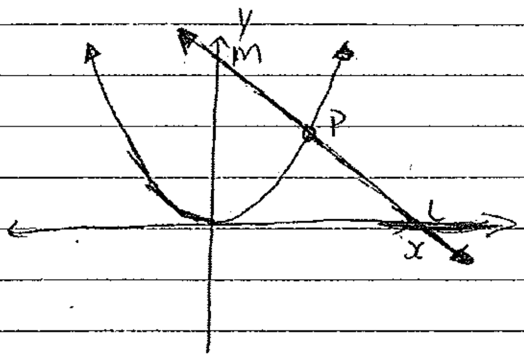
$= \frac{k}{2} \times 1$

$= \frac{k}{2}.$

$\therefore \frac{k}{2} = 5$

$k = 10.$ (1 mark.

c)



Since $x = 2p$ and $y = p^2$

$\therefore a = 1$ and $t = p$ (1)

i) From reference Sheet

At $(2at, at^2)$

Normal: $x + ty = at^3 + 2at$ (2)

$\therefore$ Equation of normal by sub (1) into (2)

$$x + py = p^3 + 2p$$ (1 mark)

Marker's Comments:
This question was done poorly by many candidates and inefficiently by majority of the candidates. Majority of the candidates derived the normal equation which was pointless when it is given on the REFERENCE SHEET. Many candidates who derived the normal equation made mistakes and received no marks.

Many candidates lost a half mark due to not recognising a = 1 from the coordinates given.

(ii) L: where $y = 0$

$x = p^3 + 2p$

$\therefore L = (p^3 + 2p, 0)$ (1 mark)

M: where $x = 0$

$py = p^3 + 2p$

$\therefore y = p^2 + 2$

$\therefore M = (0, p^2 + 2)$ (1 mark)

Marker's Comments:
Candidates lost a half mark if they did not state which coordinate was for L and which is for M.

Most candidates did well in this question.

(iii) $\text{Area}_{\triangle OLM} = \frac{1}{2} \times (p^3 + 2p) \times (p^2 + 2)$

$= \dfrac{p^5 + 2p^3 + 2p^3 + 4p}{2}$

$= \dfrac{p^5 + 4p^3 + 4p}{2}$

$= \dfrac{p(p^4 + 4p^2 + 4)}{2}$

$= \dfrac{p(p^2 + 2)^2}{2}$ units^2 (1 mark)

Marker's Comments
Majority of the candidates did well in this question.

d) $\int \cos^2 4x \, dx = \frac{1}{2}\int (1+\cos 8x)\, dx$ (1 mark)

$= \frac{1}{2}\left[x + \frac{\sin 8x}{8} \right] + c$

$= \frac{x}{2} + \frac{\sin 8x}{16} + c$ (1 mark)

Marker's Comments
- Many candidates did well in this question as they recognised $\cos^2 4x = (1+\cos 8x)/2$.
- Few candidates didn't write "+c" or "dx", which they lost a half mark.

(e) let $S(n)$ be the statement $6^{2n} - 5^{2n}$ is divisible by 11 for integer $n \geq 1$

Prove $S(1)$ is true

$6^2 - 5^2 = 36 - 25$
$= 11$

which is divisible by 11
$\therefore S(1)$ is true.

(1 mark)

Marker's Comments
*Majority of the candidates lost a half mark if they didnt not state **k is a positive integer** for the assumption.*

Assume $S(k)$ is true for an integer $k \geq 1$
i.e. assume that $6^{2k} - 5^{2k} = 11M$ where M is an integer $(*)$

Now prove $S(k+1)$ is true if $S(k)$ is true
i.e prove $6^{2(k+1)} - 5^{2(k+1)} = 11N$ where N is an integer.

$LHS = 6^{2(k+1)} - 5^{2(k+1)}$
$= 6^{2k+2} - 5^{2k+2}$
$= 36(6^{2k}) - 25(5^{2k})$
$= 25(6^{2k}) + 11(6^{2k}) - 25(5^{2k})$
$= 25(6^{2k} - 5^{2k}) + 11(6^{2k})$

Sub in $(*)$ from assumption
$= 25(11M) + 11(6^{2k})$
$= 11(25M) + 11(6^{2k})$
$= 11(25M + 6^{2k})$
$= 11N$ (where $N = 25 + 6^{2k}$ is an integer)
$= RHS$.

(1 mark)

Marker's Comments
Many candidates really struggled in proving the result holds for k+1 due to the following:
- Not using the assumption correctly.
- Writing $2(k+1) = 2k + 1$ which mad e the result false. (silly mistake but done by a significant number of candidates.)

Candidates need to take more care in specifying that $N = 25 + 6^{2k}$ is a positive integer as k is an integer.

$S(k+1)$ is true if $S(k)$ is true, $S(1)$ is true
$\therefore$ By mathematical induction, $S(n)$ is true for all integers $n \geq 1$

(1 mark for including entire process.)

(4)

f1(i) $y = \frac{x+2}{|x|}$ $x \neq 0$ ∴ Vertical Asymptote at $x = 0$.

When $x > 0$

$y = \frac{x+2}{x}$

$= 1 + \frac{2}{x}$

when $x \to \infty$

$y \Rightarrow 1$

when $x \to 0^+$

$y \Rightarrow \infty^+$ (1)

When $x < 0$

$y = \frac{x+2}{-x}$

$= -1 - \frac{2}{x}$

when $x \to -\infty$

$y \Rightarrow -1$

when $x \to 0^-$

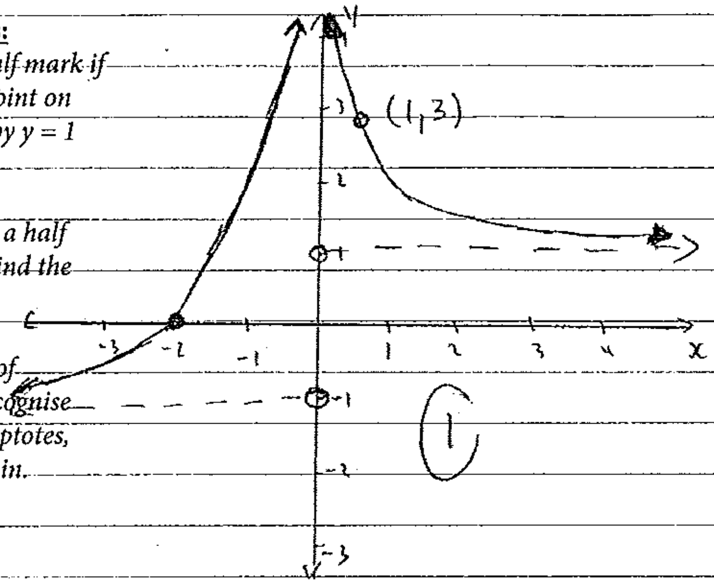
$y \Rightarrow \infty^+$

Marker's Comments:

- Candidates lost a half mark if they did not label a point on the branch bounded by y = 1 and the y-axis.

- Candidates also lost a half mark if they did not find the x-intercept.

- Significant number of candidates did not recognise the 2 horizontal asymptotes, particularly the domain.



(1)

(ii) From diagram above, the values of x for which $-1 < \frac{x+2}{x} \leq 1$

Intercept:

x-intercept:

$0 = \frac{x+2}{|x|}$

$x + 2 = 0$

$x = -2$.

when $y = 1$

$1 = \frac{x+2}{-x}$ (As it must be the branch on the left of y-axis) (1)

$-x = x + 2$

$-2x = 2$

$x = -1$

∴ $x \leq -1$ (1)

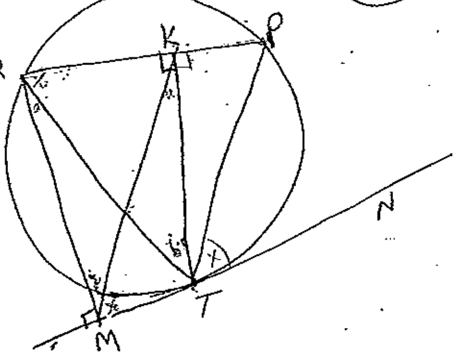
Marker's Comments:

Full marks given if candidates were able to identify the correct answer from the graph in part i).

Candidates were making errors due to attempting to solve it algebraically or not recognising that -1 and 1 was the range of the graph and not the domain.

Q8 (a)

17



(i) Show QKTM is a cyclic quad.

$\angle QKT = 90°$ (given)
$\angle QMT = 90°$ (given)
$\therefore$ QKTM is a cyclic quad. (opposite angles are supplementary)

1

(ii) $\angle KMT$ and $\angle KQT$ are both standing on chord KT in cyclic quadrilateral QKTM

1

$\Rightarrow \angle KMT = \angle KQT$ ($\angle$s in same segment)

(iii) Show MK // TP

$\angle PTN = \angle KQT$ (Alternate Segment Theorem)
$\Rightarrow \angle PTN = \angle KMT$ (since $\angle KQT = \angle KMT$ from (ii).)
$\therefore$ MK // PT (Corresponding $\angle$s)

2

Q8 (b) (i) Show $\frac{d}{dx}(\tan^3 x) = 3\sec^2 x(\sec^2 x - 1)$

$$\frac{d}{dx}(\tan^3 x)$$
$$= 3\tan^2 x \cdot \sec^2 x$$
$$= 3(\sec^2 x - 1)\sec^2 x$$
$$= 3\sec^2 x(\sec^2 x - 1)$$

2

(ii) Hence evaluate $\int_0^{\pi/4} \sec^4 x\, dx$

From (i) $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$.

$$\Rightarrow \int (3\sec^4 x - 3\sec^2 x)\, dx = \tan^3 x$$
$$\Rightarrow 3\int \sec^4 x\, dx = \tan^3 x + \int 3\sec^2 x\, dx$$
$$\int \sec^4 x\, dx = \frac{1}{3}(\tan^3 x + 3\tan x) + C$$
$$= \frac{1}{3}\tan^3 x + \tan x + C.$$

Then $\int_0^{\frac{\pi}{4}} \sec^4 x\, dx = \left[\frac{1}{3}\tan^3 x + \tan x\right]_0^{\pi/4}$

$$= \left(\frac{1}{3} \times 1 + 1\right) - 0$$
$$= \frac{4}{3}.$$

2

8(a) was done well.

Q8(b) (i) some students did not use a Pythagorean identity and so could not get the required result/

Q8(b)(ii) Even with the given derivative in (i) some students could not rearrange the integral to isolate the one required.

Q8.(c) (i) Eqn of tangent at P is

$$y = px - ap^2 \quad (1)$$

Eqn of tangent at Q is $y = qx - aq^2$ (2)

solving for R (1)−(2)

$$\Rightarrow (p-q)x - ap^2 + aq^2 = 0$$

$$(p-q)x = ap^2 - aq^2$$

$$(p-q)x = a(p-q)(p+q)$$

$$x = a(p+q) \checkmark \; (p \neq q)$$

Sub for x in (1) $\Rightarrow y = pa(p+q) - ap^2$

$$= ap(p+q-p) \checkmark$$

$$y = apq$$

(2)

$$\therefore R = (a(p+q), apq)$$

(ii) Show $\sqrt{3}|p-q| = |1+pq|$

Now $\tan\alpha = \left|\frac{m_1 - m_2}{1 - m_1 m_2}\right|$

$$\Rightarrow \tan 30 = \left|\frac{p-q}{1-pq}\right|$$

$$\frac{1}{\sqrt{3}} = \frac{|p-q|}{|1+pq|}$$

$$\Rightarrow |1+pq| = \sqrt{3}|p-q|$$

✓ (1)

Part (c)(i) was generally done well but students wasted time deriving the equation for the tangent when is was given on the formula sheet. Part (c)(ii) was generally done well.

8(iii) Locus of R $x = a(p+q)$, $y = apq$.

$$\Rightarrow \frac{x}{a} = p+q \text{ and } \frac{y}{a} = pq.$$

$$\frac{x^2}{a^2} = (p+q)^2$$

$$\frac{x^2}{a^2} = (p-q)^2 + 4pq \checkmark$$

But $p - q = \frac{1+pq}{\sqrt{3}}$ from (ii)

$$= \left(\frac{1+pq}{\sqrt{3}}\right)^2 + 4pq$$

$$\frac{x^2}{a^2} = \frac{1}{3}(1+pq)^2 + 4pq$$

$$= \frac{1}{3}\left(1 + \frac{y}{a}\right)^2 + 4\frac{y}{a} \checkmark$$

$$\frac{x^2}{a^2} = \frac{1}{3}\left(\frac{a+y}{a}\right)^2 + \frac{4y}{a}$$

(3)

$$\times a^2 \Rightarrow x^2 = \frac{1}{3}(a+y)^2 + 4ay$$

$$\underline{3x^2} = (a+y)^2 + 12ay \checkmark$$

Many students did not understand that this type of problem requires them to eliminate the parameters p and q.

8(d) $77\ldots77 \to (n+1)$ digits

$= 7 + 7\times10 + 7\times10^2 + \ldots + 7\times10^n$

Prove $7 + 77 + 777 + \ldots + \underbrace{77\ldots77}_{n\text{ digits}} = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$

Let $n=1$

LHS $= 7$

RHS $= \frac{7}{81}\left(10^2 - 9 - 10\right)$

$= \frac{7}{81} \times 81 = 7$

$\therefore$ true for $n=1$.

Assume true for $n=k$

ie. $7 + 77 + 777 + \ldots + \underbrace{77\ldots77}_{k\text{ digits}} = \frac{7}{81}\left(10^{k+1} - 9k - 10\right)$

Let $n=k+1$.

Must show

$7 + 77 + 777 + \ldots + \underbrace{77\ldots77}_{k\text{ digits}} + \underbrace{77\ldots77}_{(k+1)^{th}\text{ digit}} = \frac{7}{81}\left(10^{k+2} - 9(k+1) - 10\right)$

LHS $= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + \underbrace{77\ldots77}_{(k+1)\text{digits}}$ (from assumption)

$= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + \left(7 + 7\times10 + 7\times10^2 + \ldots + 7\times10^k\right)$

$= \left(\frac{7}{81}\times10^{k+1} - \frac{7}{9}k - \frac{70}{81}\right) + \left(7 + 7\times10 + 7\times10^2 + \ldots + 7\times10^k\right)$

The main error in 8(d) was in writing that 77...77 to (k+1) digits =7X10^k. In this case students could not proceed correctly to the GP.

8(d) cont

$= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + \underbrace{77\ldots77}_{(k+1)\text{digits}}$ (from assumption)

$= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + \left(7 + 7\times10 + 7\times10^2 + \ldots + 7\times10^k\right)$ from (i)

$= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + 7\left(1 + 10 + 10^2 + \ldots + 10^k\right)$

GP $a=1$, $n=k+1$, $r=10$

$= \frac{7}{81}\left(10^{k+1} - 9k - 10\right) + 7\left(\frac{10^{k+1} - 1}{9}\right)$

$= \frac{7}{81}\times10^{k+1} - \frac{7}{81}\times9k - \frac{7}{81}\times10 + \frac{7}{9}\left(10^{k+1}\right) - \frac{7}{9}$

$= \frac{70}{81}\left(10^{k+1}\right) - \frac{7}{81}\times9k - \frac{70}{81} - \frac{7}{9}$

$\frac{7}{81}\left(10^{k+2}\right) - \frac{7}{81}(9k) - \frac{133}{81}$

$\frac{7}{81}\left(10^{k+2}\right) - \frac{7}{81}(9k) - \frac{7}{81}(10+9)$

$\frac{7}{81}\left(10^{k+2} - 9k - 10 - 9\right)$

$= \frac{7}{81}\left(10^{k+2} - 9(k+1) - 10\right)$

3

$\therefore$ true for $n=k+1$

So, by the Principle of Mathematical Induction, true for all $n\in Z^+$.

Question 9 15 marks

(a) Let α be an approximation to a root of $x^2 = k$.

(i) Using Newton's method, show that a further approximation is given by $\dfrac{\alpha^2+k}{2\alpha}$. **2**

Let $f(x)=x^2-k$ and $x_0=\alpha$

$\therefore f'(x)=2x$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= \alpha - \frac{\alpha^2-k}{2\alpha}$$
$$= \frac{2\alpha^2-\alpha^2+k}{2\alpha}$$
$$= \frac{\alpha^2+k}{2\alpha}$$

Comment:
This is a "Show that …" problem and so students who didn't show enough were penalised. Defining $f(x)$ and quoting the formula (from the Formula booklet) went a long way to showing sufficient understanding.

(ii) Hence, find a rational approximation to $2\sqrt{2}$ using $\alpha = 3$. **1**

$\therefore x^2 = 8$ i.e. $k = 8$

$$x_1 = \frac{3^2+8}{2\times 3}$$
$$= \frac{17}{6}$$

Comment:
Surprisingly it seems that using $k = 8$, with the previous part, was not obvious.
Also, this is a "Hence …" problem and many students ignored this fact.

(b) In a set of 40 cards there are 10 red, 10 green, 10 black and 10 blue cards.
Cards of the same color are numbered separately 1, 2, 3, …, 10.

(i) Find the number of different ways three cards may be selected at random from the entire set. **1**

Choosing/selecting 3 cards from a set of 40 = ${}^{40}C_3$.

Comment:
Too many students answered with ${}^{40}P_3$.
A selection of 3 cards was asked for, not an arrangement.

(b) (ii) Show that the number of selections in part (i) for which the cards are not all of different colors is 5880. **2**

How many ways can they be of different colours?

Choose the three colours in ${}^{4}C_{3}$ ways.

There are ${}^{10}C_{1}$ ways to pick a card from one colour.

$\therefore$ there are ${}^{4}C_{3}\times{}^{10}C_{1}\times{}^{10}C_{1}\times{}^{10}C_{1}=4000$ ways to pick 3 cards of different colours

$\therefore$ there are ${}^{40}C_{3}-{}^{4}C_{3}\times{}^{10}C_{1}\times{}^{10}C_{1}\times{}^{10}C_{1}=5880$ which the cards are not all of different colors.

Comment:

This was not done well. Recognising complementary events needs more work by many students.

(b) (iii) What is the probability that the three cards selected have different colours and are not consecutive numbers? **3**

How many ways are there to pick 3 consecutive numbers of different colour?

First, consecutive numbers:

They would be:

1 2 3
2 3 4
⋮
8 9 10

For each one there are ${}^{4}P_{3} = 4!$ ways that they could be of a different colour i.e. $4!\times 8=192$ total possibilities.

How many ways of picking three cards of different colours?
From (ii) there are 4000 ways.

So there are $4000-192=3808$ ways of selected have different colours and are not consecutive numbers.

$\therefore$ the probability is $\dfrac{3808}{{}^{40}C_{3}}=\dfrac{476}{1235}$.

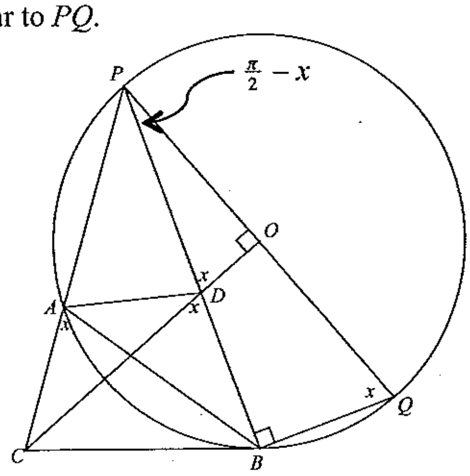
Comment:

This was not done well. Recognising complementary events needs more work by many students.

(c) In the diagram below, PA and PB are two chords on the same side of a diameter POQ, of a circle with centre O. **3**

Chord PA is produced to C, so that OC is perpendicular to PQ. OC cuts PB at D.

Prove $ACBD$ is a cyclic quadrilateral.



Let $\angle PQB = x$

$\therefore \angle QPB = \frac{\pi}{2} - x$ (angle sum of $\Delta\, PBQ$)

$\therefore \angle PDO = x$ (angle sum of $\Delta\, PDO$)

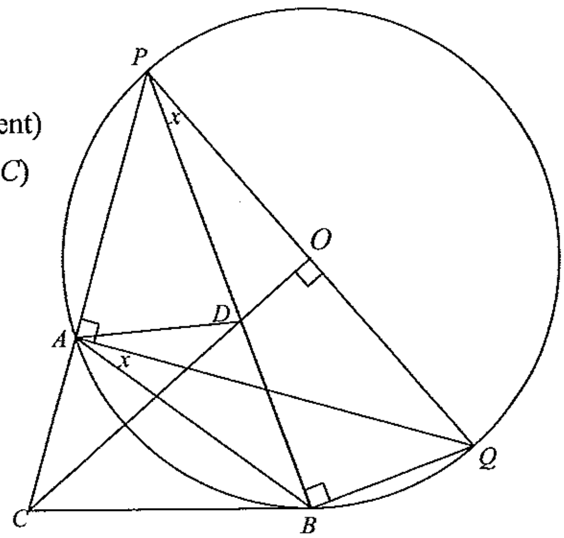
$\therefore \angle CDB = x$ (vert. opp. $\angle$s)

Also $\angle CAB = x$ (exterior angle cyclic quad $PABQ$)

$\therefore \angle CDB = \angle CAB = x$

$\therefore ACBD$ is a cyclic quad (converse of $\angle$s in the same segment)

Alternative



Let $\angle BPQ = x$

$\therefore \angle BAQ = x$ (angles in same segment)

$\therefore \angle BAC = \frac{\pi}{2} - x$ (angle sum of line PAC)

$\angle PDO = \frac{\pi}{2} - x$ (angle sum $\Delta\, PDO$)

$\therefore \angle CDB = \frac{\pi}{2} - x$ (vertically opposite)

$\therefore \angle BAC = \angle CDB = \frac{\pi}{2} - x$

$\therefore ACBD$ is a cyclic quad (converse of $\angle$s in the same segment)

Comment:

This was generally done well by those who had time to do it.

(d) The straight lines $y = mx$ and $y = kx$ meet the line $y = c$ at points P and Q respectively.

The perpendiculars to the two given lines at P and Q intersect at R, which has coordinates $R\left(\frac{cm+ck}{mk}, \frac{c(mk-1)}{mk}\right)$ (Do NOT prove).

(i) Draw a diagram to represent the above information.

Points P and Q have coordinates $\left(\frac{c}{m}, c\right)$ and $\left(\frac{c}{k}, c\right)$ respectively

(ii) If the length of the interval PQ is $3c$, find the equation of the curve on which R lies. **3**

$$PQ = 3 \Leftrightarrow \left|\frac{c}{m} - \frac{c}{k}\right| = 3c$$

$$\therefore \left|\frac{c(k-m)}{mk}\right| = 3c$$

$$\therefore \frac{c^2(k-m)^2}{m^2k^2} = 9c^2$$

$$\therefore (k-m)^2 = 9m^2k^2$$

With $R\left(\frac{cm+ck}{mk}, \frac{c(mk-1)}{mk}\right)$ let $x = \frac{c(m+k)}{mk}$ and $y = \frac{c(mk-1)}{mk}$

$$x^2 = \frac{c^2(m+k)^2}{m^2k^2}$$

$$= c^2 \times \frac{(m-k)^2 + 4mk}{m^2k^2}$$

$$= c^2 \times \frac{9m^2k^2 + 4mk}{m^2k^2}$$

$$= 9c^2 + \frac{4c^2}{mk}$$

$$y = c \times \left(1 - \frac{1}{mk}\right)$$

$$\therefore 4cy = 4c^2 - \frac{4c^2}{mk}$$

$\therefore x^2 + 4cy = 13c^2$ is the equation of the locus of R.

Comment:

Students who used the absolute value for PQ, had a better chance of recognising that x^2 would be involved with the equation of the locus.

End of solutions