SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2013
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #2

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- Each Section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 62

- Attempt questions 1 – 10.
- All questions are not of equal value.
- Unless otherwise directed give your answers in simplest exact form.

Examiner: *A.M.Gainford*

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\ x > 0$

Section A (19 Marks)

Questions 1 to 5. (5 marks)

Indicate which of the answers A, B, C, or D is the correct answer. **Marks**
Write the answer on the separate answer sheet.

(1) The gradient of the normal to the curve $y = x(x+1)$ at the point where $x = 1$ is: **1**

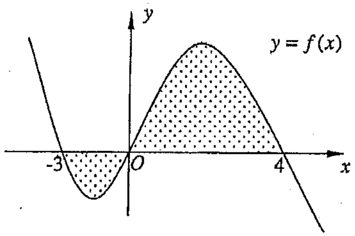
A: 3

B: $-\frac{1}{3}$

C: -3

D: $\frac{1}{3}$

(2) Consider the figure below: **1**



Which of the following represents the shaded area?

A: $\int_{-3}^{4} f(x)dx$

B: $2\int_{0}^{4} f(x)dx$

C: $\int_{0}^{4} f(x)dx - \int_{-3}^{0} f(x)dx$

D: $\int_{-3}^{0} f(x)dx + \int_{0}^{4} f(x)dx$

(3) For what values of x is the curve $f(x) = 2x^3 + x^2$ concave downwards? **1**

A: $x < -\frac{1}{6}$

B: $x > -\frac{1}{6}$

C: $x < -6$

D: $x > 6$

(4) The chance of a fisherman catching a legal length fish is 4 in 5. If he catches three fish at random, what is the probability that exactly one is of legal length? **1**

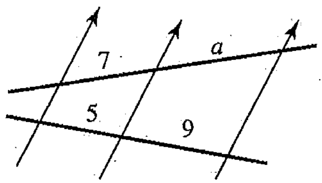
A: $\frac{4}{125}$

B: $\frac{12}{125}$

C: $\frac{16}{125}$

D: $\frac{48}{125}$

(5) **1**



The value of a in the diagram above is:

A: 9

B: 11

C: 12

D: 12.6

Question 6 (14 marks) (Start a new booklet)

Marks

(a) Differentiate the following: 4

(i) $7+2x-2x^3$

(ii) $(3x^2-1)^7$

(iii) $x\sqrt{x-1}$

(iv) $\dfrac{x}{3x+1}$

(b) Find 4

(i) $\int(4x^2+2x)dx$

(ii) $\int\frac{1-x^2}{x^2}dx$

(c) Evaluate 2

$$\int_{-1}^{3}(x^2-3x)dx$$

(d) 4

(i) Copy and complete the table for $f(x)=\dfrac{x^2}{1+x}$ correct to 4 decimal places.

x	0	1	2	3	4
$f(x)$	0				

(ii) Use Simpson's Rule with the above 5 function values to find an approximation to $\int_0^4 \frac{x^2}{1+x}dx$ correct to 4 decimal places.

Section B (21 Marks)

START A NEW BOOKLET

Question 7 (11 Marks)

Marks

(a) A certain school has 500 students. It is found that 20% are left-handed, and 40% wear glasses. It is also known that 52% of the right-handed students do not wear glasses. 3

(i) Represent this situation with an appropriate diagram.

(ii) State the probability that a student selected at random is left handed and does not wear glasses.

(b) The vertices of the triangle *OAB* are the points *O*(0,0), *A*(0,2), and *B*(3,-1). 6

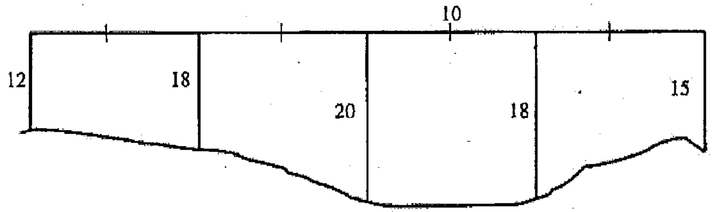
(i) Draw a sketch diagram of the triangle.

(ii) The point *K* on *AB* is such that *OK* is perpendicular to *AB*. Find the co-ordinates of *K*, and show the point *K* on your diagram.

(iii) Find the area of the triangle *OAB*.

(iv) The line through the point *B*, perpendicular to *OA*, meets *KO* produced at *S*. Find the co-ordinates of *S*.

(c) 2

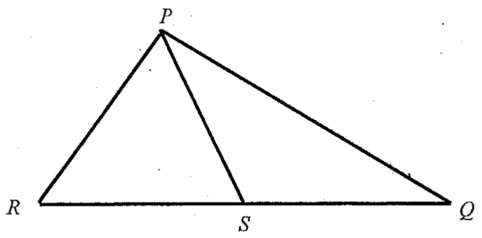


The cross-section of a river is shown above. All measurements are in metres.

Use the trapezoidal rule to estimate the area of the cross-section.

Question 8 (10 Marks)

(a) 4



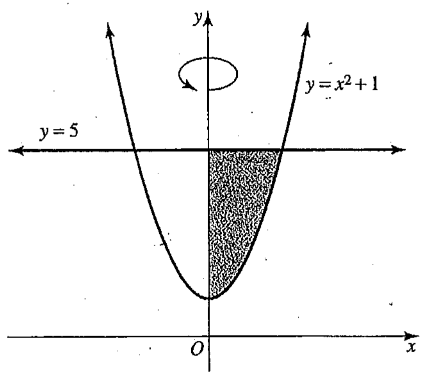
In the diagram $\angle QPR = 90^0$, $PS = SQ$.

(i) Copy the diagram to your answer booklet.

(ii) Prove that $\angle SPR = \angle SRP$.

(b) Show that the triangle whose sides satisfy $2x - y = 0$, $x + 2y = 5$ and $x - 3y = 20$ is isosceles and right-angled. 4

(c) 2



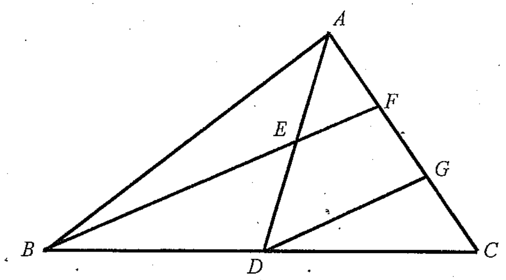
In the diagram the shaded region is bounded by the parabola $y = x^2 + 1$, the y-axis and the line $y = 5$.
Find the volume of the solid formed when then shaded region is rotated about the y-axis.

Section C (22 Marks)

START A NEW BOOKLET

Question 9 (12 Marks)

(a) 4



In the diagram above, D and E are the midpoints of BC and AD respectively, and $DG \parallel BF$.

(i) Copy the diagram to your answer booklet.

(ii) Prove that $AF = FG = GC$.

(b) Consider the curve with equation $y = x^3 - 3x^2 - 9x + 5$. 8

(i) Find the co-ordinates of the stationary points and determine their nature.

(ii) Find the co-ordinates of any points of inflexion.

(iii) Sketch the curve for the domain $-3 \le x \le 5$. (Do not attempt to find the x-intercepts.)

(iv) Mark on your curve, with the letter S, the points where the curve is increasing at the greatest rate.

Question 10 (10 Marks)

(a) The curvature at all points on a curve $y = f(x)$ is given by $f''(x) = 3x^2 - 2x - 1$. **2**

Find the equation of the curve given that $f(2) = 1$ and there is a stationary point at $x = 2$.

(b) (i) Differentiate $(x+2)\sqrt{x+1}$. **4**

(ii) Hence evaluate $\int_0^3 \frac{3x+4}{\sqrt{x+1}}\,dx$.

(c) Paul is walking along a straight road towards the town of Longueville, 15 km away. At the same time, Kirsti starts walking away from Longueville, along a straight road at right angles to the first road. **4**
If Paul walks at 5 km/h and Kirsti at 3 km/h:

(i) Show that at time t hours after they set out, their distance apart, d km, is given by $d = \sqrt{34t^2 - 150t + 225}$.

(ii) How far from Longueville are Paul and Kirsti when they are closest to each other? (Answer in kilometres, correct to one decimal place.)

This is the end of the paper.

Student Number: Answers

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Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

Section I: **Multiple choice answer sheet.**

Completely colour the cell representing your answer. Use black pen.

1. A ● B C D — **B**
2. A B ● C D — **C**
3. ● A B C D — **A**
4. A ● B C D — **B**
5. A B C ● D — **D**

Question 6.

(a) (i) $2-6x^2$ 1

(ii) $7(3x^2-1)^6 \times 6x$

$= 42x(3x^2-1)^6$ 1

(iii) $(x-1)^{\frac{1}{2}} + \frac{1}{2}(x-1)^{-\frac{1}{2}}x$

$= \sqrt{x-1} + \dfrac{x}{2\sqrt{x-1}}$ 1

(iv) $\dfrac{(3x+1)-3x}{(3x+1)^2}$

$= \dfrac{1}{(3x+1)^2}$ 1

(b) (i) $\frac{4x^3}{3} + x^2 + C$ 2

(ii) $\int \frac{1}{x^2} - 1\,dx = -x^{-1} - x + C$ 2

$= -\frac{1}{x} - x + C$

(c) $\int_{-1}^{3} (x^2-3x)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{3}$

$= \left(\frac{3^3}{3} - \frac{3^3}{2}\right) - \left(-\frac{1}{3} - \frac{3}{2}\right)$ 2

$= -\frac{8}{3}$

(d) $f(x) = \dfrac{x^2}{1+x}$

x	0	1	2	3	4	
$f(x)$	0	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{9}{4}$	$\frac{16}{5}$	1

$$\int_0^4 \frac{x^2}{1+x}\,dx \approx \frac{1}{3}\cdot\frac{4-0}{4}\left(0+\frac{16}{5}+4\left(\frac{1}{2}+\frac{9}{4}\right)+2\left(\frac{4}{3}\right)\right)$$

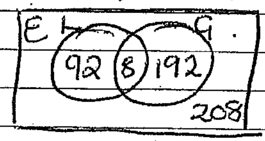
$$= \frac{1}{3}\left(\frac{16}{5}+11+\frac{8}{3}\right)$$

$$= \frac{253}{45} \qquad 3$$

$$\approx 5.6222$$

YR 12 TASK 2 – 2U MATHS – 2013.
SECTION B.

Q1 a) i)



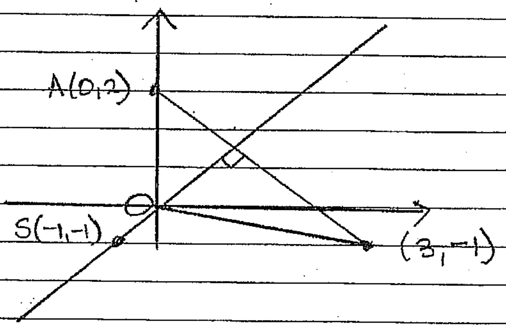
$\mathcal{E}$ = 500 students.
L = left handed students 20% = 100.
G = Wear glasses 40% = 200.

52% OF RIGHT HAND STUDENTS DON'T WEAR GLASSES
= 208

a ii) P(Left hand + no glasses)

$$= \frac{92}{500} = \frac{23}{125}.$$

b i).



b(ii)

$$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-2}{3-0} = \frac{-3}{3} = -1.$$

$$M_{OK} = \frac{-1}{M_{AB}} = \frac{-1}{-1} = 1$$

Eqtn OK: $y - y_1 = m(x-x_1)$. $(0,0)$ $m_{OK} = -1$.

$y = x$.

EQTN AB

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-2}{x-0} = \frac{-1-2}{3-0}$$

$$3(y-2) = -3x$$

$$y-2 = -x$$

$$x+y-2=0$$

INTERSECTION OF OK & AB

AB: $x+y-2=0$ —— ①

OK: $x=y$ —— ②

SUB ② INTO ①

$$x+x-2=0$$

$$2x=2$$

$$x=1$$

when $x=1$ $y=1$

$K(1,1)$

b(iii)

Area OAB $= \frac{1}{2}bh$

$= \frac{1}{2}(2)(3)$

$= 3$ units^2

(BY PERP DIST)

PERP DIST FROM $(0,0)$ TO AB $x+y-2=0$.

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{+2}{\sqrt{2}} = \sqrt{2}$$

LENGTH AB $=$

$A(0,2)$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$B(3,-1)$ $= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Area AOB $= \frac{1}{2}b \times h$

$= \frac{1}{2}(\overline{AB})(\overline{OK})$

$= \frac{1}{2} 3\sqrt{2}\ \sqrt{2}$

$= 3$ UNITS^2

b(iv) $S = (x_1, -1)$ as line through $B(3,-1)$

EQTN OK IS $y=x$

SO $S=(-1,-1)$

c) i)

x	0	10	20	30	40
$f(x)$	12	18	20	18	15

$h=10$

$$\text{Area} \doteqdot \frac{h}{2}\left(f(x_0)+f(x_4)+2[f(x_1)+f(x_2)+f(x_3)]\right)$$

$= \frac{10}{2}[12+15+2(18+20+18)]$

$= 5[27+2\times(56)]$

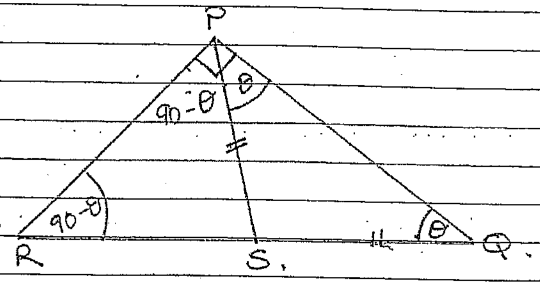
$= 5(139)$

$\doteqdot 695$ UNITS^2

YR 12 TASK 2 - 2U MATHS 2013.

SECTION B - Q8

a) i)



a(ii) Let $\angle PQS = \theta$

$\therefore \angle SPQ = \theta$ ($PS = SQ$ isosceles $\triangle$)

$\therefore \angle SPR = 90 - \theta$ (complementary $\angle$s)

In $\triangle RPQ$

$\angle QRP = 180 - \{90 - \theta\}$ ($\angle$ sum $\triangle$)

$= 90 - \theta$.

$\therefore \angle SRP = \angle QRP = 90 - \theta$. (common $\angle$)

$\therefore \angle SPR = 90 - \theta = \angle SRP$ Q.E.D.

b) i) $2x - y = 0$ $\quad x + 2y = 5 \quad x - 3y = 20$.

$y = 2x$. $\quad y = \dfrac{5-x}{2} \quad y = \dfrac{x-20}{3}$.

SOLVE SIMULTANEOUSLY.

Let A be intersection $y = 2x$. —— ①

$y = \dfrac{5-x}{2}$ —— ②.

Equating $2x = \dfrac{5-x}{2}$.

$4x = 5 - x$.

$5x = 5$

$x = 1 \quad y = 2$

A(1,2).

Let B be intersection of.

$y = \dfrac{5-x}{2}$ —— ③

$y = \dfrac{x-20}{3}$ —— ④.

EQUATING $\dfrac{5-x}{2} = \dfrac{x-20}{3}$.

$15 - 3x = 2x - 40$.

$55 = 5x$

$x = 11$.

$y = -3$

B(11,−3).

Let C be the intersection of

$y = \dfrac{x-20}{3}$.

$y = 2x$.

EQUATING. $2x = \dfrac{x-20}{3}$.

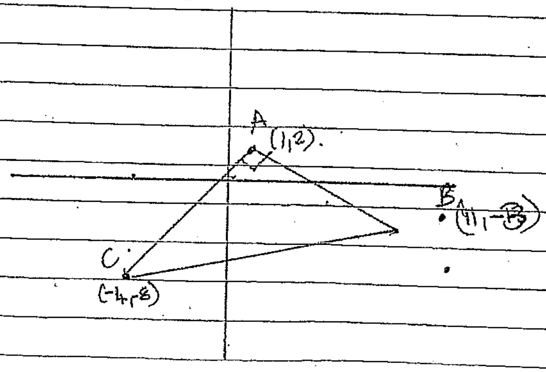
$6x = x - 20$

$5x = -20$

$x = -4$

$y = -8$

C (−4,−8)



$\overline{AC} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

A $(1,2)$ $= \sqrt{(1+4)^2 + (2+8)^2}$

C $(-4,-8)$ $= \sqrt{5^2+10^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$.

B$(11,-3)$ $\overline{AB} = \sqrt{(1-11)^2 + (-8+3)^2}$

$= \sqrt{10^2 + 5^2}$

$= 5\sqrt{5}$.

Since $\overline{AC} = \overline{AB}$ the $\triangle ABC$ is isosceles.

$m_{AC} = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-{}^-8}{1-{}^-4}$

$= \dfrac{10}{5}$

$= 2$.

$m_{AB} = \cancel{\dfrac{(-8+3)^2}{(-1-11)}} = \dfrac{2-{}^-3}{1-11} = \dfrac{5}{-10} = -\dfrac{1}{2}$.

Since $m_{AB} = -\dfrac{1}{m_{AC}}$.

AB and AC are perpendicular and

$\triangle ABC$ is right $\angle$d.

$\therefore \triangle ABC$ is right angled + isosceles.

c) $V = \pi \int_1^5 x^2 . dy$

$y = x^2+1$.

$x^2 = y-1$.

$V = \pi \int_1^5 (y-1)\, dy$.

$= \pi \int_1^5 \frac{y^2}{2} - y$

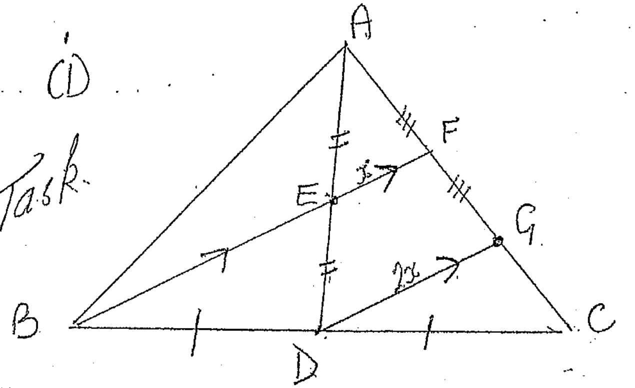
$= \pi \left[\frac{5^2}{2} - 5 - \left(\frac{1^2}{2} - 1\right) \right]$.

$= \pi \left[\frac{15}{2} - \left(-\frac{1}{2}\right) \right]$.

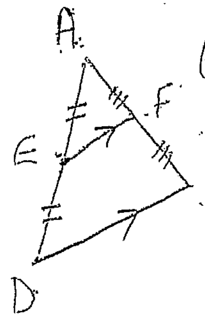
$= \frac{16}{2}\pi = 8\pi$ units^3.

1 9 (a) (i)

2 unit
April YR 12
2nd Assess Task



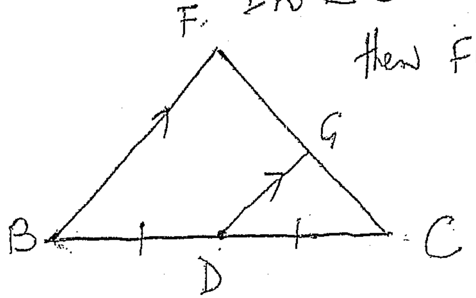
(1)



(ii) In $\triangle ADG$, because $AE = ED$ (given)

then $AF = FG$ a line through the midpt of 1 side of a triangle $\parallel$ to another side, bisects the 3rd side.

($1\frac{1}{2}$)



In $\triangle BFC$ because $BD = DC$ (given)

then $FG = GC$ explanation as above

($1\frac{1}{2}$)

$\therefore AF = FG$ and $FG = GC$

$\therefore AF = FG = GC$.

1 9 (b) $y = x^3 - 3x^2 - 9x + 5$

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

(i) when $y' = 0$ $3x^2 - 6x - 9 = 0$

$\div 3$ $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, \ x = -1$

when $x = 3$, $y = \cancel{27} - \cancel{27} - 27 + 5 = -22$

$(3, -22)$ (1) $y'' = 18 - 6 = 12 > 0$ min s. pt (1)

when $x = -1$ $y = -1 - 3 + 9 + 5 = 10$

$(-1, 10)$ (1) $y'' = -6 - 6 = -12 < 0$ MAX s. pt (1)

(ii) $y'' = 6x - 6 = 0$

$6x = 6$

$x = 1$

when $x = 1$, $y = 1 - 3 - 9 + 5 = -6$

$(1, -6)$ (1)

sign change?

at $x = 1 - \varepsilon$ $y'' < 0$

$x = 1 + \varepsilon$ $y'' > 0$ } yes (1)

P.O.I $(1, -6)$

at $x = -3$

$y = -27 - \cancel{27} + \cancel{27} + 5$

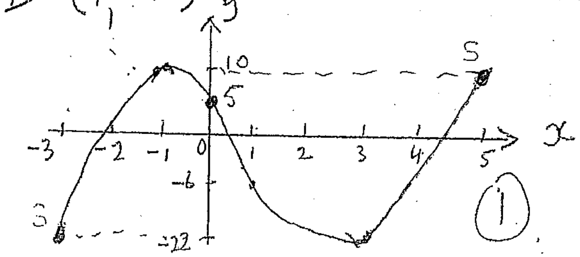
$= -22$

at $x = 5$

$y = 125 - 75 - 45 + 5$

$= 10$

(iii)



(1) + S (1)

10 (a) $f''(x) = 3x^2 - 2x - 1$

$f'(x) = \int (3x^2 - 2x - 1)\,dx$

$f'(x) = \cancel{\frac{3x^3}{3}} - \cancel{\frac{2x^2}{2}} - 1x + C$

$f'(x) = x^3 - x^2 - x + C$

<u>data</u> $x = 2,\ f'(x) = 0$

$0 = 8 - 4 - 2 + C$

$0 = 2 + C$

$C = -2$ 1

$f'(x) = x^3 - x^2 - x - 2$

$f(x) = \int (x^3 - x^2 - x - 2)\,dx$

$f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - 2x + K$

<u>data</u> $(2, 1)$ $1 = \cancel{4} - \frac{8}{3} - 2 - \cancel{4} + K$

$1 = -4\frac{2}{3} + K$

$K = 5\frac{2}{3}$ 1

$f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} - 2x + 5\frac{2}{3}$

10 (b) (i) $\frac{d}{dx}(x+2)(x+1)^{\frac{1}{2}}$

$= (x+2) \times \frac{1}{2}(x+1)^{-\frac{1}{2}} \times 1 + (x+1)^{\frac{1}{2}} \times 1$

$= \frac{x+2}{2\sqrt{x+1}} + \frac{\sqrt{x+1}}{1}$

$= \frac{x+2+2(x+1)}{2\sqrt{x+1}}$

$= \frac{3x+4}{2\sqrt{x+1}}$ 2

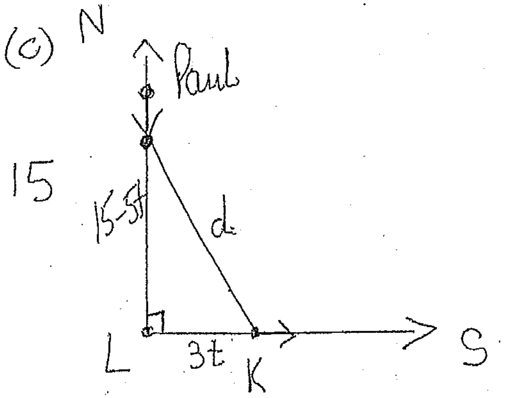
(ii) now $2\int_0^3 \frac{3x+4}{2\sqrt{x+1}}\,dx$

$= 2.(x+2)\sqrt{x+1}\Big]_0^3$

$= 2 \times 5 \times 2 - 2 \times 2 \times 1$

$= 20 - 4 = 16.$ 2

10 (c)



Paul 5 km/h after t hours 5t km.
Kirsti 3 km/h after t hours 3t km.

(i) $d^2 = (3t)^2 + (15-5t)^2$
$= 9t^2 + 225 - 150t + 25t^2$
$d^2 = 34t^2 - 150t + 225$
$d = \sqrt{34t^2 - 150t + 225}$ (1)

(ii) $\frac{dD}{dt} = \frac{1}{2}(34t^2 - 150t + 225)^{-\frac{1}{2}} \times 68t - 150$
$= \frac{34t - 75}{\sqrt{34t^2 - 150t + 225}}$ (1)

let $\frac{dD}{dt} = 0$, $34t - 75 = 0$
$34t = 75$
$t = \frac{75}{34} = 2.2$ hours. (1)

check $t = 2.2 - \varepsilon$ $\frac{dD}{dt} < 0$
$t = 2.2 + \varepsilon$ $\frac{dD}{dt} > 0$ } min satisfied

Paul is $15 - 5 \times 2.2$ km away $= 4$ km. (1)

and Kirsti is $3 \times 2.2 = 6.6$ km away

They are 7.72 km away on hypotenuse after 2.2 hr.