NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL

2014 Annual Examination

FORM V

MATHEMATICS 2 UNIT

Wednesday 27th August 2014

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 9 Marks

- Questions 1–9 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 91 Marks

- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Ten.
- Write your name and master on this question paper and submit it with your answers.

5A: BDD	5B: MLS	5C: LYL	5D: LRP
5E: PKH	5F: BR	5G: SG	5P: REJ
5Q: NL	5R: TCW		

Checklist

- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Candidature — 185 boys

Examiner
PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

The gradient of the line $3x - 2y = 6$ is

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

QUESTION TWO

The exact value of $\cos 210°$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

QUESTION THREE

The expression $\sqrt{16x^{16}}$ simplifies to

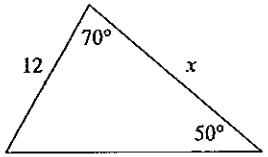
(A) $8x^4$ (B) $4x^8$ (C) $4x^4$ (D) $8x^8$

QUESTION FOUR

Which of the following is rational?

(A) $\sqrt[3]{18}$ (B) $\sqrt{8}$ (C) $0.\dot{4}\dot{9}$ (D) π

QUESTION FIVE



In the diagram above which of the following is correct?

(A) $x = \frac{12\sin 50°}{\sin 70°}$ (B) $x = \frac{6\sqrt{3}}{\sin 50°}$

(C) $x = \frac{12\sin 70°}{\sin 50°}$ (D) $x = \frac{6}{\sin 50°}$

Exam continues next page ...

QUESTION SIX

The quadratic equation $2x^2 - 4x + 6 = 0$ has roots α and β. Which of the following is true?

(A) $\alpha + \beta = 4$ (B) $\alpha + \beta = 2$

(C) $\alpha\beta = 6$ (D) $\alpha\beta = -6$

QUESTION SEVEN

What is the domain of $y = \sqrt{x^2 - 1}$?

(A) $x \geq 1$ (B) $x \geq 1$ or $x \leq -1$

(C) $x > 1$ (D) $-1 \leq x \leq 1$

QUESTION EIGHT

For the geometric sequence $2, -4, 8, -16, \ldots$ which of the statements below is true?

(A) The limiting sum is $\frac{2}{3}$.

(B) The sequence does not have a limiting sum.

(C) The nth term is never greater than 100 000.

(D) The sum of the first n terms is never less than $-100\,000$.

QUESTION NINE

Let $y = \frac{x}{1+x}$. A correct expression for the derivative $\frac{dy}{dx}$ is:

(A) $-\frac{1}{(1+x)^2}$ (B) $\frac{2x+1}{(1+x)^2}$

(C) $\frac{1}{(1+x)^2}$ (D) $\frac{2x-1}{(1+x)^2}$

End of Section I

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN (13 marks) Use a separate writing booklet. **Marks**

(a) Solve $\frac{x}{2} + 2x = 5$. [2]

(b) Differentiate $y = x^3 - 5x$. [1]

(c) Sketch the line $y = 2x - 6$, marking the intercepts with the axes. [2]

(d) Expand and simplify $(2x - 3)^2$. [1]

(e) Simplify:

(i) $\log_2 30 - \log_2 6$ [1]

(ii) $\log_3 81$ [1]

(f) Differentiate $y = x^{\frac{1}{2}} + x^{-1}$. [2]

(g) Given $f(x) = x^{\frac{3}{2}} - x$, evaluate $f(9)$. [2]

(h) Write down the range of $y = \sqrt{4 - x}$. [1]

Exam continues next page ...

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

(a) Solve the equation $\tan\theta = -0{\cdot}64$, for $0° \leq \theta \leq 360°$. Leave your answers correct to the nearest minute. [3]

(b) Solve $x^2 + 2x - 24 = 0$. [2]

(c) Find the gradient of the line passing through $A(2,-4)$ and $B(-6,2)$. [2]

(d) Solve $|x-5| = 3$. [2]

(e) Differentiate $y = (2x+3)^5$. [1]

(f) Rationalise the denominator of $\dfrac{6}{3-\sqrt{6}}$, giving your answer in simplest form. [3]

QUESTION TWELVE (13 marks) Use a separate writing booklet. **Marks**

(a) Find the gradient of the tangent to the curve $y = x^2 - 4x$ when $x = -4$. [2]

(b) Simplify $\sqrt{50} - 2\sqrt{8}$. [2]

(c) Solve:

(i) $8^x = 2^{2x+1}$ [2]

(ii) $\log_2 x = 3$ [1]

(iii) $2 - x \leq 5$ [1]

(d) For the arithmetic sequence $60, 56, 52\ldots$ find:

(i) the twenty-first term, [2]

(ii) the sum of the first twenty one terms. [2]

(e) The numbers $3, x$ and 12 are consecutive terms of a geometric sequence. What are the possible values for x? [1]

Exam continues overleaf ...

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Sketch the following functions on separate axes, showing any x-intercepts, y-intercepts and asymptotes:

(i) $y = \dfrac{4}{x-2}$ [2]

(ii) $y = -\sqrt{9-x^2}$ [2]

(b) For what values of k does the quadratic equation $2x^2 - 4x + (k+2) = 0$ have no real roots? [2]

(c) Differentiate $y = \dfrac{1}{(2x-5)^3}$. [2]

(d) Use the product rule to differentiate $y = x^3(2+x)^5$. Leave your answer in fully factored form. [3]

(e) Find the equation of the tangent to $y = 4x - x^3$ at the point when $x = -2$. [2]

QUESTION FOURTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Simplify $2^x \times 3^x$. [1]

(b) The first term of a geometric sequence is 5 and the eighth term is 640.

(i) Find the 12th term. [2]

(ii) Find the sum of the first 8 terms. [2]

(c) Solve the equation $2\cos^2\theta = 1$, for $0° \leq \theta \leq 360°$. [3]

(d) Find the points of intersection of the straight line $y = 2x+5$ and the hyperbola $y = \dfrac{3}{x}$. [3]

(e) Use the quotient rule to differentiate $y = \dfrac{x}{x^2+1}$. [2]

Exam continues next page ...

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Using the perpendicular distance formula from a point to a line, find the values of a for which the line $4x + 3y - a = 0$ is 4 units from the point $(2, -1)$. [3]

(b) In triangle ABC, $\angle BAC = 24°$, $AB = 2{\cdot}6$ and $BC = 1{\cdot}1$. Find the possible values of $\angle ACB$ to the nearest degree. [3]

(c) Prove that $(1 - \cos A)(1 + \sec A) \doteqdot \sin A \tan A$. [3]

(d) Two men are at an intersection of two straight roads which cross at an angle of 58°. They set off at the same time with one man walking at 6 km/h along one road and the other walking along the other road at 5 km/h. [4]

How long before they are 20 km apart, as the crow flies, to the nearest minute?

QUESTION SIXTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Find the value of $7 + 14 + 21 + \ldots + 1008$. [3]

(b) Using first principles differentiation, find the derivative of $f(x) = 4x - x^2$. [3]

(c) Simplify $\log_a b^2 \times \log_b a^3$. [1]

(d) (i) Find the equation of the normal to the hyperbola $y = \dfrac{1}{x}$ at the point A where $x = 2$. [2]

(ii) The normal at A meets the hyperbola again at B. Find the co-ordinates of M the mid-point of interval AB. [3]

(iii) How many times does the circle with diameter AB meet this hyperbola? You must justify your answer. [1]

End of Section II

END OF EXAMINATION

SOLUTION 2 UNIT MATHS YEARLY 2014

ONE

$3x - 2y = 6$

$2y = 3x - 6$

$y = \frac{3}{2}x - 3$

$m = \frac{3}{2}$ (C)

TWO

$\cos 210^\circ$

$= \cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$ (D)

THREE

$\sqrt{16x^{16}}$

$= 4x^8$ (B)

FOUR (C)

FIVE

$\frac{x}{\sin 60^\circ} = \frac{12}{\sin 50^\circ}$

$x = \frac{6\sqrt{3}}{\sin 50^\circ}$ (B)

SIX

$\alpha + \beta = 2$

$\alpha\beta = 3$ (B)

SEVEN

D: $x^2 - 1 \geq 0$

$x \leq -1$ or $x \geq 1$

(B)

EIGHT

$2, -4, 8, -16$

$|H| = 2 > 1$ (B)

Soo does not exist

NINE

$y = \frac{x}{1+x}$

$y' = \frac{(1+x)1 - x \cdot 1}{(1+x)^2}$

$= \frac{1}{(1+x)^2}$ (C)

9

TEN

(a) $\frac{x}{2} + 2x = 5$

$x + 4x = 10$ ✓

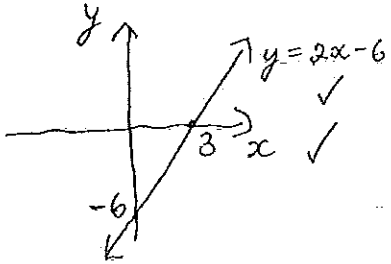
$5x = 10$ ✓

$x = 2$

(b) $y = x^3 - 5x$

$y' = 3x^2 - 5$ ✓

(c)



✓ ✓

(d) $(2x-3)^2$

$= 4x^2 - 12x + 9$ ✓

(e) (i) $\log_2 30 - \log_2 6$

$= \log_2 5$ ✓

(ii) $\log_3 81$ ✓

$= 4$

(f) $y = x^{\frac{1}{2}} + x^{-1}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} - x^{-2}$ ✓ ✓

or $y' = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

(g) $f(x) = x^{\frac{3}{2}} - x$

$f(9) = 9^{\frac{3}{2}} - 9$ ✓

$= 27 - 9$

$= 18$ ✓

(h) $y = \sqrt{4-x}$

Range is $y \geq 0$ ✓

13

ELEVEN

(a) $\tan\theta = -0.64$ ✓

Relate angle is $\tan^{-1}(0.64) = 32^\circ 37'$

$\tan\theta$ is negative

$\theta = 180^\circ - 32^\circ 37'$ ✓

or $360^\circ 37'$ ✓

$\theta = 147^\circ 23'$ or $327^\circ 23'$

(b) $x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$ ✓

$x = -6$ or $x = 4$ ✓

(c) $A(2,-4)$ $B(-6,2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m(AB) = \frac{2+4}{-6-2} = \frac{-3}{4}$ ✓ ✓

(d) $|x-5| = 3$

$x - 5 = 3$ or $x - 5 = -3$

$x = 8$ or $x = 2$ ✓ ✓

(e) $y = (2x+3)^5$

$\frac{dy}{dx} = 5(2x+3)^4 \times 2$

$= 10(2x+3)^4$ ✓

(f) $\frac{6}{3-\sqrt{6}} \times \frac{3+\sqrt{6}}{3+\sqrt{6}}$ ✓

$= \frac{6(3+\sqrt{6})}{9-6}$ ✓

$= 2(3+\sqrt{6})$

or $6 + 2\sqrt{6}$ ✓

13

TWELVE

(a) $y = x^2 - 4x$ ✓

$y' = 2x - 4$

When $x = -4$, $m = -8 - 4$

$= -12$ ✓

(b) $\sqrt{50} - 2\sqrt{8}$

$= 5\sqrt{2}$ ✓ $- 4\sqrt{2}$

$= \sqrt{2}$ ✓

(c) (i) $8^x = 2^{2x+1}$

$2^{3x} = 2^{2x+1}$ ✓

$3x = 2x + 1$

$x = 1$ ✓

(ii) $\log_2 x = 3$

$x = 8$ ✓

(iii) $2 - x \leq 5$

$-x \leq 3$

$x \geq -3$ ✓

(d) $60, 56, 52, \ldots$

$a = 60$, $d = -4$

(i) $T_n = a + (n-1)d$

$T_{21} = 60 + 20(-4)$ ✓

$= -20$ ✓

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$ ✓

$S_{21} = \frac{21}{2}[120 + 20x - 4]$ ✓

$= 420$

OR Easier

$S_n = \frac{n}{2}(a + l)$

$= \frac{21}{2}(60 + -20)$

$= 21 \times 20$

$= 420$

(e) 3, x and 12 in GP

$\frac{x}{3} = \frac{12}{x}$

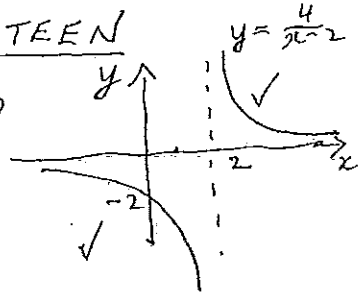
$x^2 = 36$

$x = \pm 6$ ✓

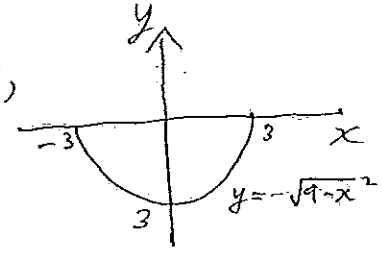
13

THIRTEEN

(a) (i)



(ii)



(b) $2x^2 - 4x + k + 2 = 0$

No real roots when

$\Delta < 0$ ✓

$16 - 4 \times 2(k+2) < 0$

$16 - 8k - 16 < 0$ ✓

$k > 0$

(c) $y = (2x-5)^{-3}$

$y' = -3(2x-5)^{-4} \times 2$ ✓ ✓

$y' = -\frac{6}{(2x-5)^4}$

(d) $y = x^3(2+x)^5$

$y = x^3 \times 5(2+x)^4 + (2+x)^5 \times 3x^2$ ✓

$= x^2(2+x)^4(5x + 6 + 3x)$

$= x^2(2+x)^4(8x+6)$ ✓

$= 2x^2(2+x^4)(4x+3)$ ✓

(e) $y = 4x - x^3$

$y' = 4 - 3x^2$

When $x = -2$

$m = 4 - 3 \times 4$

$= -8$

$y = -8 + 8 = 0$

Eqn of tangent is

$y - y_1 = m(x - x_1)$

$y - 0 = -8(x+2)$

$y = -8x - 16$

FOURTEEN

(a) $2^x \times 3^x = 6^x$ ✓

(b) $a = 5$

$T_n = ar^{n-1}$

$T_8 = ar^7$

$640 = 5r^7$

$128 = r^7$

$r = 2$ ✓

(i) $T_{12} = ar^{11}$

$= 5 \times 2^{11}$

$= 5 \times 2048$ ✓

$= 10240$

(ii) $S_n = \frac{a(r^n - 1)}{r-1}$

$S_8 = \frac{5(2^8 - 1)}{2-1}$ ✓

$= 5 \times 255$

$= 1275$ ✓

(c) $2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$ ✓

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 45°, 135°, 315°, 225°$ ✓ ✓

(d) $y = 2x + 5$

$y = \frac{3}{x}$

$\frac{3}{x} = 2x + 5$ ✓

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$ ✓

$x = \frac{1}{2}, x = -3$ ✓

Pts of intersection are

$(\frac{1}{2}, 6)$ and $(-3, -1)$

✓

(e) $y = \frac{x}{x^2+1}$

$y' = \frac{(x^2+1) \times 1 - x(2x)}{(x^2+1)^2}$ ✓

$= \frac{1 - x^2}{(x^2+1)^2}$ ✓

13

FIFTEEN

(a) Line is $4x+3y-a=0$

$(2,-1)$

$$d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}} \checkmark$$

$$4 = \frac{|4\times 2+3\times -1 - a|}{\sqrt{3^2+4^2}}$$

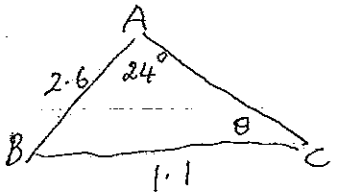
$$4 = \frac{|5-a|}{5} \checkmark$$

$$|5-a| = 20$$

$5-a = 20$ or $5-a=-20$

$a=-15$ or $a=25$ ✓

(b)



$$\frac{\sin\theta}{2.6} = \frac{\sin 24^\circ}{1.1} \checkmark$$

$$\sin\theta = \frac{2.6\sin 24^\circ}{1.1}$$

$\theta = 74^\circ$ or 106° ✓

(Both work since $24^\circ + 106^\circ < 180^\circ$)

(c) LHS $= (1-\cos A)(1+\sec A)$

$= 1 + \sec A - \cos A - 1$

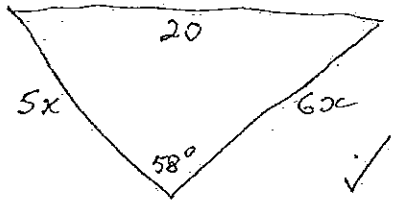
$= \frac{1}{\cos A} - \frac{\cos A}{1}$ ✓

$= \frac{1-\cos^2 A}{\cos A}$ ✓

$= \frac{\sin^2 A}{\cos A}$

$= \sin A \frac{\sin A}{\cos A}$

$= \sin A \tan A$ ✓



✓

Let x be the number of hours

Using the cosine rule: ✓

$20^2 = (5x)^2 + (6x)^2 - 60x^2\cos 58^\circ$ ✓

$400 = 25x^2 + 36x^2 - 60x^2\cos 58^\circ$

$400 = x^2(61 - 60\cos 58^\circ)$ ✓

$x = \sqrt{\frac{400}{61-60\cos 58^\circ}}$ ✓

$x = 3.7008\ldots$

$t \doteq 3$hrs 42mins

T3

SIXTEEN

(a) $7+14+21+\ldots 1008$

$T_n = a+(n-1)d$

$1008 = a+(n-1)7$

$1008 = 7+7n-7$ ✓

$7n = 1008$

$n = 144$

$S_n = \frac{n}{2}(a+l)$

$= \frac{144}{2}(7+1008)$ ✓

$= 73080$ ✓

(b) $f(x) = 4x-x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{4(x+h)-(x+h)^2-(4x-x^2)}{h} \checkmark$$

$$= \frac{4x+4h-x^2-2xh-h^2-4x+x^2}{h}$$

$$\frac{f(x+h)-f(x)}{h} = 4-2x-h \checkmark$$

Now $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}(4-2x-h)$ ✓

$= 4-2x$

(c) $\log_a b^2 \times \log_b a^3$

$= 2\log_a b \times \frac{\log_a a^3}{\log_b a}$

$= 2\times 3\log_a a$ ✓

$= 6$

(d) (i) $y = x^{-1}$

$\frac{dy}{dx} = -\frac{1}{x^2}$

When $x = 2$, $m = -\frac{1}{4}$ $\quad y = \frac{1}{2}$

Grad of normal is 4

Eqn of normal is $y - \frac{1}{2} = 4(x-2)$ — *

$y = 4x - \frac{15}{2}$

(ii) Solve $\frac{1}{x} = y$ and * simultaneously

$\frac{1}{x} = 4x - 8 + \frac{1}{2}$

$\times 2x$: $2 = 8x^2 - 16x + x$ ✓

$8x^2 - 15x - 2 = 0$

$(8x+1)(x-2) = 0$

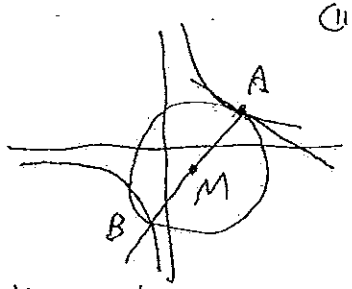
$x = -\frac{1}{8}$ or $x = 2$ ✓

$y = -8$ $\quad y = \frac{1}{2}$

Now $x_M = \frac{2 + -\frac{1}{8}}{2} = \frac{15}{16}$

$y_M = \frac{\frac{1}{2} - 8}{2} = -\frac{15}{4}$

So $M = \left(\frac{15}{16}, -\frac{15}{4}\right)$ ✓



Alternative using sum of the roots is a bit easier in terms of calculations

$x_M = \frac{x_A + x_B}{2}$

$= \frac{\frac{15}{8}}{2}$

$= \frac{15}{16}$

(iii) See diagram, the circle with AB as diameter touches the hyperbola at A. It cuts the hyperbola twice on the other branch.

So the circle with diameter A meets the hyperbola 3 times.