NAME _______________ MASTER

Sydney Grammar School

2016 Half-Yearly Examination

FORM V

MATHEMATICS EXTENSION 1

Monday 23rd May 2016

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 80 Marks

- All questions may be attempted.

Section I – 8 Marks

- Questions 1–8 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 72 Marks

- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

5A: DNW	5B: PKH	5C: LRP	5D: FMW
5E: WJM	5F: GMC	5G: NL	5H: SO

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 144 boys

Examiner
FMW

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

If $a = -3$, the value of $a^2 - a^3$ is equal to:

(A) -18

(B) -36

(C) 18

(D) 36

QUESTION TWO

When expressed with a rational denominator, $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}$ is equal to:

(A) $3 - 2\sqrt{2}$

(B) $2\sqrt{2} - 3$

(C) 1

(D) $\dfrac{3-2\sqrt{2}}{3}$

QUESTION THREE

Given $f(x) = \dfrac{2}{3x^2}$, then $f'(x)$ is equal to:

(A) $-\dfrac{4}{3x^3}$

(B) $\dfrac{2}{3x^3}$

(C) $\dfrac{2}{3x}$

(D) $-\dfrac{4}{3x}$

$(3x^2)^{-1}$

$-(3x^2)^{-2} \times 6x$

$\dfrac{6x}{9x^4}$ $\dfrac{2}{3x^3}$

Examination continues next page ...

QUESTION FOUR

The point P divides the interval from $A(-3,-1)$ to $B(4,9)$ internally in the ratio $3:2$. What is the x coordinate of P?

(A) $\frac{1}{2}$

(B) $-\frac{1}{5}$

(C) 5

(D) $1\frac{1}{5}$

$x = \frac{l x_1 + k x_2}{l + k}$

$x = \frac{-6 + 12}{5}$

QUESTION FIVE

In factored form, $x^3 - 8$ is equal to:

(A) $(x+2)(x^2-2x+4)$

(B) $(x-2)^3$

(C) $(x-2)(x^2+2x+4)$

(D) $(x-2)(x^2+4x+4)$

QUESTION SIX

Consider the sequence $\log_b 3$, $\log_b 9$, $\log_b 27$, ...

Which of the following statements is TRUE?

(A) The sequence is arithmetic with common difference $d = \log_b 3$.

(B) The sequence is geometric with common ratio $r = \log_b 3$.

(C) The sequence is neither geometric nor arithmetic.

(D) The sequence has a limiting sum.

QUESTION SEVEN

The solution of $\sin^2\theta = \sin\theta$, for $0° \le \theta \le 360°$, is:

(A) $\theta = 90°$

(B) $\theta = 90°$ or $270°$

(C) $\theta = 0°$, $90°$, $180°$ or $360°$

(D) $\theta = 0°$, $90°$, $180°$, $270°$ or $360°$

Examination continues overleaf ...

QUESTION EIGHT

The expression $\dfrac{2^x - 2^{x+1}}{2}$ in simplest form is equal to:

(A) $-\frac{1}{2}$

(B) -2^{x-1}

(C) $x - 2^{x+1}$

(D) 2^{x-1}

$\frac{2^x}{2^1} - \frac{2^{x+1}}{2^1}$

$= 2^{x-1} - 2^x$

$= 2^x(2^{-1} - 1)$

$= 2^x \cancel{(-\frac{1}{2})} \; (-\frac{1}{2})$

$= 2^x \times -2^{-1}$

$= -2^{x-1}$

End of Section I

Examination continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION NINE (12 marks) Use a separate writing booklet. Marks

(a) (i) Solve $x^2 - 2x - 3 = 0$. 1

(ii) Solve $x^2 - 2x - 3 < 0$. 1

(b) Write down the natural domain of the function $f(x) = \sqrt{10 - x}$. 1

(c) Solve $\tan\theta = -\sqrt{3}$, for $0° \leq \theta \leq 360°$. 2

(d) Find the gradient of the tangent to the curve $y = 4 - 5x^3$ at the point $T(-2, 44)$. 1

(e) Find the equation of the perpendicular bisector of the interval joining the points $A(1, -3)$ and $B(-3, 5)$. 3

(f) Find the sum of the first 22 terms of the geometric sequence $3, 6, 12, \ldots$. 2

(g) Sketch $y = |x| - 1$, showing any x and y intercepts. 1

QUESTION TEN (12 marks) Use a separate writing booklet. Marks

(a) Differentiate:

(i) $y = 10x^4 - 3x^2 + 2x + 1$ 1

(ii) $y = (3x - 4)^5$ 2

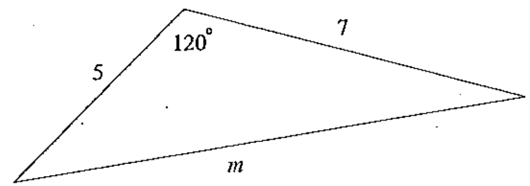
(iii) $y = \dfrac{2x - 1}{2x + 1}$ 2

(iv) $y = 5\sqrt{x}$ 1

(b) Given the points $A(-2, 7)$ and $B(4, -10)$, find the coordinates of the point $P(x, y)$ that divides AB externally in the ratio $2 : 5$. 2

(c) Evaluate $\displaystyle\sum_{k=3}^{6}(10 - 4k)$. 2

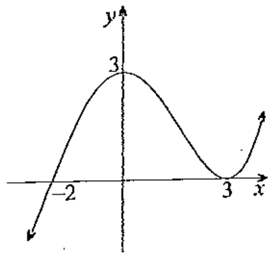
(d) 2



Find the exact value of m in the diagram above.

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a)



The graph of $y = f(x)$ is sketched above. On separate number planes, sketch:

(i) $y = f(x+1)$ $\boxed{1}$

(ii) $y = f(2x)$ $\boxed{1}$

(b) The fourth term of a geometric series is 48 and the eighth term is 768. Find all possible values for the first term and the common ratio. $\boxed{3}$

(c) The side length x of a cube is increasing at the constant rate of $\frac{1}{2}$ cm/s. Find the rate of increase of the volume of the cube when the sides have length 5 cm. $\boxed{3}$

(d) Solve the inequation $\dfrac{2}{x+1} \leq 3$. $\boxed{4}$

QUESTION TWELVE (12 marks) Use a separate writing booklet. **Marks**

(a) Solve $\cos(x - 50°) = \dfrac{1}{\sqrt{2}}$, for $0° \leq x \leq 360°$. $\boxed{3}$

(b) Given $\sec\theta = 3$, find the possible values for $\tan\theta$. $\boxed{2}$

(c) Differentiate $f(x) = 2x - x^2$, from first principles. $\boxed{3}$

(d) The line $2x + 3y + k = 0$ is a tangent to the circle $(x+1)^2 + (y-4)^2 = 13$.

(i) Use the perpendicular distance formula to show that $|k + 10| = 13$. $\boxed{2}$

(ii) Hence find the equations of the two tangents. $\boxed{2}$

QUESTION THIRTEEN (12 marks) Use a separate writing booklet. Marks

(a) Prove that $\dfrac{\cot\theta}{\operatorname{cosec}\theta - 1} - \dfrac{\cos\theta}{1+\sin\theta} = 2\tan\theta$. 3

(b) Consider the graph of $y = \dfrac{x(x+2)}{(x-1)^2}$.

(i) Find any intercepts with the x and y axes. 1

(ii) Write down the equation of the vertical asymptote. 1

(iii) Find the equation of the horizontal asymptote. 1

(iv) Copy and complete the following table: 1

x	-1	$\frac{1}{2}$	2
y			

(v) Use Parts (i)-(iv) to sketch the curve. 2

(c) (i) Sketch the graphs of $y = x$ and $y = |2x-1|$ on the same number plane. 2

(ii) Hence, or otherwise, determine the values of c for which the equation $|2x-1| = x + c$ has exactly two solutions. 1

Examination continues overleaf ...

QUESTION FOURTEEN (12 marks) Use a separate writing booklet. Marks

(a) A sequence is defined recursively by:

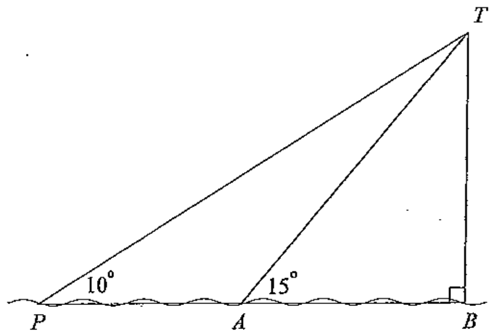
$$T_1 = 3$$
$$T_n = T_{n-1} + 3^{-n}, \text{ for } n = 2, 3, 4, \ldots$$

(i) Show that $T_3 = 3 + \frac{1}{9} + \frac{1}{27}$. 1

(ii) Find a simplified expression for T_n in terms of n. 2

(iii) Find the limit of T_n as $n \to \infty$. 1

(b) A boat is moving at a constant speed of k km/min, along a straight river towards a bridge at B. 3



When the boat is at P, the angle of elevation to the top of the bridge T is $10°$. Ten minutes later the boat has moved to A and the angle of elevation to T is $15°$.
How long will it take for the boat to travel from A to B? Give your answer correct to the nearest second.

(c) Determine the range of the function $y = \sec^2 x - 2\tan x + 1$. 2

(d) Let P be the point on the curve $y = \dfrac{1}{a^2+x^2}$ where $x = \dfrac{a}{\sqrt{3}}$ and $a > 0$. The tangent at P meets the y-axis at T and the x-axis at R. Find the ratio of the area of $\triangle OPT$ to the area of $\triangle OPR$. 3

End of Section II

END OF EXAMINATION

NAME _______________ MASTER _______

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Solutions

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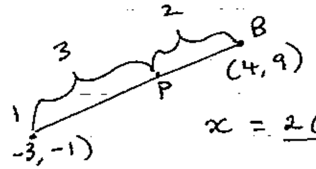
Examiner
FMW

Form V Ext 1 Solutions

(1) $a^2 - a^3 = (-3)^2 - (-3)^3$
$= 9 + 27$
$= 36$ $\boxed{D}$ ✓

(2) $\frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-2\sqrt{2}+2}{1-2}$
$= \frac{3-2\sqrt{2}}{-1}$ $\boxed{B}$ ✓
$= 2\sqrt{2}-3$

(3) $f(x) = \frac{2}{3x^2}$
$= \frac{2}{3}x^{-2}$ $\boxed{A}$ ✓
$f'(x) = -2 \times \frac{2}{3}x^{-3}$
$= -\frac{4}{3x^3}$

(4)


$x = \frac{2(-3)+3(4)}{3+2}$ $\boxed{D}$ ✓
$= \frac{-6+12}{5}$
$= \frac{6}{5}$

(5) $x^3 - 8 = x^3 - 2^3$ $\boxed{C}$ ✓
$= (x-2)(x^2+2x+4)$

⑥ $\log_b 3$

$\log_b 9 = \log_b 3^2$
$= 2\log_b 3$

$\log_b 27 = \log_b 3^3$
$= 3\log_b 3$

$$\underbrace{\log_b 3, 2\log_b 3, 3\log_b 3, \ldots}$$

this is an A.P with $d = \log_b 3$

$\boxed{A}$ ✓

⑦ $\sin^2\theta = \sin\theta$

$\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

$\sin\theta = 0$, $\sin\theta = 1$

$\theta = 0°, 180°, 360°$ $\theta = 90°$

$\boxed{C}$ ✓

⑧ $\dfrac{2^x - 2^{x+1}}{2} = \dfrac{2^x(1-2)}{2}$

$= -\frac{1}{2} \times 2^x$

$= -2^{x-1}$

$\boxed{B}$ ✓

⑨ /12

(a) (i) $x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$ ✓

(ii) $x^2 - 2x - 3 < 0$

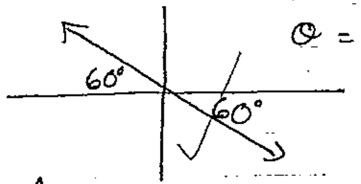
$(x-3)(x+1) < 0$

$-1 < x < 3$ ✓

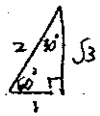
(b) $10 - x \geq 0$

$x \leq 10$ ✓

(c) $\tan\theta = -\sqrt{3}$



$\theta = 120°$ or $300°$ ✓



(d) $y = 4 - 5x^3$

$y' = -15x^2$

at $x = -2$

$y' = -15(-2)^2$

$= -60$ ✓

(e) $M = \left(\frac{1-3}{2}, \frac{-3+5}{2}\right)$ ✓

$= (-1, 1)$

$m = \dfrac{5+3}{-3-1}$

$= \dfrac{8}{-4}$

$= -2$

gradient of bisector is $+\frac{1}{2}$ ✓

equation:

$y - 1 = \frac{1}{2}(x+1)$ ✓

$2y - 2 = x + 1$

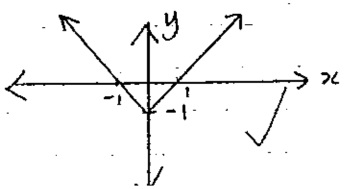
$x - 2y + 3 = 0$ (or $y = \frac{x}{2} + \frac{3}{2}$)

(f) $a = 3$, $r = 2$

$S_{22} = \dfrac{3(2^{22} - 1)}{2 - 1}$ ✓

$= 12\,582\,909$ ✓

(g)



✓

(10) /12

(a) (i) $y = 10x^4 - 3x^2 + 2x + 1$

$y' = 40x^3 - 6x + 2$

(ii) $y = (3x-4)^5$

$y' = 5(3x-4)^4 \times 3$

$= 15(3x-4)^4$

(iii) $y = \dfrac{2x-1}{2x+1}$

$y' = \dfrac{(2x+1)\times 2 - (2x-1)\times 2}{(2x+1)^2}$

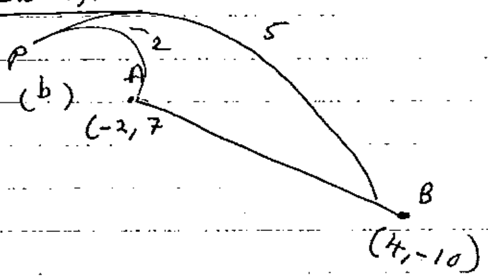
$= \dfrac{4}{(2x+1)^2}$

(iv) $y = 5x^{\frac{1}{2}}$

$y' = \frac{5}{2}x^{-\frac{1}{2}}$ } either

$= \dfrac{5}{2\sqrt{x}}$

(b)



$x = \dfrac{5(-2) - 2(4)}{-2+5}$

$= -6$

$y = \dfrac{5(7) - 2(-10)}{-2+5}$

$= \dfrac{55}{3}$

P is $(-6, 18\frac{1}{3})$

(c) $\sum_{k=3}^{6} (10-4k)$

$= (10-12) + (10-16) + (10-20) + (10-24)$

$= -32$

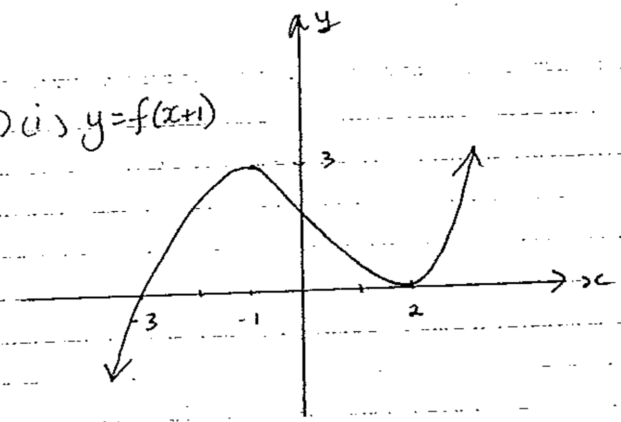
(d) $m^2 = 5^2 + 7^2 - 2\times 5\times 7 \times \cos 120°$

$= 25 + 49 - 70\times(-\frac{1}{2})$

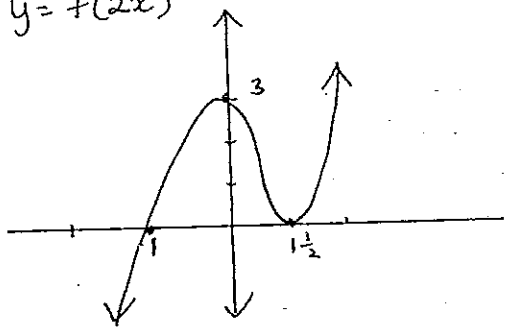
$= 109$

$m = \sqrt{109}$, $m > 0$

(11) (a) (i) $y = f(x+1)$



(ii) $y = f(2x)$



(b) $ar^3 = 48$ $ar^7 = 768$

$a = \dfrac{48}{r^3}$ $a = \dfrac{768}{r^7}$

$\dfrac{48}{r^3} = \dfrac{768}{r^7}$

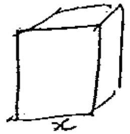
$r^4 = 16$

$r = 2$ or -2

$a = 6$ or -6

(award ✓✓x if ignore negative solution)

(c)



$V = x^3$

$\dfrac{dV}{dx} = 3x^2$

$\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt}$

$= 3x^2 \times \frac{1}{2}$

$= 3(5)^2 \times \frac{1}{2}$

$= 37\frac{1}{2}$ cm^3/s

(d) $\frac{2}{x+1} \leq 3$

$(x+1)^2 \times \frac{2}{x+1} \leq 3(x+1)^2, \quad x \neq -1$ ✓

$2(x+1) \leq 3(x+1)^2$

$3(x+1)^2 - 2(x+1) \geq 0$

$(x+1)(3(x+1) - 2) \geq 0$

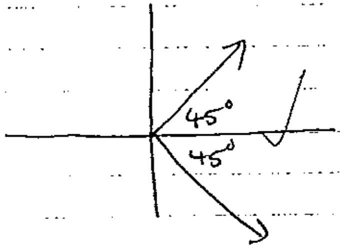
$(x+1)(3x+1) \geq 0$ ✓

$x < -1$ or $x \geq -\frac{1}{3}$ ✓✓

(12)

(a) $\cos(x - 50°) = \frac{1}{\sqrt{2}}, \quad 0° \leq x \leq 360°$

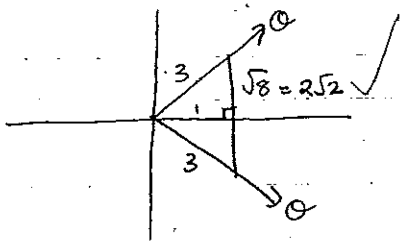
$-50° \leq x - 50 \leq 310°$



$x - 50° = -45°$ or $45°$

$x = 5°$ ✓ or $95°$ ✓

(b) $\sec\theta = 3$



$\tan\theta = 2\sqrt{2}$ or $-2\sqrt{2}$ ✓

(accept $\sqrt{8}, -\sqrt{8}$)

(c) $f'(x) = \lim_{h \to 0} \frac{2(x+h) - (x+h)^2 - (2x - x^2)}{h}$ ✓

$= \lim_{h \to 0} \frac{2x + 2h - (x^2 + 2xh + h^2) - 2x + x^2}{h}$ ✓

$= \lim_{h \to 0} \frac{2h - 2xh - h^2}{h}$

$= \lim_{h \to 0} 2 - 2x - h$ ✓

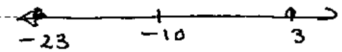
$= 2 - 2x$

(d) (i) $\frac{|2(-1) + 3(4) + k|}{\sqrt{2^2 + 3^2}} = \sqrt{13}$ ✓✓

$\frac{|-2 + 12 + k|}{\sqrt{13}} = \sqrt{13}$

$|k + 10| = 13$

(ii) $|k+10| = 13$



$k = -23$ or $k = 3$

tangents are $2x + 3y - 23 = 0$
and $2x + 3y + 3 = 0$

(13) (a) $$LHS = \frac{\cot\theta}{\operatorname{cosec}\theta - 1} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta} - 1} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1-\sin\theta}{\sin\theta}} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta) - \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2\cos\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2\cos\theta\sin\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta}$$

$$= 2\tan\theta$$

$$= RHS$$

(b) $$y = \frac{x(x+2)}{(x-1)^2}$$

(i) if $x = 0$, $y = 0$ (both)
if $y = 0$, $x(x+2) = 0$
$x = 0$ or $x = -2$

(ii) $x = 1$

(iii) $$y = \frac{x^2 + 2x}{x^2 - 2x + 1}$$

$$= \frac{\frac{x^2}{x^2} + \frac{2x}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}}$$

$$= \frac{1 + \frac{2}{x}}{1 - \frac{2}{x} + \frac{1}{x^2}}$$

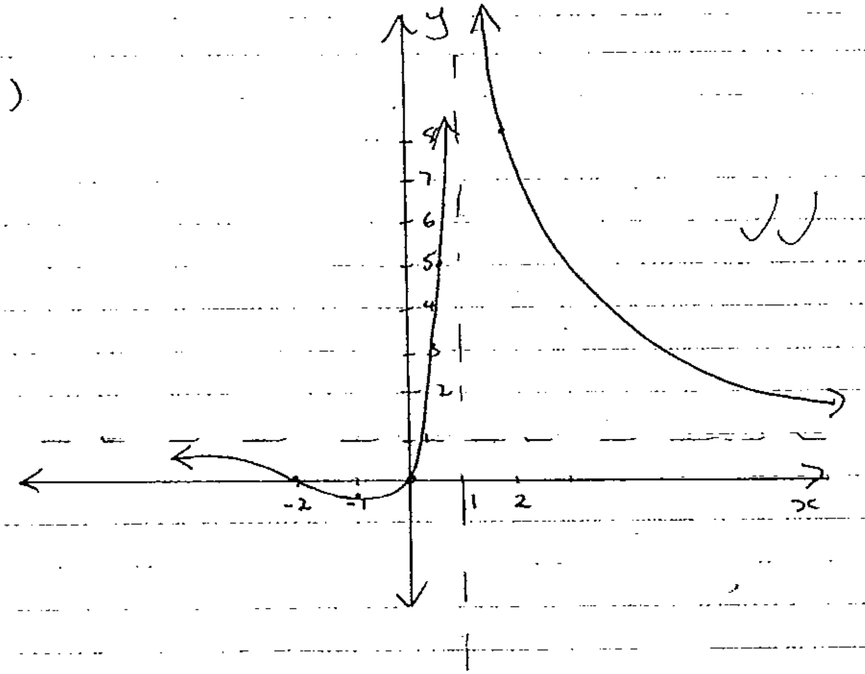
as $x \to \infty$, $y \to 1$

the asymptote is $y = 1$

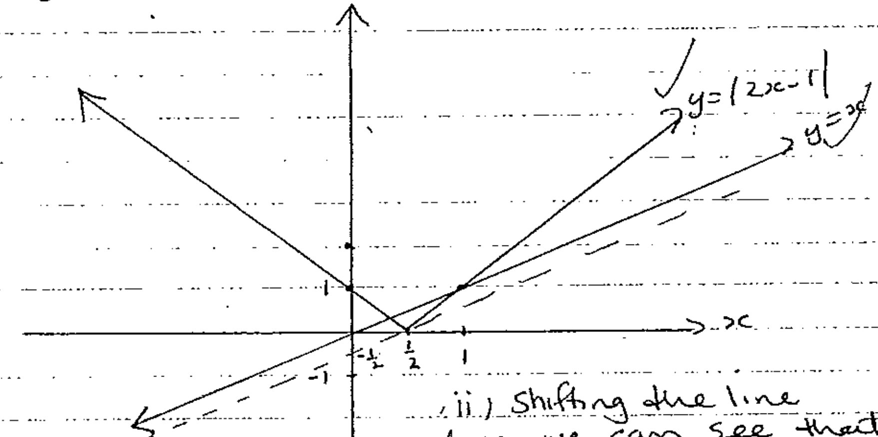
(iv)

x	-1	$\frac{1}{2}$	2
y	$-\frac{1}{4}$	5	8

(v)



(c) (i) $y = x$ and $y = |2x-1|$



(ii) Shifting the line down we can see that $c > -\frac{1}{2}$ for 2 solutions

(14) (a) $T_1 = 3$

$\frac{}{12}$ $T_n = T_{n-1} + 3^{-n}$, for $n = 2, 3, 4, \ldots$

(i) $T_2 = T_1 + 3^{-2}$
$= 3 + \frac{1}{3^2}$
$= 3 + \frac{1}{9}$

$T_3 = T_2 + 3^{-3}$
$= 3 + \frac{1}{9} + \frac{1}{3^3}$
$= 3 + \frac{1}{9} + \frac{1}{27}$

(ii) $T_n = 3 + \underbrace{\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots}$

this is a G.P with $a = \frac{1}{9}$, $r = \frac{1}{3}$ with $(n-1)$ terms

$S_n = \frac{\frac{1}{9}\left(\left(\frac{1}{3}\right)^{n-1} - 1\right)}{\frac{1}{3} - 1}$

$= -\frac{1}{6}\left(\left(\frac{1}{3}\right)^{n-1} - 1\right)$

$T_n = 3 - \frac{1}{6}\left(\left(\frac{1}{3}\right)^{n-1} - 1\right)$

$= 3 + \frac{1}{6}\left(1 - \left(\frac{1}{3}\right)^{n-1}\right)$

alternatively:

$T_n = \left(3^1 + 3^0 + 3^{-1} + \ldots + 3^{-n}\right) - \left(3^0 + 3^{-1}\right)$

$= \frac{3\left(\left(\frac{1}{3}\right)^{n+2} - 1\right)}{\frac{1}{3} - 1} - \left(1 + \frac{1}{3}\right)$

$= -\frac{9}{2}\left(\left(\frac{1}{3}\right)^{n+2} - 1\right) - \frac{4}{3}$

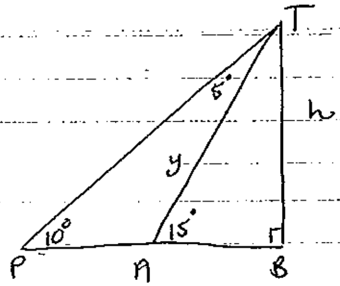
(iii) as $n \to \infty$

$T_n = 3 - \frac{1}{6}(-1)$
$= 3\frac{1}{6}$

$\left(\text{or } 3 + \frac{\frac{1}{9}}{1 - \frac{1}{3}} = 3\frac{1}{6}\right)$

$\left(\text{or } -\frac{9}{2} \times -1 - \frac{4}{3} = 3\frac{1}{6}\right)$

(b)



$\tan 15^\circ = \dfrac{h}{AB}$ $\qquad AB = \dfrac{h}{\tan 15^\circ}$

$\tan 10^\circ = \dfrac{h}{PB}$ $\qquad PB = \dfrac{h}{\tan 10^\circ}$

$PA = \dfrac{h}{\tan 10^\circ} - \dfrac{h}{\tan 15^\circ}$

$= h\left(\dfrac{1}{\tan 10^\circ} - \dfrac{1}{\tan 15^\circ}\right)$

let t be the time taken,

$$\frac{t}{10} = \frac{\frac{h}{\tan 15^\circ}}{h\left(\frac{1}{\tan 10^\circ} - \frac{1}{\tan 15^\circ}\right)}$$

$$t = \frac{\frac{10}{\tan 15^\circ}}{\frac{1}{\tan 10^\circ} - \frac{1}{\tan 15^\circ}}$$

$= 19.245\ldots$

$\doteqdot 19$ min 15 s

alternatively: $\dfrac{PA}{\sin 5^\circ} = \dfrac{y}{\sin 10^\circ}$

$PA = \dfrac{y \sin 5^\circ}{\sin 10^\circ}$

also $\cos 15^\circ = \dfrac{AB}{y}$

$AB = y \cos 15^\circ$

so $\dfrac{t}{10} = \dfrac{\cancel{y}\cos 15^\circ}{\cancel{y}\frac{\sin 5^\circ}{\sin 10^\circ}}$

$t = \dfrac{10 \cos 15^\circ \sin 10^\circ}{\sin 5^\circ}$

$= 19.245\ldots$

$\doteqdot 19$ min 15 s

(c) $y = \sec^2 x - 2\tan x + 1$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$= \tan^2 x + 1 - 2\tan x + 1$

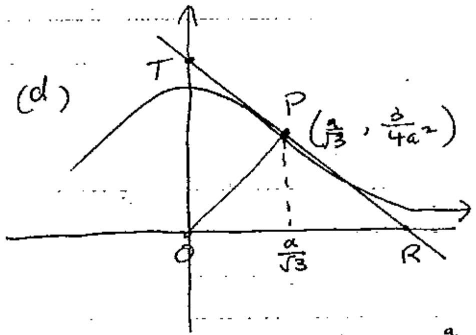
$= \tan^2 x - 2\tan x + 1 + 1$

$= (\tan x - 1)^2 + 1$

as $(\tan x - 1)^2 \geq 0$ for all x

range is $y \geq 1$

(d)



area ratio $= \dfrac{\frac{1}{2} OT \times \frac{a}{\sqrt{3}}}{\frac{1}{2} OR \times \frac{3}{4a^2}}$

$= \dfrac{OT}{OR} \times \dfrac{4a^3}{3\sqrt{3}}$

$y = (a^2 + x^2)^{-1}$

$y' = -1(a^2 + x^2)^{-2} \times 2x$

$= -\dfrac{2x}{(a^2 + x^2)^2}$

at P, $y' = -\dfrac{2\left(\frac{a}{\sqrt{3}}\right)}{\left(a^2 + \left(\frac{a}{\sqrt{3}}\right)^2\right)^2}$

$= \dfrac{-2a}{\sqrt{3}\left(a^2 + \frac{a^2}{3}\right)^2}$

$= \dfrac{-2a}{\sqrt{3} \times \left(\frac{4a^2}{3}\right)^2}$

$= \dfrac{-2a}{\sqrt{3}} \times \dfrac{9}{16a^4}$

$= -3\sqrt{3}$

now $\dfrac{dy}{dx}$ = gradient of tangent

$= -\dfrac{OT}{OR}$

so area ratio

$= \dfrac{3\cancel{\sqrt{3}}}{8a^3} \times \dfrac{4a^3}{3\cancel{\sqrt{3}}}$

$= \dfrac{1}{2}$