NAME _____ MASTER _____

SYDNEY GRAMMAR SCHOOL

2016 Half-Yearly Examination

FORM V

MATHEMATICS 2 UNIT

Tuesday 17th May 2016

General Instructions
- Writing time — 1 hour 30 minutes
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 80 Marks
- All questions may be attempted.

Section I – 8 Marks
- Questions 1–8 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 72 Marks
- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection
- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

5A: DNW	5B: PKH	5C: LRP	5D: FMW
5E: WJM	5F: GMC	5G: NL	5H: SO
5P: TCW	5Q: SDP	5R: RCF	

Checklist
- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 178 boys

Examiner
NL

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following bearings is due west?

(A) 000° T

(B) 090° T

(C) 180° T

(D) 270° T

QUESTION TWO

What is the correct expansion of $(x-1)(x+4)$?

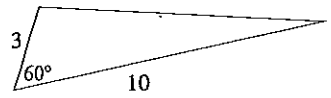
(A) x^2-3x+4

(B) x^2+4x-4

(C) x^2+3x-4

(D) x^2+4x-3

QUESTION THREE



Which expression is the exact area of the triangle above?

(A) $5\sqrt{2}$

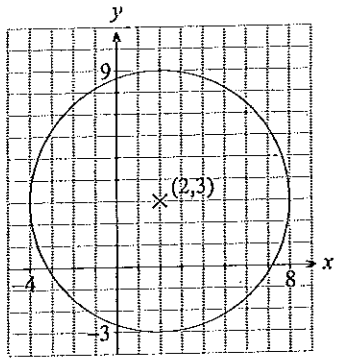
(B) $\frac{20\sqrt{2}}{3}$

(C) $\frac{15\sqrt{3}}{2}$

(D) $10\sqrt{2}$

Examination continues next page ...

QUESTION FOUR



The equation of the circle drawn above is:

(A) $(x+2)^2+(y+3)^2=6$

(B) $(x-3)^2+(y-2)^2=6$

(C) $(x+3)^2+(y+2)^2=36$

(D) $(x-2)^2+(y-3)^2=36$

QUESTION FIVE

Which expression is equivalent to x^2+4x+9?

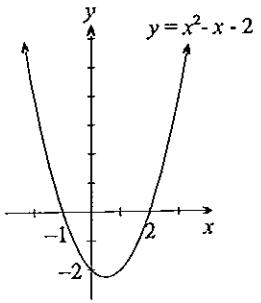
(A) $(x+2)^2+13$

(B) $(x-2)^2+5$

(C) $(x+2)^2+9$

(D) $(x+2)^2+5$

QUESTION SIX



Using the graph above, the solution to the inequation $x^2-x-2>0$ is:

(A) $-1\leq x\leq 2$

(B) $x<-1$ or $x>2$

(C) $-1>x>2$

(D) $x<-1$ or $x<2$

QUESTION SEVEN

Which of these functions is even?

(A) $f(x)=x^2-8x+15$

(B) $f(x)=(x+2)^2-4$

(C) $f(x)=\sqrt{9-x^2}$

(D) $f(x)=x^3$

QUESTION EIGHT

Which of the following identities is <u>not</u> true?

(A) $\sec^2\theta+1=\cot^2\theta$

(B) $\sin^2\theta+\cos^2\theta=1$

(C) $1+\cot^2\theta=\operatorname{cosec}^2\theta$

(D) $\tan^2\theta+1=\sec^2\theta$

End of Section I

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) If $f(x) = 4x - 3$, find $f(2)$. [1]

(b) Evaluate $|-5| - |-7|$. [1]

(c) Write $\frac{2}{b} + \frac{1}{3}$ as a single fraction. [1]

(d) Express $\frac{3}{2-\sqrt{2}}$ as a simplified fraction with a rational denominator. [2]

(e) What is the exact value of $\sqrt{27} + \sqrt{12}$? [1]

(f) (i) Factorise $x^2 - x - 6$. [1]

(ii) Hence solve $x^2 - x - 6 = 0$. [1]

(g) Consider the points $A(6, 9)$ and $B(2, 1)$.

(i) Find the coordinates of the midpoint of the interval AB. [1]

(ii) Find the gradient of AB. [1]

(iii) Find the equation of the line which passes through points A and B. [2]

Examination continues overleaf ...

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) Show that the lines $y = 3x - 4$ and $x + 3y = 12$ are perpendicular. [2]

(b) Solve $2x - 4y = -12$ and $3x + 2y - 2 = 0$ simultaneously. [2]

(c) Consider the parabola with equation $y = x^2 - 6x + 7$.

(i) Show that the parabola has x-intercepts $x = 3 + \sqrt{2}$ and $x = 3 - \sqrt{2}$. [1]

(ii) Write down the y-intercept. [1]

(iii) Write down the equation of the axis of symmetry. [1]

(iv) Find the coordinates of the vertex. [1]

(v) Sketch the graph of $y = x^2 - 6x + 7$, clearly marking all of the above features. [2]

(d) Given that $\cos\theta = \frac{3}{7}$ and $\tan\theta > 0$, find $\sin\theta$. Leave your answer in exact form. [2]

Examination continues next page ...

QUESTION ELEVEN (12 marks) Use a separate writing booklet. Marks

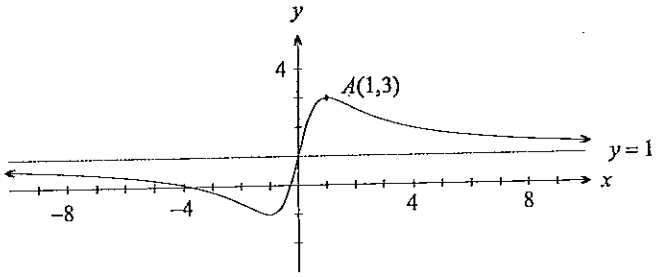
(a) (i) Solve $|x-1|=5$. [2]

(ii) Solve $|3x+2|>1$. [2]

(b) Solve $\cos x = 0{\cdot}6$, for $0° \leq x \leq 360°$. Give your answer to the nearest degree. [2]

(c) Find the perpendicular distance from the point $P(2,3)$ to the line $x+2y+3=0$. [2]

(d)



The graph of $y=g(x)$ is shown above. It has horizontal asymptote $y=1$. State the coordinates of the point A and the equation of the asymptote after each of the following transformations.

(i) $y=g(x)+1$ [2]

(ii) $y=-g(x)$ [2]

QUESTION TWELVE (12 marks) Use a separate writing booklet. Marks

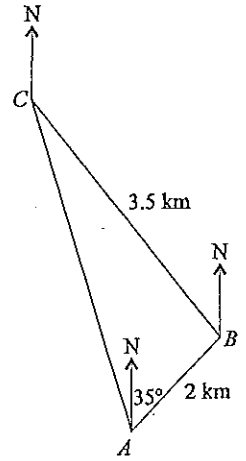
(a) Sketch each function below on separate axes, including labelled asymptotes and intercepts with axes if they exist.

(i) $y=\sqrt{x+4}$ [1]

(ii) $y=|x+5|$ [2]

(iii) $y=\dfrac{1}{x-1}+3$ [3]

(b)



The diagram above shows the path of a hiker. The hiker starts from point A and walks on a bearing of 035° T for 2 km to point B. She leaves point B on a bearing of 320° T and walks for 3·5 km until she reaches point C.

(i) Show that $\angle ABC$ is 105°. Justify your answer with geometrical reasoning. [2]

(ii) Find the distance AC. Give your answer correct to the nearest metre. [2]

(iii) Find the bearing of C from A. Give your answer correct to the nearest degree. [2]

QUESTION THIRTEEN (12 marks) Use a separate writing booklet. Marks

(a) Consider the function $f(x) = 2^x + 1$.

(i) Sketch the graph of $y = f(x)$. [2]

(ii) State the range of the function. [1]

(b) Find the angle of inclination of the line $y = -4x + 2$. Give your answer correct to the nearest degree. [2]

(c) Solve $\cos 2\alpha = \dfrac{\sqrt{3}}{2}$, for $0° \leq \alpha \leq 360°$. [3]

(d) Consider the points $A(3,9)$ and $B(1,1)$ and the circle with diameter AB.

(i) Show that the equation of the circle is $(x-2)^2 + (y-5)^2 = 17$. [2]

(ii) The line $y = x + 6$ cuts the circle at point A and again at a second point C. Find the coordinates of the point C. [2]

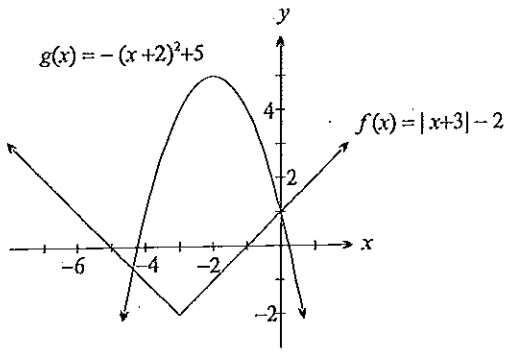
Examination continues overleaf ...

QUESTION FOURTEEN (12 marks) Use a separate writing booklet. Marks

(a) (i) Express $A^3 + B^3$ as a product of two factors. [1]

(ii) Hence prove the identity $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$. [2]

(b)



Consider the functions $f(x) = |x+3| - 2$ and $g(x) = -(x+2)^2 + 5$ in the diagram above.

(i) Show that $y = f(x)$ and $y = g(x)$ intersect at $(0,1)$. [1]

(ii) Find the exact values of the coordinates of the second point of intersection of $y = f(x)$ and $y = g(x)$. [3]

(iii) Hence solve $|x+3| - 2 \leq -(x+2)^2 + 5$. [1]

(c) Two points A and B lie on a line l. The midpoint of AB is $M(6,7)$. The line $2x + 5y - 15 = 0$ passes through A and the line $3x - 2y + 5 = 0$ passes through B.

Let A have coordinates $A(h,k)$.

(i) Show that the coordinates of B are $B(12-h, 14-k)$. [1]

(ii) Find the equation of l. [3]

End of Section II

END OF EXAMINATION

2016 Form V Mathematics Solutions

1) D
2) C
3) C
4) D
5) D
6) B
7) C
8) A

Q9)

a) $f(2) = 4 \times 2 - 3$
$= 5$

b) $|-5| - |-7|$
$= -2$

c) $\frac{2}{6} + \frac{1}{3}$
$= \frac{6}{36} + \frac{6}{36}$
$= \frac{6+6}{36}$

d) $\frac{3}{2-\sqrt{2}}$
$= \frac{3(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$ ✓
$= \frac{6+3\sqrt{2}}{2}$ ✓ (or factorised)

e) $3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$

f) i) $x^2 - x - 6 = (x+2)(x-3)$
ii) $x = -2$ or $x = 3$

g) i) $M = \left(\frac{6+2}{2}, \frac{9+1}{2}\right)$
$= \left(\frac{8}{2}, \frac{10}{2}\right)$
$= (4, 5)$ ✓

ii) $m = \frac{9-1}{6-2}$
$m = 2$ ✓

iii) $y - 1 = 2(x-2)$ ✓
$y = 2x - 4 + 1$
$y = 2x - 3$ ✓

Q10.

a) $y = 3x - 4$ $\quad m_1 = 3$ $\quad x + 3y = 12$

$3y = -x + 12$

$y = -\frac{1}{3}x + 4$

either ✓ $\rightarrow m_2 = -\frac{1}{3}$

$\therefore \quad 3 \times -\frac{1}{3} = -1$ ✓

or $m_1 \times m_2 = -1$ (if explicitly stated m_1 and m_2 above)

b) ① $2x - 4y = -12$ $\quad$ ② $3x + 2y = 2$

② $6x + 4y = 4$

②+① $\quad 6x + 4y = 4$

$+ (2x - 4y = -12)$

$8x = -8$

$x = -1$ ✓

(✓ for correctly forming an equation with one variable; $3(2y-6)+2y-2=0$)

Sub: $\quad -2 - 4y = -12$

$y = 5/2$ ✓

c) $y = x^2 - 6x + 7$

i) $a = 1 \quad b = -6 \quad c = 7$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 7}}{2 \times 1}$ ✓ (or $(x-3)^2 = 2$ ✓, $x = 3 \pm \sqrt{2}$)

$x = \frac{6 \pm \sqrt{8}}{2}$

$x = 3 \pm \sqrt{2}$

ii) 7 or $(0, 7)$

iii) $x = \frac{-b}{2a}$

$= \frac{6}{2}$

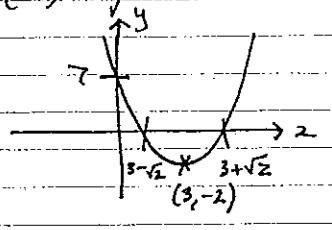
$= 3$ ✓

iv) $y = 3^2 - 6 \times 3 + 7$

$= -2$

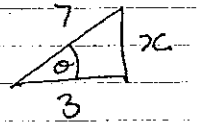
$\therefore (3, -2)$ ✓

v)



−1 for missing −intercepts
− poor shape
− axis labels
− vertex label

d)



$x = 2\sqrt{10}$ ✓ (or 1st quadrant ✓)

$\sin\theta = \frac{2\sqrt{10}}{7}$ ✓

(Accept $\frac{\sqrt{40}}{7}$)

Q11.

a) i) $x-1=5$ or $x-1=-5$

$x=6$ ✓ $x=-4$ ✓

(or other valid method (mark for working))

ii) $3x+2>1$ or $3x+2<-1$

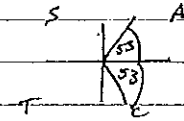
$3x>-1$ $3x<-3$

$x>-1/3$ $x<-1$

$\therefore$ $x<-1$ ✓ or $x>-1/3$ ✓

b) $\cos x = 0.6$.

$x \approx 53.13^\circ$ ✓ $x = 53^\circ$ (to nearest degree)



and $x \approx 306.86^\circ$ $x = 307^\circ$ ✓

(to nearest degree).

c) $P(2,3)$ $x+2y+3=0$ $x_1=2$ $y_1=3$

$a=1$ $b=2$ $c=3$

$$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|1(2)+2(3)+3|}{\sqrt{1^2+2^2}}$$ ✓

$$= \frac{|11|}{\sqrt{5}}$$ ✓

$$= \frac{11\sqrt{5}}{5}$$

d) i) $(1,4)$ ✓

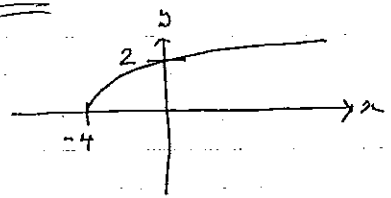
$y=2$ ✓

ii) $(1,-3)$ ✓

$y=-1$ ✓

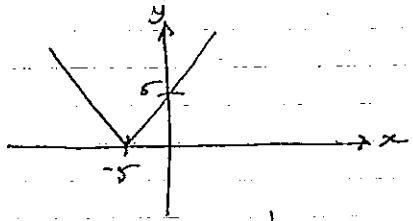
Q12

a) i)



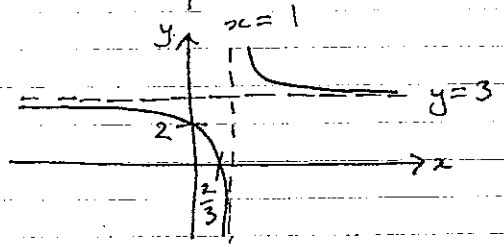
✓

ii)



Shape ✓
intercepts ✓

iii)



Shape ✓
asymptotes ✓
intercepts ✓

b) i) $35^\circ + x = 180^\circ$ ✓ (Cointerior angles between ∥lines)

$x = 145^\circ$ ✓

$360^\circ - 320^\circ = 40^\circ$

$145^\circ - 40^\circ = 105^\circ$ (Adjacent Angles) ✓

ii) $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 3500^2 + 2000^2 - 2 \times 3500 \times 2000 \times \cos 105^\circ$ ✓

$a = 4458\text{m}$ (to nearest m) ✓

iii) $\frac{\sin A}{3500} = \frac{\sin 105^\circ}{4458}$

$A = 49.3197\ldots$ ✓

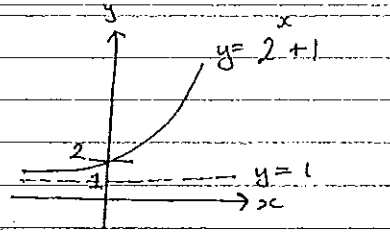
∴ Bearing $= 860 - 49.3197\ldots$

$= 310.68^\circ$

$= 311^\circ$ (to nearest degree) ✓

Q13

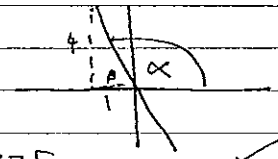
a) i)



asymptote, labelled; ✓
Shape + intercept; ✓

ii) $y > 1$ ✓

b)



$\tan\beta = 4$

$\beta = 75.96375\ldots$ ✓

$\alpha = 104.036$

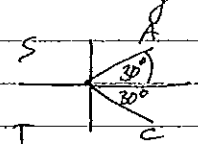
$= 104^\circ$ ✓ (to nearest degree)

c) $\cos 2x = \frac{\sqrt{3}}{2}$ for $0^\circ \le x \le 360^\circ$

let $u = 2x$

$\cos u = \frac{\sqrt{3}}{2}$ for $0^\circ \le u \le 720^\circ$

related angle: $u = 30^\circ$ ✓



$u = 30^\circ, 330^\circ, 390^\circ, 690^\circ$ ✓

$x = 15^\circ, 165^\circ, 195^\circ, 345^\circ$ ✓

(d) i) $A(3,9)$ $B(1,1)$.

$M = \left(\frac{3+1}{2}, \frac{9+1}{2}\right)$ ✓

$M = (2,5)$ centre of circle.

$MB = \sqrt{(2-1)^2 + (5-1)^2}$ ✓

$= \sqrt{17}$

$\therefore$ Radius is $\sqrt{17}$

ii) $y = x + 6$

$(x-2)^2 + ((x+6) - 5)^2 = 17$ ✓

$x^2 - 4x + 4 + x^2 + 2x + 1 = 17$

$2x^2 - 2x + 5 = 17$

$2x^2 - 2x - 12 = 0$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$, $y = 4$

or $x = 3$

$\therefore (-2, 4)$ ✓ is the other point of intersection.

Q14)

a) i) $A^3 + B^3 = (A+B)(A^2 - AB + B^2)$ ✓

ii) LHS $= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$ ✓

$= \sin^2 x + \cos^2 x - \sin x \cos x$

$= 1 - \sin x \cos x$ ✓

$=$ RHS as required

b) i) $f(x) = |x+3| - 2$

$f(0) = |0+3| - 2$

$= 1$

$g(x) = -(x+2)^2 + 5$

$g(0) = -(0+2)^2 + 5$

$g(0) = 1$ ✓ (both)

ii) when $x < -3$ $f(x) = -(x+3) - 2$

$-(x+3) - 2 = -(x+2)^2 + 5$ ✓

$-x - 3 - 5 = -(x^2 + 4x + 4) + 5$

$-x - 5 = -x^2 - 4x + 1$

$x^2 + 3x - 6 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times -6}}{2 \times 1}$

$x = \frac{-3 \pm \sqrt{33}}{2}$

as $x < -3$, $x = \frac{-3 - \sqrt{33}}{2}$ ✓

$f\left(\frac{-3-\sqrt{33}}{2}\right) = \left|\frac{-3-\sqrt{33}}{2} + 3\right| - 2$

$= \left|\frac{3 - \sqrt{33}}{2}\right| - 2$

$= \frac{\sqrt{33} - 7}{2}$ ✓

$\left(\frac{-3-\sqrt{33}}{2}, \frac{\sqrt{33}-7}{2}\right)$

iii) $\frac{-3-\sqrt{33}}{2} \leq x \leq 0$ ✓

2i) $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (6, 7)$.

$A(h,k)$.

let B have coordinates $B(x_1, y_1)$.

$6 = \frac{x_1+h}{2}$

$12 = x_1 + h$

$x_1 = 12 - h$

$7 = \frac{y_1+k}{2}$

$14 = y_1 + k$

$y_1 = 14 - k$.

(or equivalent) ✓

$\therefore B(12-h, 14-k)$

ii) $A(h,k)$ lies on line $2x + 5y - 15 = 0$.

$\therefore$ ① $2h + 5k - 15 = 0$

$B(12-h, 14-k)$ lies on line $3x - 2y + 5 = 0$

$\therefore$ ② $3(12-h) - 2(14-k) + 5 = 0$

$36 - 3h - 28 + 2k + 5 = 0$

$-3h + 2k + 18 = 0$. ✓

① $\times 3$ + ② $\times 2$:

$6h + 15k - 45 = 0$

$-6h + 4k + 26 = 0$

$19k - 19 = 0$

$k = 1$

$h = 5$ ✓

sub into ①

$\therefore A(5,1) \quad M(6,7)$

$y - 1 = \frac{7-1}{6-5}(x-5)$

$y = 6x - 29$ ✓