Adele Liew

12ml Maths Test No 1 Differentiation and Integration of Trig and Logs

1.Find $\frac{dy}{dx}$ in the following: a) $y = e^{5-4x}$ b) $y = x^2e^{-3x}$ c) $y = x \ln x$

d) $y = e^{2x}\sin 3x$ e) $y = \ln\sqrt{1-x^2}$ f) $y = \cos^3 2x$

g) $y = 4^x$ h) $y = \ln x(1+2x)^3$

2. Find the following integrals: a) $\int e^{2-4x}.dx$ b) $\int 4^x.dx$

c) $\int 2xe^{-x^2}.dx$ d) $\int \frac{2x^2}{1+x^3}dx$ e) $\int \frac{4x+3}{2x-1}.dx$

f) $\int \sin 3x.dx$ g) $\int \sec^2 x.dx$ h) $\int \tan^2 x.dx$

3. Consider the curve y = x ln x
a) Find the stationary point,
b) show there are no inflexion points and
c) use your calculator to find what has as x approaches zero
d) then sketch the curve.

4. Find the area enclosed by y = sin 2x, the Y axis and the line y = 1.

5.If $y = \{x + \sqrt{x^2+1}\}$, find the simplest expression for $\frac{dy}{dx}$.

6. Find $\frac{d}{dx}\{x\sin x\}$ and hence evaluate $\int_0^{\frac{\pi}{2}} x\cos x.dx$

1. (a) $\frac{dy}{dx} = -4e^{5-4x}$ ✓

(b) $\frac{dy}{dx} = 2xe^{-3x} + x^2 \cdot -3e^{-3x}$

$= 2xe^{-3x} - 3x^2e^{-3x}$

$= xe^{-3x}(2-3x)$ ✓

(c) $\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x}$

$= 1 + \ln x$ ✓

(d) $\frac{dy}{dx} = e^{2x} \cdot 3\cos 3x + 2e^{2x}\sin 3x$

$= e^{2x}(3\cos 3x + 2\sin 3x)$ ✓

(e) $y = \frac{1}{2}\ln(1-x^2)$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{-2x}{1-x^2}\right)$

$= \frac{-x}{1-x^2}$ ✓

(f) $\frac{dy}{dx} = 3\cos^2 2x \cdot -2\sin 2x$

$= -6\cos^2 2x \sin 2x$ ✓

(g) ~~$\frac{dy}{dx}$~~ [illegible]

$y = 4^x$

$\log_e y = x\log_e 4$

$\frac{\log_e y}{\log_e 4} = x$

$\frac{1}{\log_e 4} \cdot \frac{1}{y} \cdot \frac{dy}{dx} = 1 \therefore \frac{dy}{dx} = y\log_e 4$

$\frac{dy}{dx} = 4^x \log_e 4$

(h) $y = \ln x + \ln(1+2x)^3$

$= \ln x + 3\ln(1+2x)$

$\frac{dy}{dx} = \frac{1}{x} + \frac{6}{1+2x}$ ✓

2. (a) $\frac{-1}{4}e^{2-4x} + c$ ✓

(b) $\frac{d}{dx}4^x = 4^x \log_e 4$

$\therefore \int \frac{d}{dx}4^x\,dx = \int 4^x \log_e 4\,dx$

$4^x = \int 4^x \cdot \log_e 4\,dx$

$\therefore$ since $\log_e 4$ is a constant ✓

$\int 4^x = \frac{4^x}{\log_e 4} + c$

(c) $-e^{-x^2} + c$ ✓

(d) $\frac{2}{3}\int \frac{3x^2}{1+x^3}\,dx = \frac{2}{3}\ln(1+x^3) + c$ ✓

(e)
$$\begin{array}{r} 2 \\ 2x-1\,\overline{)\,4x+3} \\ \underline{4x-2} \\ 5 \end{array}$$

$\therefore 2 + \frac{5}{2x-1}$ ✓

$\therefore \int \frac{4x+3}{2x-1}\,dx = \int 2\,dx + \int \frac{5}{2x-1}\,dx$

$= 2x + \frac{5}{2}\int \frac{2}{2x-1}\,dx$

$= 2x + \frac{5}{2}\ln(2x-1) + C$

(f) $\frac{-1}{3}\cos 3x + c$ ✓

(g) $\tan x$ ✓

(h) $1 + \tan^2 x = \sec^2 x$ ✓

$\therefore \int \sec^2 x - 1\,dx$

$\int \sec^2 x\,dx - \int 1\,dx$

$= \tan x - x + C$ ✓

3. (a) $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$

$= 1 + \ln x$

$\therefore$ stat point $\ln x = -1$

$\therefore \log_e x = -1$

~~$x = e^{-1}$~~ $\frac{1}{e}$ ✓

$\therefore y = \frac{1}{e}(\log_e \frac{1}{e})$

$= \frac{-1}{e}$

$\therefore$ stat point $(\frac{1}{e}, -\frac{1}{e})$

$\frac{d^2y}{dx^2} = \frac{1}{x}$

when $x = \frac{1}{e}$ $\frac{d^2y}{dx^2} = e > 0 \therefore \cup$ concave

$\therefore$ minimum at $(\frac{1}{e}, \frac{-1}{e})$

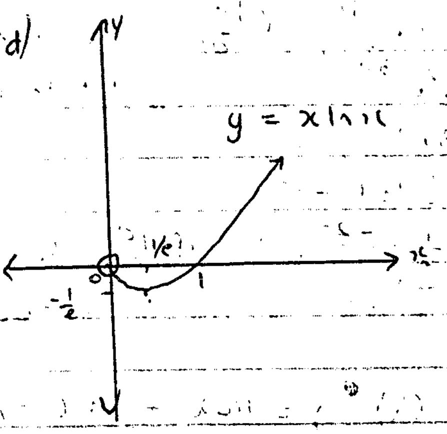
(b) $\frac{d^2y}{dx^2} = 0$ is inflexion point

But $\frac{1}{x} = 0$

$\therefore x$ does not exist for that ✓

(c) As $x \to 0$, $y \to 0$

(d)



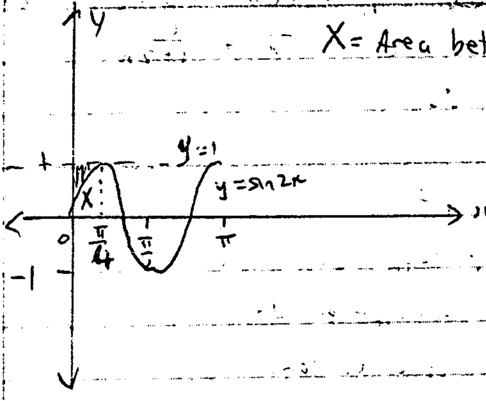
$x \ln x = 0$

$x \log_e x = 0$

$e^{0} = x$ ✓

$\therefore x = 1$ is x-intercept

4.



X = Area between curve & x co-ordinates 0 & π/4 & x-axis

$\therefore \int_0^{\pi/4} \sin 2x \, dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/4}$

$= 0 - \left(-\frac{1}{2}\right)$

$= \frac{1}{2}$ ✓

$\therefore$ Area $= \left(\pi/4 - \frac{1}{2}\right)$ units^2

5. $\frac{dy}{dx} = 1 + \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$= 1 + \frac{x}{\sqrt{x^2+1}} = \cancel{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}} \cancel{\frac{x^2+1+x\sqrt{x^2+1}}{x^2+1}}$ ✓

$= 1 + \frac{x\sqrt{x^2+1}}{x^2+1}$

6. $\frac{d}{dx}(x \sin x) = x\cos x + \sin x$

$\therefore \int \frac{d}{dx} x \sin x \, dx = \int x \cos x \, dx + \int \sin x \, dx$

$x \sin x - \int \sin x \, dx = \int x \cos x \, dx$

$\therefore \int x \cos x \, dx = x \sin x + \cos x + c$

$\therefore \left[x \sin x + \cos x\right]_0^{\pi/2} = \left[\frac{\pi}{2}[1] + 0\right] - \left[0 + 1\right]$ ✓

$= \frac{\pi}{2} - 1$