Student:____________________________

Teacher:____________________________

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FORTITER ET SUAVITER

Preliminary Course Task 2 Half Yearly

Mathematics

2017

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown on every question
- Write your name & teacher's name on top of every section

Multiple Choice (10 marks)

1. What is the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures?

(A) 0.040 (B) 0.041

(C) 0.0409 (D) 0.0410

2. What is the solution to the equation $\dfrac{x+4}{3} = \dfrac{x}{2} - 2$?

(A) $x = 2$ (B) $x = 5$

(C) $x = 6$ (D) $x = 20$

3. If $2x^2 - 6x - 3 = 0$, then $x =$

(A) $\dfrac{3 \pm \sqrt{3}}{2}$ (B) $\dfrac{3 \pm \sqrt{15}}{2}$

(C) $\dfrac{-3 \pm \sqrt{3}}{2}$ (D) $\dfrac{-3 \pm \sqrt{15}}{2}$

4. The formula $H = 5m(Y - X)$ is used to calculate the heat (H) required to raise the temperature of a steel rod, of mass m, from a temperature of X to a temperature of Y. Rearrange the formula to make X the subject.

(A) $X = \dfrac{5mY - H}{5m}$ (B) $X = \dfrac{H - 5m}{Y}$

(C) $X = \dfrac{H - 5mY}{5m}$ (D) $X = \dfrac{5m - H}{Y}$

5. If $f(x) = 2x^2 - 3x - 5$ then $\dfrac{f(a)}{2a + 2}$ equals

(A) $\dfrac{2a-5}{2}$

(B) $\dfrac{2x-5}{2}$

(C) $2a-5$

(D) $x-5$

6. Which of the following expressions does NOT have $m+1$ as a factor?

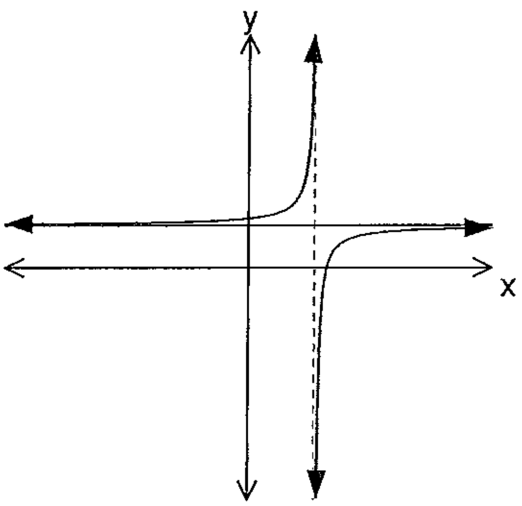
(A) m^2-1

(B) m^2+1

(C) m^2+m

(D) m^2+2m+1

7.



The graph could be represented by the equation:

(A) $y = -\dfrac{1}{x-3} - 2$

(B) $y = -\dfrac{1}{x-3} + 2$

(C) $y = -\dfrac{1}{x+3} + 2$

(D) $y = -\dfrac{1}{x+3} - 2$

8. The minimum value of $y = x^2 - 7x + 10$ is:

(A) 2

(B) $3\frac{1}{2}$

(C) $-2\frac{1}{4}$

(D) $2\frac{1}{4}$

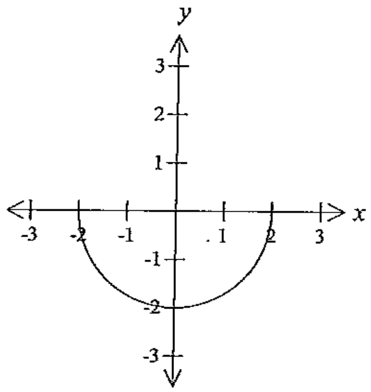
9. Which of the following is true for the function $f(x) = 3x^2 - x$?

(A) Even function (B) Odd function

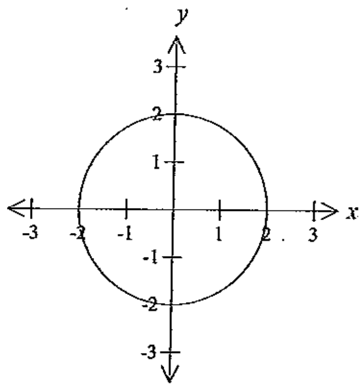
(C) Neither odd or even (D) Zero function

10. Which graph best represents $y = \sqrt{4 - x^2}$?

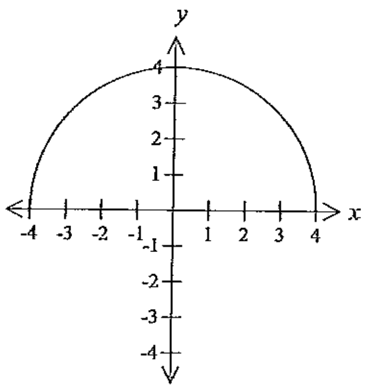
(A)



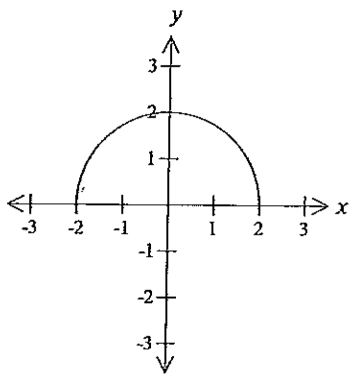
(B)



(C)



(D)



Question 11 (Start a new page – 18 marks)

a. Simplify $\frac{x^2-1}{x-3}\times\frac{x^2-3x}{2x-2}$ as a single fraction in simplest form. **2**

b. Solve $3x=x^2$ **2**

c. Completely factorise the following

(i) $6x^2-3xy-4xz+2yz$. **2**

(ii) x^4-1 **2**

(iii) $8x^3-125y^3$ **2**

d. Expand and simplify: $(2x-3y)^2-5x(x-2y)$. **2**

e. Solve for x and y : $\begin{aligned} x+3y&=2 \\ 2x-y&=11 \end{aligned}$ **3**

f. Solve the equation by completing the square: **3**

$x^2-10x=11$

Question 12 (Start a new page – 21 marks)

a. Simplify

(i) $\sqrt{32}$ **1**

(ii) $\sqrt{3}+\sqrt{27}-\sqrt{18}$ **2**

b. Express $0.1\dot{2}\dot{5}$ as a fraction in simplest form, showing all working. **2**

c. Expand and simplify: $(2\sqrt{7}+\sqrt{11})(2\sqrt{7}-\sqrt{11})$ **2**

d. Express $\dfrac{3\sqrt{2}+1}{2\sqrt{3}}$ with a rational denominator. **2**

e. Find the value of p and q such that: $\dfrac{\sqrt{5}}{\sqrt{5}-2}=p+q\sqrt{5}$ **3**

f. Sketch the following function, stating their domain and range: **3**

$$f(x)=\frac{3}{x-4}$$

g. A function is defined as $y=15-7x-2x^2$

(i) Determine the x intercepts. **2**

(ii) Determine the vertex. **1**

(iii) Sketch the function, showing the important features. **1**

(iv) Hence solve $15-7x-2x^2\geq 0$ **1**

(v) State the range of the function. **1**

Question 13 (Start a new page – 22 marks)

a. Solve for x:

(i) $1 < 1 + x < 2$ **1**

(ii) $|x+1| = 3x+2$ **2**

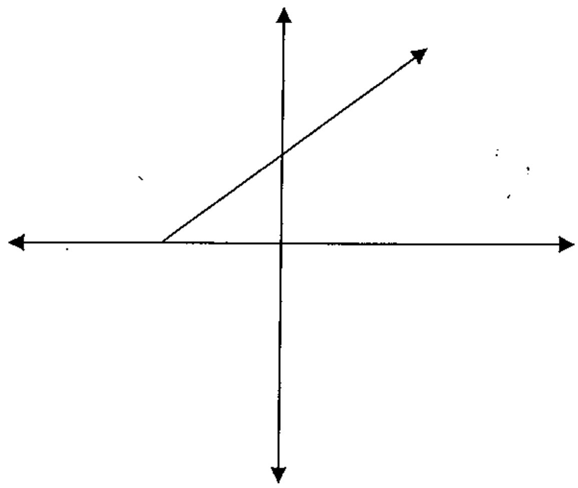
b. A function is defined by the rule $p(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ -1, & \text{if } -2 < x < 1 \\ |x|, & \text{if } x \leq -2 \end{cases}$

Find:

(i) $p(0) + p(-2)$ **2**

(ii) Sketch the function. **3**

c. Copy and complete the graph below, given that it is not a function. **1**



d. Show the region of the number plane where the following hold simultaneously: **3**

$(x-2)^2 + y^2 \leq 4$ and $y < 2 - x$

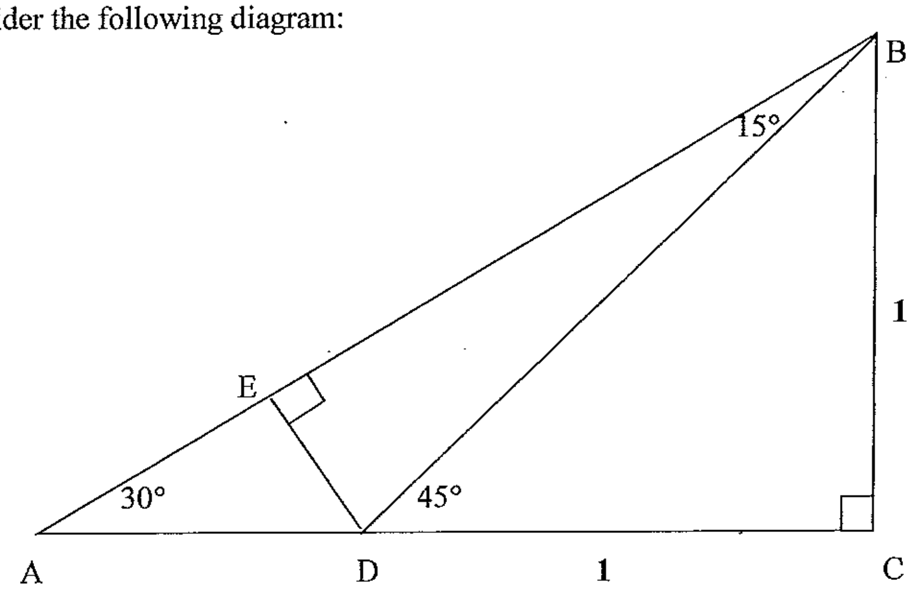
e. Show that the function $f(x) = 8x^3 - 7x$ is odd. **1**

f. Solve for θ over the domain $0° \leq \theta \leq 360°$:

(i) $\sin 40° = \cos(90 - \theta)°$. **1**

(ii) $\sin\theta = -\dfrac{1}{2}$ **2**

g. Consider the following diagram:



(i) On about one third of a page, copy the above diagram.

(ii) Find the length of AC in exact form. **1**

(iii) Hence show that $ED = \dfrac{\sqrt{3}-1}{2}$ **2**

(v) Hence, using exact values, show that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ **3**

END OF EXAM

MC

Brigidine 2u Prelim Task 2 Sample Solutions.

1. $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$

$= \dfrac{0.2401}{5.857}$

$= 0.0410$ (3 sig fig) (D)

2. $\dfrac{x+4}{3} = \dfrac{x}{2} - 2$

$\dfrac{2(x+4)}{6} = \dfrac{3x}{6} - \dfrac{12}{6}$

$\dfrac{2(x+4)}{6} = \dfrac{3x-12}{6}$

$2(x+4) = 3x - 12$

$2x + 8 = 3x - 12$

$x = 8 + 12 = 20$

(D)

3. $2x^2 - 6x - 3 = 0.$

$a = 2, \quad b = -6, \quad c = -3.$

$\dfrac{6 \pm \sqrt{36 - 4(-3)(2)}}{4} = \dfrac{6 \pm \sqrt{12}}{4} = \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{3 \pm \sqrt{3}}{2}$ (A)

4. $H = 5m(Y - X)$

$\dfrac{H}{5m} = Y - X$

$Y - \dfrac{H}{5m} = X$

$X = \dfrac{5mY - H}{5m.}$ (A)

CHCEK BY REARRANGING.

$5mX = 5mY - H$

$X = Y - \dfrac{H}{5m.}$

$\dfrac{H}{5m} = Y - X$

$H = 5m(Y - X).$ ✓

5. $f(x) = 2x^2 - 3x - 5$

$\dfrac{f(a)}{2a+2} = \dfrac{2a^2 - 3a - 5}{2(a+1).}$

$= \dfrac{(2a-5)(a+1)}{2(a+1)}$

$= \dfrac{2a-5}{2.}$ (A)

6 m^2-1

$= (m+1)(m-1)$ ✗

m^2+1 ✓

$m^2+m = m(m+1)$ ✗

$m^2+2m+1 = (m+1)^2$

⇒ (B)

7. horizontal asymptote at positive y and positive x.

graph is shifted up 2 units

∴ ⇒ $y = -\frac{1}{x-3} + 2$.

Check horizontal asymptote.

$y-2 = -\frac{1}{x-3}$

$\frac{1}{y-2} = -(x-3)$

↳ Asymptote at $y=2$!

(B)

8 $y = x^2-7x+10$

↳ Alt method $\frac{-b}{2a} = \frac{7}{2} = 3\frac{1}{2}$

$\frac{dy}{dx} = 2x-7=0$

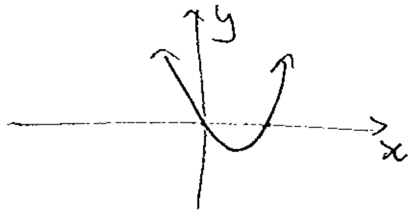
$2x = 7$

$x = \frac{7}{2} = 3\frac{1}{2}$ ⇒ (B)

9. $f(x) = 3x^2-x$

$= 3x(x-1)$

root at 0 and 1



Graph is neither, because the axis of symmetry/rotation is not centred at the axis ⇒ (C)

10. $y = \sqrt{4-x^2}$

Range: $y \geq 0$.

Domain $4-x^2 \geq 0$

$x^2 \leq 4$

$x \leq \pm 2$.

⇒ (D)

11\.

a) $\frac{x^2-1}{x-3} \times \frac{x^2-3x}{2x-2}$

$= \frac{(x+1)(x-1)\,x(x-3)}{(x-3)(2(x-1))}$

$= \frac{x(x+1)}{2} \Rightarrow \frac{x^2+x}{2}$ expanded.

b) $3x = x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ OR $x = 3$.

c) i) $6x^2 - 3xy - 4xz + 2yz$

$3x(2x-y) - 2z(2x-y)$

$= (3x-2z)(2x-y)$.

ii) $x^4 - 1$

$= (x^2)^2 - 1$

$= (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

iii) $8x^3 - 125y^3$

$= (2x)^3 - (5y)^3$

$= (2x-5y)(4x^2+25y^2+10xy)$

d) $(2x-3y)^2 - 5x(x-2y)$.

$4x^2 + 9y^2 - 12xy - 5x^2 + 10xy$.

$= 9y^2 - x^2 - 2xy$.

e) $x + 3y = 2$

$2x - y = 11$

$x = 2 - 3y$

$2(2-3y) - y = 11$

$4 - 6y - y = 11$

$-7y = 7$

$y = -1$

$x + 3(-1) = 2$

$x = 5$

f) $x^2 - 10x - 11 = 0$

$(x-5)^2 - 25 - 11 = 0$

$(x-5)^2 = 36$

$x - 5 = \pm 6$

$x = -1$ OR $x = 11$

Double check by factorising

$(x+1)(x-11)$

$= x^2 - 11 - 10x$ ✓

12. a) i) $\sqrt{32} = \sqrt{16 \times 2}$

$= \sqrt{16}\sqrt{2}$

$= 4\sqrt{2}$.

ii) $3 + \sqrt{27} - \sqrt{18}$

$\sqrt{3} + \sqrt{9}\sqrt{3} - \sqrt{9}\sqrt{2}$

$\sqrt{3} + 3\sqrt{3} - 3\sqrt{2}$

$=$

b. $0.1\dot{2}\dot{5} \Rightarrow 0.1252525\ldots$

let this be x.

$10x = 1.2525\ldots$

$1000x = 125.2525\ldots$

$1000x - 10x = 124$

$990x = 124$ → $x = \frac{62}{495}$

c) $(2\sqrt{7} + \sqrt{11})(2\sqrt{7} - \sqrt{11})$

$= (2\sqrt{7})^2 - (\sqrt{11})^2$

Difference of 2 squares

$= 4(7) - 11$

$= 28 - 11 = 17$

d) $\frac{3\sqrt{2}+1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ Rationalise.

$= \frac{3\sqrt{6} + \sqrt{3}}{6}$

e) $\frac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$.

$\frac{\sqrt{5}(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$= \frac{5 + 2\sqrt{5}}{5 - 4} = 5 + 2\sqrt{5}$

$p = 5,\ q = 2$.

f. let $y = f(x)$

$y = \frac{3}{x-4}$ Vertical asymptote at $x = 4$.

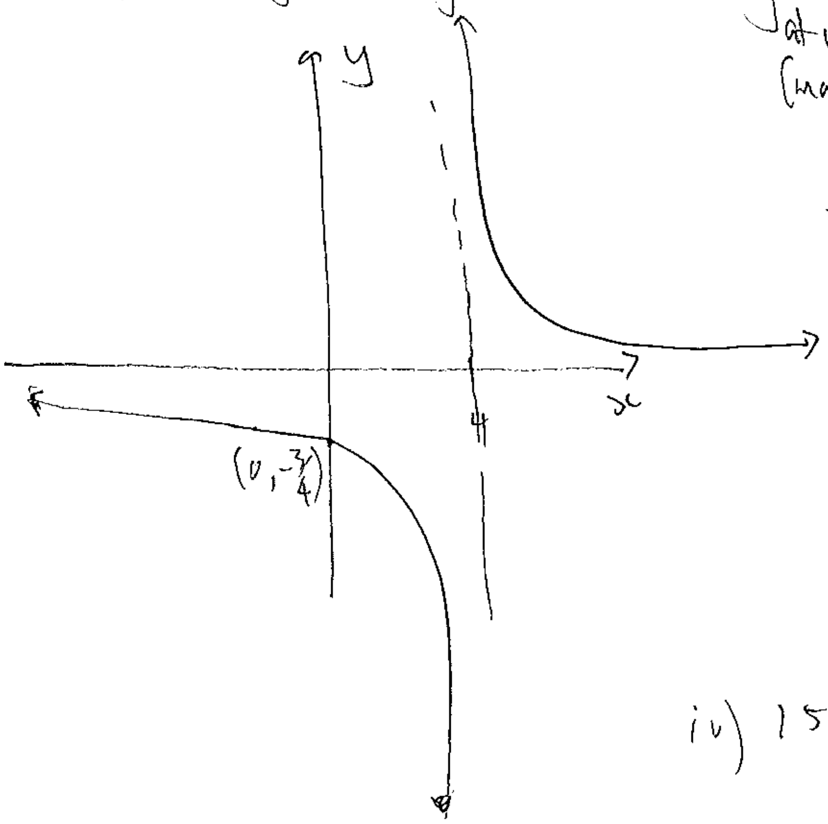
$\frac{x-4}{3} = \frac{1}{y}$

$x - 4 = \frac{3}{y}$

$x = \frac{3}{y} + 4$. horizontal asymptote at $y = 0$

domain: $x \in \mathbb{R}; x \neq 0$

Range: $y \in \mathbb{R}, y \neq 0$



g) $y = 15 - 7x - 2x^2$

$(3 - 2x)(5 + x)$

Roots

$x = -5$, $x = \frac{3}{2}$.

ii) Vertex

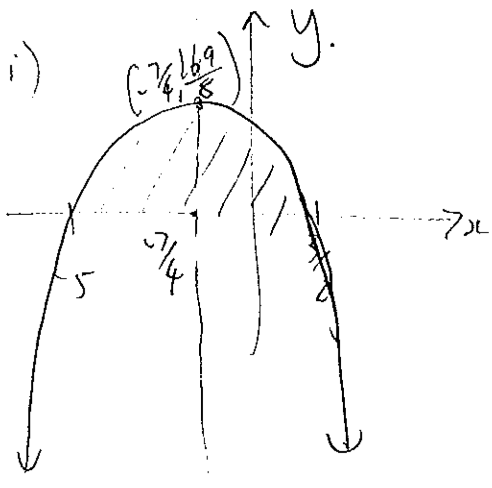
$a = -2$
$b = -7$
$c = 15$

$\frac{-b}{2a} = \frac{7}{-4}$

$15 - 7\left(\frac{7}{-4}\right) - 2\left(\frac{49}{16}\right)$

y-coordinate at vertex (maximum) $= \frac{169}{8}$

iii)



iv) $15 - 7x - 2x^2 \geq 0$

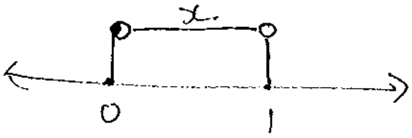
$-5 \leq x \leq \frac{3}{2}$.

v. $y \leq \frac{169}{8}$

13.

a. i) $1 < 1 + x < 2$

$0 < x < 1$



ii) $|x+1| = 3x+2$

$(x+1)^2 = (3x+2)^2$

$x^2 + 1 + 2x = 9x^2 + 4 + 12x$

$8x^2 + 10x + 3$

$(2x+1)(4x+3)$

$x = -\frac{1}{2}$ OR $x = -\frac{3}{4}$

Substitute back to find correct answer

$\left|-\frac{1}{2}+1\right| = 3\left(-\frac{1}{2}\right)+2$ ✓

$\left|-\frac{3}{4}+1\right| = 3\left(-\frac{3}{4}\right)+2$ ✗

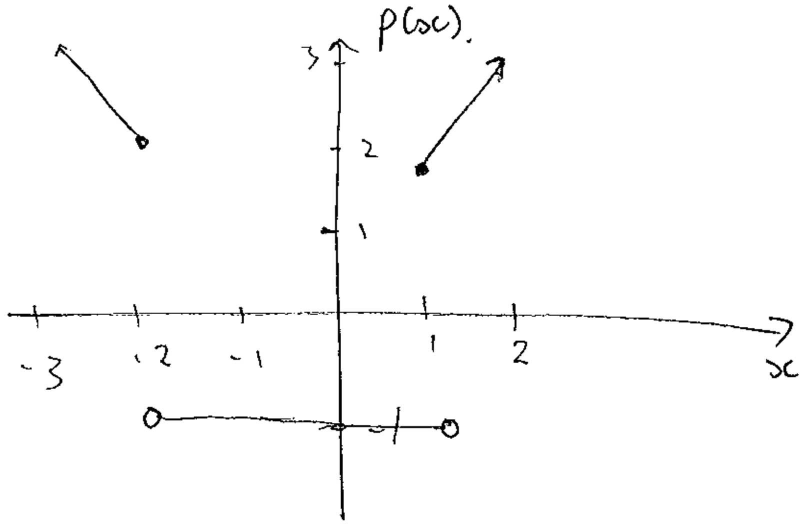
So the only solution is $x = -\frac{1}{2}$

b. $p(x) = \begin{cases} x+1 & ; x \geq 1 \\ -1 & ; -2 < x < 1 \\ |x| & x \leq -2 \end{cases}$

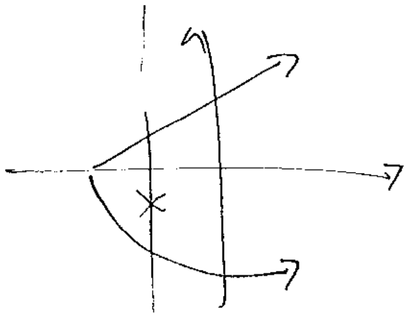
$p(0) = -1$

$p(-2) = |-2| = 2$

$p(0) + p(-2) = 1$

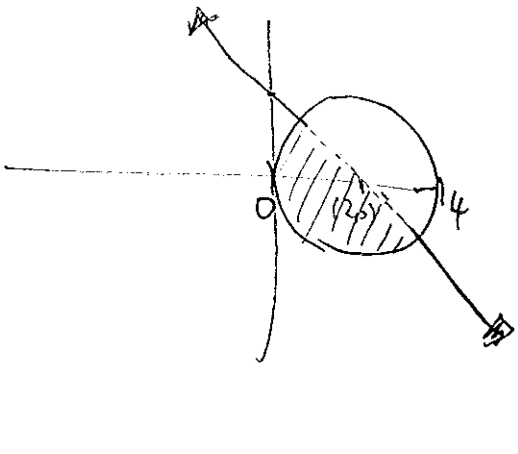


c).



Any line drawn that does not pass the straight line test will suffice.

d).



e) $f(x) = 8x^3 - 7x$.

$f(-x) = 8(-x)^3 - 7(-x)$

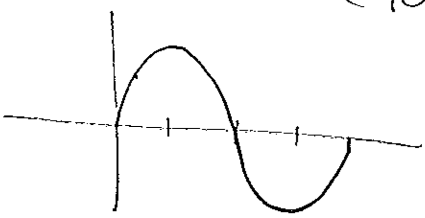
$= -8x^3 + 7x$

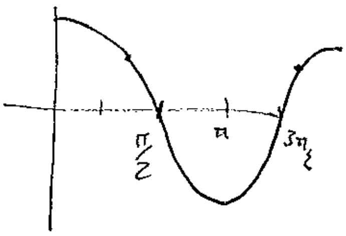
$= -(8x^3 - 7x)$

$= -f(x)$

$\therefore$ ODD FUNCTION.

f. $\sin 40 = \cos(90 - \theta)$





$\theta = 40, 140$

ii) $\sin\theta = -\frac{1}{2}$.

$\theta: 210°, 330°$.

g) i) Check it yourself!

ii) $\frac{1}{AC} = \tan 30°$

$\frac{1}{AC} = \frac{1}{\sqrt{3}}$

$AC = \sqrt{3}$ units

$AD = \sqrt{3} - 1$

$\frac{ED}{\sqrt{3}-1} = \sin 30$

$ED = \frac{1}{2}(\sqrt{3}-1)$

$= \frac{\sqrt{3}-1}{2}$.

V. $\sin 15 = \frac{ED}{(\sqrt{2})}$

$\frac{\frac{\sqrt{3}-1}{2}}{\sqrt{2}}$.

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$