

SYDNEY BOYS HIGH SCHOOL HOORE PARK, SURRY HILLS

2015 HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK # 2

Mathematics Extension 1

General Instructions

- Reading time 5 minutes
- Working time 90 Minutes
- Write using black or blue pen. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 8 10, show ALL relevant mathematical reasoning and/or calculations.
- Marks may NOT be awarded for untidy or badly arranged work.
- Answer in simplest EXACT form unless otherwise instructed.

Total marks - 52

Section I Pages 2-4 (7 marks)

- Attempt Questions 1–7 on the Multiple Choice answer sheet provided.
- · Allow about 10 minutes for this section.

Section II Pages 5-15 (45 marks)

- Attempt Questions 8–10.
- Start a new answer booklet for each question.
- Allow about 1 hours and 20 minutes for this section.

Examiner: V. Likourezos

This is an assessment task only and does not necessarily reflect the content of the Higher School Certificate.

Section I
7 marks
Attempt Questions 1–7
Allow about 10 minutes for this section.

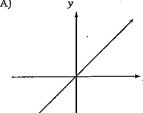
Use the multiple-choice answer sheet for Questions 1-7.

- 1 Seven people are to be seated around a circular table. If two particular people must be seated together, how many seating arrangements are possible?
 - (A) 7!
 - (B) 5!×2
 - (C) 6!×2
 - (D) 6!
- 2 It is known that an approximate root to the curve $y = e^x 3x^2$ is x = 3.8. Using Newtons Method of Approximation with one application what is an approximation of the root?

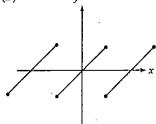
Round your answer correct to 2 decimal places.

- (A) x = 3.74
- (B) x = 4.22
- (C) x = -12.06
- (D) x = 3.70
- What is the amplitude and period of the graph $y = 2\pi \cos\left(2x + \frac{\pi}{3}\right)$?
 - (A) amplitude = 2 and period = π
 - (B) amplitude = 2π and period = π
 - (C) amplitude = 2π and period = 2
 - (D) amplitude = 2 and period = 2

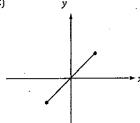
- Four female and four male athletes are arranged in a row for the presentation of prizes. In how many ways can this be done if the males and females must alternate?
 - (A) 4!×4!
 - (B) 2×4!×4!
 - (C) 4!x5!
 - (D) 2×41×5!
- 5 Which diagram best represents the function $y = \sin^{-1}(\sin x)$?
 - (A)



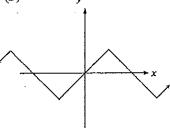
(B)



(C)



(D)



- What is the value of $\frac{(n+r+1)!}{(n+r-1)!}$ in simplest form?
 - (A) n(n+r)
 - (B) n+r+1
 - (C) n(n+r-1)
 - (D) (n+r)(n+r+1)

- 7 What is the derivative of $y = 3^{3x+1}$?
 - (A) $\frac{dy}{dx} = (3x+1) \times 3^{3x}$
 - (B) $\frac{dy}{dx} = 3 \times 3^{3x+3} \ln 3$
 - (C) $\frac{dy}{dx} = 3^{3x+1} \ln 3$
 - (D) $\frac{dy}{dx} = 3 \times 3^{3x+2} \ln 3$

End of Section I

Section II

45 marks

Attempt Questions 8-10

Allow about 1 hour and 20 minutes for this section.

Answer each question in a new writing booklet. Extra writing booklets are available.

In Questions 8-10, your responses should include ALL relevant mathematical reasoning and/or calculations.

Question 8 (15 marks) Start a new writing booklet for each question.

(a) Using the table of standard integrals, find

$$\int 3\sec\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)dx$$

(b) Differentiate $y = x \cos(x^2 + 1)$ with respect to x.

(c) Find
$$\int \frac{\cos x}{1+\sin x} dx$$

(d) (i) Let
$$f(x) = \ln(x + \sqrt{x^2 - 1})$$
, what is the domain of $f(x)$?

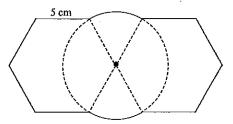
(ii) Show that the derivative
$$f'(x) = \frac{1}{\sqrt{x^2 - 1}}$$

(e) (i) Find
$$\frac{d}{dx}(x^2e^{x^2})$$

(ii) Hence, evaluate
$$\int_0^1 x^3 e^{x^2} dx$$
 3

Question 8 continued

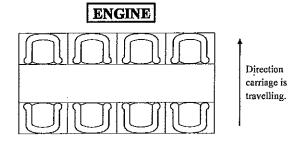
(f) The diagram above shows two regular hexagons joined to both sides of a circle. The corner of the hexagons meets the circle at its centre. The side length of the hexagon is 5 cm. Find the exact perimeter of the resulting shape.



End of Question 8

Question 9 (15 marks) Start a new writing booklet for each question.

(a) Five people enter a railway carriage in which there are 8 empty seats. Below is a diagram showing the layout of the seats.



In how many ways can they take places if:

(i) Any person can sit in any seat?

1

(ii) If one of the passengers, Chloe, sits in a corner?

1

1

(iii) Chloe sits with her back to the engine while her friend, Vicki, sits facing the engine?

acing

(b) (i) Show that
$$\int_{\frac{\pi}{8}}^{\frac{2\pi}{8}} \sin^2 2x \, dx = \frac{1}{8} (\pi + 2)$$

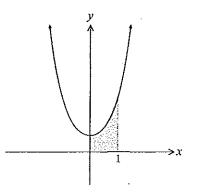
3

3

(ii) Hence or otherwise find the volume of the solid of revolution formed when the curve $y=1+\sin 2x$ is rotated about the x-axis between $x = \frac{\pi}{8}$ and $x=\frac{3\pi}{8}$. Give your answer correct to 3 significant figures.

Question 9 continued

(c) In physics and geometry, a catenary is the curve that an idealised hanging chain or cable assumes under its own weight when supported only at its ends. The diagram below shows the region bounded by the catenary, the x-axis, the y-axis and the line x=1.



The equation of the catenary is given by $y = \frac{e^x + e^{-x}}{2}$.

(i) Find the exact area of the shaded region.

2

(ii) Using Simpson's rule with three function values, find an approximation to the shaded area correct to 2 decimal places.

2

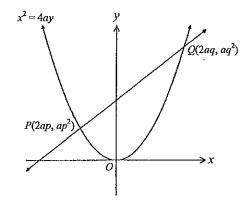
(iii) Hence deduce that $e \approx 2.7$ correct to 1 decimal place.

2

End of Question 9

Question 10 (15 marks) Start a new writing booklet for each question.

- (a) Evaluate $\lim_{x\to 0} \frac{\sin 2x}{5x}$
- (b) Prove by mathematical induction $7^n + 3n \times 7^n 1$ is divisible by 9 for all positive 3 integers n.
- (c) The points $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ lie on the parabola $x^2 = 4ay$.



The variable chord PQ is such that it is always parallel to the line y = 2x.

Show that p+q=4.

1

- (ii) Show that the equation of the normal at P is $x + py = 2ap + ap^3$.
- (iii) The normals at P and Q intersect at $R(-apq(p+q), 2a+a(p^2+pq+q^2))$. Find the locus of R. (Ignore any restrictions).
- (iv) Find the domain of the locus of R.

Question 10 continued

(d) Consider the pair of simultaneous equations

$$y = \sin x \cos x$$
$$y = mx$$

- Suppose m is positive. By sketching, or otherwise, find any restrictions on m so that it will have a unique simultaneous solution.
- (ii) Suppose m is negative. Show that the pair of equations have a unique simultaneous solution if $m < \cos \theta$, where θ satisfies the equation $\tan \theta = \theta$ for $\pi < \theta < \frac{3\pi}{2}$.

2

End of paper

2015 SBHS X1 Task 2

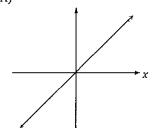
Section I Solutions

- Seven people are to be seated around a circular table, If two particular people must be seated together, how many seating arrangements are possible?
 - (A) 7!
 - (B)
- 5!×2
- (C) 6!×2
- (D) 6!
- It is known that an approximate root to the curve $y=e^x-3x^2$ is x=3.8. Using Newtons Method of Approximation with one application what is an approximation of the root?
 - Round your answer correct to 2 decimal places.
 - (A)
- x = 3.74
- (B) x = 4.22
- (C) x = -12.06
- (D) x = 3.70

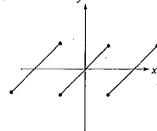
- What is the amplitude and period of the graph $y = 2\pi \cos\left(2x + \frac{\pi}{3}\right)$?
 - (A) amplitude = 2 and period = π
 - (B)
- amplitude = 2π and period = π
- (C) amplitude = 2π and period = 2
- (D) amplitude = 2 and period = 2

- Four female and four male athletes are arranged in a row for the presentation of prizes. In how many ways can this be done if the males and females must alternate?
 - (A) 4!×4!
 - (B)
- $2 \times 4! \times 4!$
- (C) 41×5!
- D) 2×4!×5!

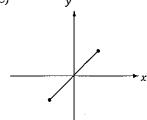
- Which diagram best represents the function $y = \sin^{-1}(\sin x)$?
 - (A)



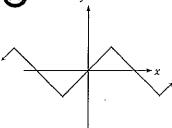
(B)



(C)







- 6 What is the value of $\frac{(n+r+1)!}{(n+r-1)!}$ in simplest form?
 - (A) n(n+r)
 - (B) n+r+1
 - (C) n(n+r-1)
 - (n+r)(n+r+1)
- 7 What is the derivative of $y = 3^{3x+1}$?
 - (A) $\frac{dy}{dx} = (3x+1) \times 3^{3x}$
 - (B) $\frac{dy}{dx} = 3 \times 3^{3x+1} \ln 3$
 - (C) $\frac{dy}{dx} = 3^{3x+1} \ln 3$
 - (D) $\frac{dy}{dx} = 3 \times 3^{3x+2} \ln 3$

COMMENTS & SOLUTIONS FOR QUESTION 8.

a gree =

comment. Well done. Bery few were unable to arene werethy.

 $y' = \cos(2^2+1) - 2x^2 \sin(2^2+1)$

COMMINT Well doe.

6, \(\left(\text{conx} \) \deft(\text{conx} \) + c.

COMMENT. Very Sew students failed to get full marks.

d (1) x > 1.

COMMENT. He common ener was

to counder x2-130

N. 25-1, X71.

Clearly if x 5-1 Then x + \(\si^2 - 1 \) to 4 herak lu \(\frac{2}{2} \times + \si^2 \si^2 - 1 \)

is undefined

(11) If
$$f(x) = \ln \left(x + \sqrt{x^2 - 1}\right)$$

$$f(x) = \frac{1 + \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}} \cdot 2x}{x + \sqrt{x^2 - 1}}$$

$$= \frac{1 + \frac{x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}}$$

$$= \sqrt{x^2 - 1} + \frac{x}{x + \sqrt{x^2 - 1}}$$

$$= \frac{1}{\sqrt{x^2 - 1}}$$

Comment.
A significant number of students used the Standard Integrals to find the arms.

Well dene.

(e) (1) Hence
$$\int_{0}^{1} 2xe^{x^{2}} \left(x^{2}+1\right) = x^{2} \times 2xe^{x^{2}} \left(x^{2}+1\right)$$
(11) Hence
$$\int_{0}^{1} 2xe^{x^{2}} \left(x^{2}+1\right) dx = \left[x^{2}e^{x^{2}}\right]_{0}^{1}$$

$$\therefore 2\int_{0}^{1} x^{3}e^{x^{2}} dx + \int_{0}^{1} 2xe^{x^{2}} dx = e$$

ie. $2\int_0^3 e^2 dn + \left[e^2 \right]_0^7 = e.$ $2\int_{0}^{3}x^{3}e^{2}dx + (e-1) = e$ $2\int_{2}^{2}e^{x}dx=1$ $\int_{\mathbb{R}} x^3 e^{2x} dx = \frac{1}{a}.$

COMMENT. Part (1) was well done. Some struggled with (11).

a) Permeter = 8×5 cm + 1 y 27×5. = (40 + 10 T) Cen.

COMMISMI. Most were able to gain full machs.

GENERAL COMMENT Nery few marks below 10 out 415. a seguificant number attained full marks.

$$Q.9. ExtL.$$
(a) (i) ${}^{8}P_{5} = 6720 \text{ ways}$
(ii) $4 \times {}^{7}P_{4} = 3360 \text{ ways}$
(iii) $4 \times 4 \times 6P_{3} = 1920 \text{ ways}$

Q9(a) For the most part students either did all correctly aor all incorrectly. Errors included using combinations or using nl instead of "P,

(b)
$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{2x}{2x} dx$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (1 - \cos 4x) dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{2}} (1 - \cos 4x) dx$$

$$= \int$$

(b)(i) Mostly correct. Some errors with signs of trig functions.

(b) (i)
$$V = \int_{\frac{\pi}{2}}^{\pi} \int_{\frac{\pi}{2}}^{\pi} (1+2\pi i \pi^{2} x)^{2} dx$$

 $= \pi \int_{\frac{\pi}{2}}^{\pi} \int_{-\infty}^{\infty} \int_{\frac{\pi}{2}}^{\infty} \int_{-\infty}^{\infty} \int_{\frac{\pi}{2}}^{\infty} \int_{-\infty}^{\infty} \int_{\frac{\pi}{2}}^{\infty} \int_{-\infty}^{\infty} \int_{\frac{\pi}{2}}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{$

(ii) Again mostly correct. Errors tended to be arithmetic.

(c)
$$y = \frac{e^x + e^{-x}}{2}$$

Three $= \frac{1}{2} (\frac{1}{16} + e^{-x}) dx$
 $= \frac{1}{2} (e^{-x} - e^{-x}) - (1-i)$
 $= \frac{1}{2} (e^{-x} - e^{-x}) - (1-i)$

Area $= \frac{1}{2} (e^{-x} - e^{-x}) - (1-i)$
 $= \frac{1}{2} (e^{-x} - e^{x}) - (1-i)$
 $= \frac{1}{2} (e^{-x} - e^{x})$

- (c) (i) Done well.
- (ii) Done well
- (iii) Most equated (i) and (ii) to get equation. Most errors involved using the value of e from the calculator in order to solve the equation to find e.

Q10 Y12 Ext 1 Task 2 2015

The mean score for this question was 7.15/15

(a) Most students were able to receive full marks.

0	0.5	I	Av
15	41	107	0.78

(b) Most students found this Mathematical Induction question difficult. Identifying an appropriate way to use the S(k) expression was difficult.

Stepl: Show S(1) is true.

Step 21 Assume SIK) is true he 7k (3kH)-1=914x, MKanintyr

Thous S(kti) is the

LHS = (3KH) . 7 x . 7+3. 7 kH -1 = 7 ((3k+1), 7 1)+3.7 k+1 + 6 = 7.9Mx +3.7.7k+6 = 1.9Mx + 3.7 (9Mx - 3k.74+1)+6 = 7.9Mx +21.9Mx -7.9K.7k +21+6 = 9 (7MK +21MK -7K 7K +3) = 9 MKH MINH ON POLEGET

as the set of integer is chosen under +, -, x,

: If s(k) 11 true, s(kt) 11 tre

S(1) is true, therefore by the process of mothernatical Deduction sly is true for all integral not.

Other (better) uses of S(k) were:

0	0.5	1	1.5	2	2.5	3	Αv	l
1	0	45	75	4	4	34	1.70	l

(c) (i) Most students were able to receive full marks.

(c) (i)
$$m_{PQ} = \frac{aq^2 - ap^2}{2eq^{-2ap}}$$

$$= \frac{a(q-p)(q+p)}{2a(q-p)}$$

$$= \frac{p+q}{2}$$

At Pall
$$y=2x$$

$$PT = 2$$

$$PT = 4$$

[0	0.5	1	Av
[21	4	138	0.86

(c) (ii) This part was done well.

(i)
$$y' = \frac{2x}{4x}$$

At P, $y' = \frac{2 \cdot 2xp}{4x}$

: Equ of normal is
$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

0	0 0.5		1 1.5		Αv
5	0	1	0	157	1.93

(c) (iii) Students who remembered that p + q = 4usually did well in this part.

(iii) Eqn of normal at a is

$$x+qy = 2eq + aq^3$$

 $x = -apq(p+q)$
 $y = 2a+le(p^2+pq+q^2)$

$$x = -apy. 4$$

$$\therefore pq = -\frac{x}{4a}$$

$$y = 2a + a \left[(p+q)^2 - pq \right]$$

$$= 2a + a \left[16 + \frac{x}{4a} \right]$$

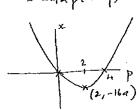
$$= 2a + 16a + \frac{x}{4}$$

$$= 18a + \frac{x}{4}$$

(c) (iv) No students received marks in this part.

Again it was useful to remember that p + q = 4.

$$(N) = -4a p (4-p)$$



Pa har gratical 2

Tanget occurs when 10 = 2 = 9

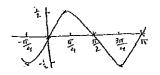
p<2 (and 1>2)

	スフール	,	
0	0.5	1	Av
163	0	0	0

(d) (i) A number of students realised that the tangent at the origin was important in this question but did not then deduce that m=1 was the lowest value of m that would give the required result.

(d)
$$y = \sin x \cos x = \frac{1}{2} \sin 2x$$

 $y = mx$

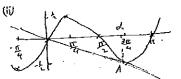


A single root is possible if m = gradient of target at D.

Al x=0, g'=1
: For right solution mol.

0	0.5	1	1.5	2	Av
61	17	26	5	24	0.55

(d) (ii) This part was poorly done; possibly students had run out of time at this stage of the paper.



Fingle robotion if time dorroot to the touch out surve Cother than at D.

11 = cor2x

$$y' = cor2x$$
Let A be $(x, \frac{1}{2} sin2x)$

y-2 sind = cook (2-d)

He line passes through the

origin

-1 sind = -1 cook

in If m is negative for single (3) solution, m < cor &

į	0	0.5	1	1.5	2	2.5	3	Av
	144	1	14	0	2	Γī	1	0.15