Student Number: ___

Year 12 Mathematics

Assessment Task 2

14th March 2017

General Instructions

- Reading Time – 5 minutes
- Working Time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Marks may be deducted for careless or badly arranged work
- Show all necessary working
- Task Weighting – 25%
- Total Marks – 60
- Attempt Questions 1 – 6 on Multiple Choice Answer Sheet.
- Start Question 7 in a new booklet.
- Start Question 8 in a new booklet.
- Start Question 9 in a new booklet

Question 1 – 6	/6
Question 7	/18
Question 8	/18
Question 9	/18
TOTAL	/60

ANSWER QUESTIONS 1-6 ON MULTIPLE CHOICE ANSWER SHEET

For Questions 1-6, answer either A, B, C or D. Choose the best answer for each question. Questions 1-6 are 1 mark each.

Q1. Joanne wants to use the trapezoidal rule to approximate $\int_1^5 f(x)dx$. She decides to use 5 function values. The width h she should use for this calculation is:

(A) $h = 0.8$ (B) $h = 1$

(C) $h = 1.2$ (D) $h = 2$

Q2. Francis is examining a point $(3,5)$ which lies on the curve $f(x)$. She uses the following table:

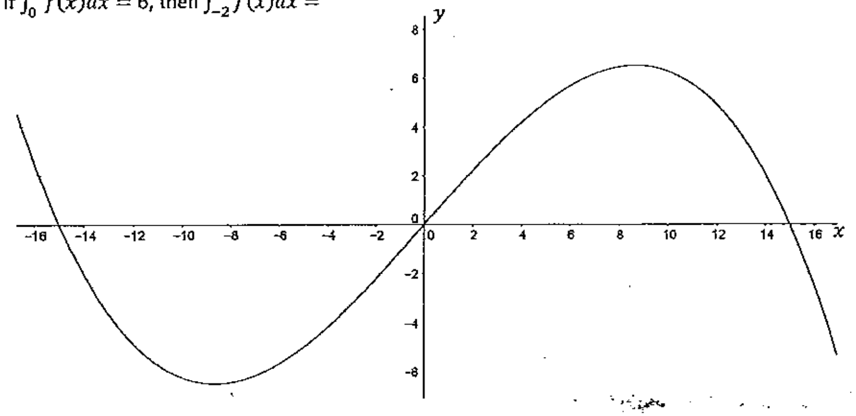
x	2.9	3	3.1
$f''(x)$	-0.5	0	0.5

What conclusion can Francis make about the nature of point $(3,5)$ using the data in the table?

(A) $(3,5)$ is a maximum turning point. (B) $(3,5)$ is a minimum turning point.

(C) $(3,5)$ is an inflexion point. (D) $(3,5)$ is a horizontal inflexion point.

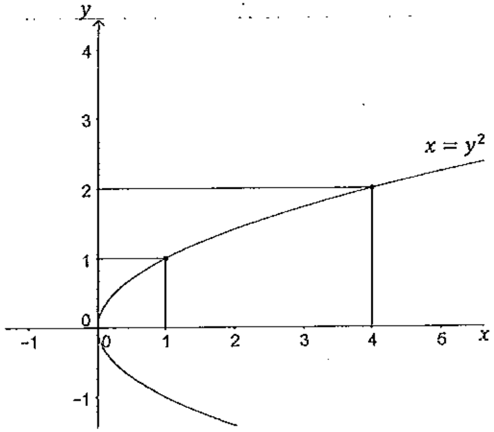
Q3. The diagram below shows a sketch of $y = f(x)$. $f(x)$ is an odd function.
If $\int_0^2 f(x)dx = 6$, then $\int_{-2}^2 f(x)dx =$



(A) 6 (B) 12

(C) 0 (D) 36

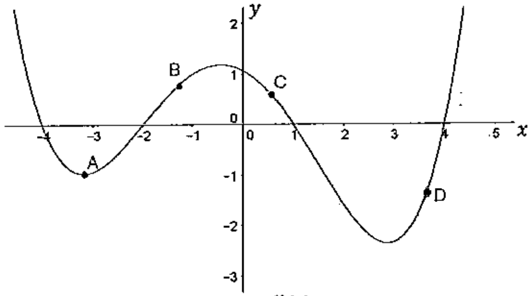
Q4. Which integral will correctly determine the area between the curve $x = y^2$, the y-axis and the lines $y = 1$ and $y = 2$.



(A) $\int_1^4 x^2 dx$ (B) $\int_1^2 y^2 dx$

(C) $\int_1^4 y^2 dy$ (D) $\int_1^2 y^2 dy$

Q5.



For which point on the curve above is $f'(x) > 0$ and $f''(x) < 0$?

(A) Point A (B) Point B

(C) Point C (D) Point D

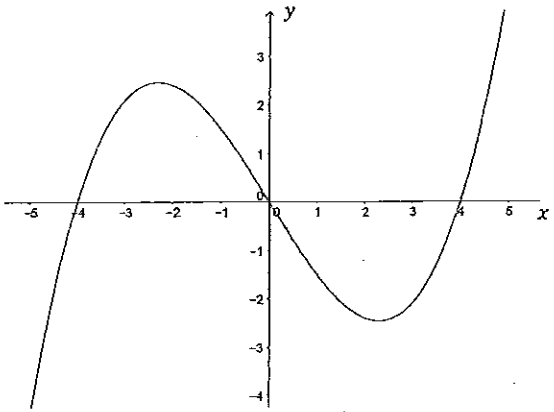
Q6. Find $\int (3x + 2)^4 dx$

(A) $\frac{(3x+2)^5}{15} + c$ (B) $\frac{(3x+2)^5}{5} + c$

(C) $\frac{(3x+2)^5}{3} + c$ (D) $5(3x + 2)^5 + c$

Question 7 (18 marks) START A NEW BOOKLET **Marks**

(a) For the function $f(x)$ shown below, copy or trace the curve into your answer booklet. On the same set of axes, sketch the gradient function for this curve. 2



(b) Find the first derivative of the following:

(i) $y = (5x^2 - 14)^7$ 2

(ii) $y = x^2\sqrt{x - 1}$ 2

(iii) $f(x) = \frac{2x^2+1}{x-1}$ 2

(c) For the function $f(x) = 2x^3 - 3x^2 + 2$

(i) Find the coordinates of any stationary points and determine their nature. 3

(ii) Find the coordinates of any inflexion point(s). 2

(iii) Sketch the curve, labelling stationary point(s), inflexion point(s) and the y-intercept. 2

(d) Use Simpson's Rule to find an approximation of $\int_2^6 \sqrt{x}\, dx$ using 4 subintervals. 3

END OF QUESTION 7

Question 8 **(18 marks)** **START A NEW BOOKLET** **Marks**

(a) Find the following integrals:

(i) $\int_0^1 (5x^4 + x^2)dx$ 2

(ii) $\int \frac{5x^3+2x^2}{x} dx$ 2

(iii) $\int (2x-4)(x+1)dx$ 2

(b) $f(x)$ is a curve that passes through the point (2,7) 3
If $f'(x) = 3x^2 - 4x + 2$, find $f(-1)$

(c) A truck driver is planning his journey from Sydney to Melbourne, which is a distance of 1000km
The cost of running a truck at an average speed v km/h is $49 + \frac{1}{100}v^2$ dollars per hour.

(i) Write an expression for the time taken for the journey in terms of v. 1

(ii) Show that the cost C (dollars) of the journey of 1000km at a speed v km/h is given by 1

$$C = 10v + \frac{49\,000}{v}$$

(iii) Determine the speed v that will minimise the cost of the journey. 3

(iv) What is the minimum cost of the journey? 1

(d) Find the volume of the solid of revolution when $y = x^2 + x$ is rotated about the x-axis between $x = 1$ and $x = 3$. 3

END OF QUESTION 8

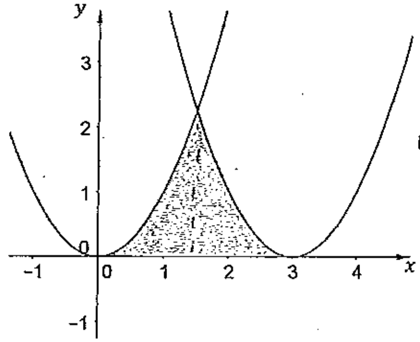
Question 9 **(18 marks)** **START A NEW BOOKLET** **Marks**

(a) (i) Show that the curves $y = x^2$ and $y = 8 - x^2$ intersect when $x = -2$ and $x = 2$. 2

(ii) Sketch the curve $y = x^2$ and $y = 8 - x^2$ on the same set of axes. 1

(iii) Hence, or otherwise, find the area bounded by the curves $y = x^2$ and $y = 8 - x^2$. 3

(b) The following sketch shows the curves $y = x^2$ and $y = (x-3)^2$. 3



Calculate the area bounded by the curves $y = x^2$, $y = (x-3)^2$ and the x-axis.

(c) State the domain where the function $f(x) = \frac{x^2+1}{x^2-1}$ is decreasing. 3

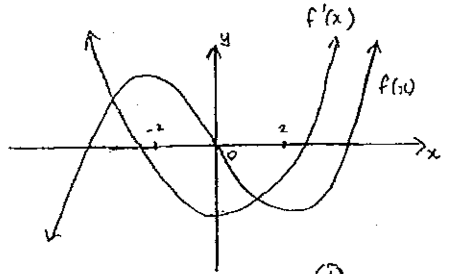
(d) For the curve $y = 8x^3 - x^4$

(i) Find the coordinates of any stationary points and determine their nature. 4

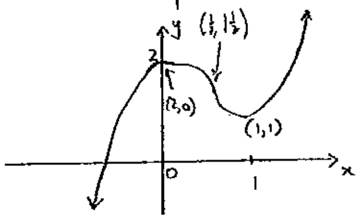
(ii) Sketch the curve, clearly labelling any stationary point(s), the x-intercept and y-intercept. 2

END OF ASSESSMENT TASK

Year 12 Mathematics 2017 : Task 2 Solutions

Qn	Solutions	Marks	Comments: Criteria
1	5 function values = 4 subintervals $h = \frac{5-1}{4}$ $= 1 \Rightarrow$ B		
2	C - (3,5) is an inflexion point since concavity changes		
3	C - $\int_{-2}^{2} f(x) = 0$ since $f(x)$ is odd		
4	D $\quad$ Area $= \int_1^2 y^2\,dy$		
5	B		
6	A		
7	(a) 	1 1	shape x-intercepts
7	(b) (i) $y' = 7(5x^2-14)^6 (10x)$ ① ① $= 70x(5x^2-14)^6$		(No penalties for simplification errors)
	(ii) $y = x^2\sqrt{x-1}$ Let $u = x^2$ $\quad v = (x-1)^{\frac{1}{2}}$ $u' = 2x$ $\quad v' = \frac{1}{2}(x-1)^{-\frac{1}{2}}$	1	Fully differentiate u & v
	$\frac{dy}{dx} = uv' + vu'$ $= \frac{x^2}{2\sqrt{x-1}} + 2x\sqrt{x-1}$	1	Correct application of product rule (No penalties for simplification errors)

Qn	Solutions	Marks	Comments: Criteria
7	(b) (iii) $f(x) = \frac{2x^2+1}{x-1}$		
	Let $u = 2x^2+1$ $\quad v = x-1$ $u' = 4x$ $\quad v' = 1$	①	Fully differentiate u & v
	$f'(x) = \frac{4x(x-1) - (2x^2+1)}{(x-1)^2}$ $= \frac{4x^2-4x-2x^2-1}{(x-1)^2}$ $= \frac{2x^2-4x-1}{(x-1)^2}$	①	Correct application of quotient rule
7	(c) (i) $f(x) = 2x^3 - 3x^2 + 2$ $f'(x) = 6x^2 - 6x$ $= 6x(x-1)$ $f''(x) = 12x - 6$ $= 6(2x-1)$		
	Stationary points when $f'(x) = 0$ i.e. $6x(x-1) = 0$		
	$\therefore x = 0$ $\quad$ or $\quad x = 1$	1	Finding solutions to $f'(x) = 0$ ($\frac{1}{2}$ each)
	$f(0) = 2$ $\quad f(1) = 1$	1	Finding coordinates ($\frac{1}{2}$ each)
	For $(0,2) \Rightarrow f''(0) = -6$ $\quad \therefore (0,2)$ is a maximum turning point For $(1,1) \Rightarrow f''(1) = 6$ $\quad \therefore (1,1)$ is a minimum turning point	1	Classifying turning points ($\frac{1}{2}$ each)

<table>
<tr><th>Qn</th><th>Solutions</th><th>Marks</th><th>Comments: Criteria</th></tr>
<tr><td>7</td><td>(c) (ii) Inflexion point when $f''(x)=0$
i.e. $12x-6=0$
$12x=6$
$x=\frac{1}{2}$
$f(\frac{1}{2}) = 2(\frac{1}{8})-3(\frac{1}{4})+2$
$=1\frac{1}{2}$</td><td></td><td>($\frac{1}{2}$ mark for finding $x=\frac{1}{2}$ only)</td></tr>
<tr><td></td><td>$\therefore (\frac{1}{2}, 1\frac{1}{2})$ is a possible inflexion point</td><td>1</td><td>Identify $(\frac{1}{2}, 1\frac{1}{2})$</td></tr>
<tr><td></td><td><table><tr><td>x</td><td>$\frac{1}{4}$</td><td>$\frac{1}{2}$</td><td>$\frac{3}{4}$</td></tr><tr><td>$f''(x)$</td><td>-ve</td><td>0</td><td>+ve</td></tr></table></td><td></td><td></td></tr>
<tr><td></td><td>Since concavity changes, $(\frac{1}{2}, 1\frac{1}{2})$ is an inflexion point</td><td>1</td><td>Showing change in concavity</td></tr>
<tr><td>7</td><td>(c)(iii)
</td><td>$\frac{1}{2}$
$1\frac{1}{2}$</td><td>y-intercept
shape correct</td></tr>
<tr><td>7</td><td>(d) $h=\frac{6-2}{4}$
$=1$
$\therefore \int_2^6 \sqrt{x} \doteqdot \frac{1}{3}\left[\sqrt{2}+\sqrt{6}+4(\sqrt{3}+\sqrt{5})+2(\sqrt{4})\right]$
$\doteqdot 7.91$ (2dp)</td><td>1
1

1</td><td>correct h
correct application of Simpson's rule
correct calculation</td></tr>
</table>

<table>
<tr><th>Qn</th><th>Solutions</th><th>Marks</th><th>Comments: Criteria</th></tr>
<tr><td>8</td><td>(a) (i) $\int_0^1 (5x^4+x^2)\,dx = \left[x^5+\frac{1}{3}x^3\right]_0^1$
$=\left(1^5+\frac{1}{3}(1)^3\right)-(0+0)$
$=1\frac{1}{3}\quad(=\frac{4}{3})$</td><td>1
$\frac{1}{2}$
$\frac{1}{2}$</td><td>primitive function</td></tr>
<tr><td>8</td><td>(a) (ii) $\int \frac{5x^3+2x^2}{x}\,dx = \int(5x^2+2x)\,dx$
$=\frac{5}{3}x^3+x^2+c$</td><td>1
1</td><td>$\frac{1}{2}$ for mechanical errors</td></tr>
<tr><td>8</td><td>(a) (iii) $\int(2x-4)(x+1)\,dx = \int(2x^2-2x-4)\,dx$
$=\frac{2x^3}{3}-x^2-4x+c$</td><td>1
1</td><td></td></tr>
<tr><td>8</td><td>(b) $f'(x)=3x^2-4x+2$
$\therefore f(x)=\int(3x^2-4x+2)\,dx$
$=x^3-2x^2+2x+c$
$f(2)=7$
So $7=2^3-2(2)^2+2(2)+c$
$c=7-8+8-4$
$=3$
So $f(x)=x^3-2x^2+2x+3$
$\therefore f(-1)=(-1)^3-2(-1)^2+2(-1)+3$
$=-2$</td><td>

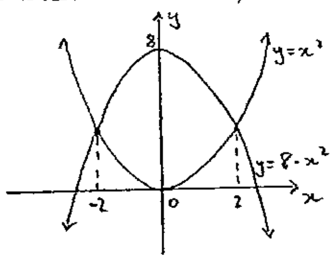
1

1

1</td><td>$\frac{1}{2}$ marks off for mechanical errors</td></tr>
</table>

Qn	Solutions	Marks	Comments: Criteria
8	(c) (i) Time taken $= \frac{\text{distance}}{\text{speed}}$		
	$= \frac{1000}{v}$	1	
	(ii) $C = \left(49 + \frac{v^2}{100}\right) \times$ time taken		
	$= \left(49 + \frac{v^2}{100}\right) \times \frac{1000}{v}$		
	$= \frac{49\,000}{v} + 10v$		
	$= 10v + \frac{49\,000}{v}$	1	
	(iii) For minimum turning point, $C' = 0$		
	$C = 10v + 49\,000v^{-1}$		
	$\frac{dC}{dv} = 10 - \frac{49\,000}{v^2}$	1	differentiation
	$C' = 0$ when $0 = 10 - \frac{49\,000}{v^2}$		
	$\frac{49\,000}{v^2} = 10$		
	$4900 = v^2$		(no penalty for justifying +ve solution)
	$\therefore v = \pm 70$		
	$= +70$ only, since $v > 0$	1	Showing $v = 70$
	when $v = 70$, test C'': If $C' = 10 - 49\,000v^{-2}$, $C'' = +98\,000v^{-3}$		
	when $v = 70$, $C'' > 0$		
	So $v = 70$ km/h is a minimum turning point	1	Showing min. T.P
	Hence $v = 70$ km/h will minimise cost of journey		

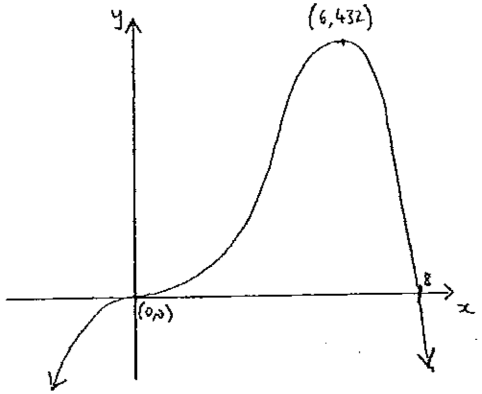
Qn	Solutions	Marks	Comments: Criteria
8	(c) (iv) Minimum cost when $v = 70$.		
	i.e. $C = 10(70) + \frac{49\,000}{(70)}$		
	$= \$1400$	1	
8	(d) Volume $= \pi \int_1^3 y^2\,dx$		
	$= \pi \int_1^3 (x^2+x)^2\,dx$		
	$= \pi \int_1^3 (x^4 + 2x^3 + x^2)\,dx$	1	
	$= \pi \left[\frac{x^5}{5} + \frac{2x^4}{4} + \frac{x^3}{3}\right]_1^3$	1	
	$= \pi \left[\left(\frac{243}{5} + \frac{162}{4} + \frac{27}{3}\right) - \left(\frac{1}{5} + \frac{2}{4} + \frac{1}{3}\right)\right]$		No marks if integrated incorrectly
	$= \pi \left[\frac{981}{10} - \frac{31}{30}\right]$		
	$= \frac{1456\pi}{15}$ units^3	1	
	$= 304.94$ units^3 (2dp)		

Qn	Solutions	Marks	Comments: Criteria
9	(a) (i) Solve simultaneously		
	$y = x^2 \dots (1)$		
	$y = 8 - x^2 \dots (2)$		
	(1)→(2) $x^2 = 8 - x^2$	1	
	$2x^2 = 8$		
	$x^2 = 4$		
	$x = \pm 2$	1	
	∴ Point of intersection when $x = 2$, $x = -2$		
9	(a) (ii) 		
	(a)(iii) Area $= \int_{-2}^{2}(8-x^2)\,dx - \int_{-2}^{2} x^2\,dx$	1	
	$= \left[8x - \frac{x^3}{3}\right]_{-2}^{2} - \left[\frac{x^3}{3}\right]_{-2}^{2}$	1	
	$= \left[\left(16 - \frac{8}{3}\right) - \left(-16 - \frac{-8}{3}\right)\right] - \left[\frac{8}{3} - \frac{-8}{3}\right]$		
	$= 26\frac{2}{3} - 5\frac{1}{3}$		
	$= 21\frac{1}{3}$ units²	1	

Qn	Solutions	Marks	Comments: Criteria
9	(b) To find point of intersection, solve simultaneously.		
	$y = x^2 \dots (1)$		
	$y = (x-3)^2 \dots (2)$		
	(1)→(2) $x^2 = (x-3)^2$		
	$\cancel{x^2} = \cancel{x^2} - 6x + 9$		
	$6x = 9$		
	$x = \frac{9}{6}$		
	$x = 1\frac{1}{2}$	$\frac{1}{2}$	
	Area $= \int_0^{1\frac{1}{2}} x^2\,dx + \int_{1\frac{1}{2}}^{3} (x-3)^2\,dx$	1	
	$= \left[\frac{x^3}{3}\right]_0^{1\frac{1}{2}} + \left[\frac{(x-3)^3}{3}\right]_{1\frac{1}{2}}^{3}$	$\frac{1}{2}$	
	$= \left[\frac{(1\frac{1}{2})^3}{3} - 0\right] + \left[\frac{0}{3} - \frac{(1\frac{1}{2}-3)^3}{3}\right]$		
	$= 1\frac{1}{8} + 1\frac{1}{8}$		
	$= 2\frac{1}{4}$ units²	1	
9	(c) $f(x) = \frac{x^2+1}{x^2-1}$		
	Let $u = x^2+1$, $v = x^2-1$		
	$u' = 2x$, $v' = 2x$		
	So $f'(x) = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}$	1	$-\frac{1}{2}$ if not justifying $f'(x) < 0 \Rightarrow -4x < 0$ because $(x^2-1)^2 > 0$
	$= \frac{2x\left[x^2-1-x^2-1\right]}{(x^2-1)^2}$		
	$= \frac{2x(-2)}{(x^2-1)^2}$		
	$= \frac{-4x}{(x^2-1)^2}$	1	
	$f(x)$ is decreasing when $f'(x) < 0$ i.e. $\frac{-4x}{(x^2-1)^2} < 0$		
	Note denominator is always positive (when $x \neq 1$), So $f'(x) < 0$ when $-4x < 0$ i.e. $x > 0$	1	

∴ Function is decreasing for $0 < x < 1$ ($\frac{1}{2}$) and $x > 1$ ($\frac{1}{2}$)

($\frac{1}{2}$) if only the $x > 0$

Qn	Solutions	Marks	Comments: Criteria
9	(d) (i) $y = 8x^3 - x^4$ $y' = 24x^2 - 4x^3$ $y'' = 48x - 12x^2$ Stationary points when $y' = 0$ ie. $24x^2 - 4x^3 = 0$ $4x^2(6 - x) = 0$ $x = 0$ or $x = 6$ When $x = 0$, $y = 0 \Rightarrow (0, 0)$ $y'' = 0 \rightarrow$ po test When $x = 6$, $y = 432 \Rightarrow (6, 432)$ $y'' = -144$ $\therefore (6, 432)$ is a maximum turning point	1 $1\frac{1}{2}$ 1 $\frac{1}{2}$	Finding (0,0) Classifying (0,0) Finding (6,432) Classifying (6,432)
	| x | -1 | 0 | 1 | | y'' | $-$ve | 0 | $+$ve | Since concavity changes, (0,0) is an inflexion point	1	-1 if not showing change in concavity (or gradient)
	(d)(ii) For x-intercepts $y = 0$ ie $8x^3 - x^4 = 0$ $x^3(8 - x) = 0$ $\therefore x = 0, 8$		
		1	$-\frac{1}{2}$ if no $x = 8$ (x-intercept)