Student Number:________________

St. Catherine's School

Waverley

August 2010

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Complete each section in a separate booklet

- Attempt Questions 1 – 7
- All questions are of equal value
- Questions to presented in Sections:

Booklet 1 – Questions 1-2
Booklet 2 – Questions 3-4
Booklet 3 – Questions 5-6
Booklet 4 – Question 7

- **Total Marks – 84**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ if\ n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(x+\sqrt{x^2-a^2}),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln(x+\sqrt{x^2+a^2})$$

NOTE : $\ln x = \log_e x,\quad x>0$

Total marks -120
Attempt Questions 1-10
All questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 1 (12 marks) **(Use Writing Booklet 1)** **Marks**

(a) Differentiate $\tan^{-1}\frac{x}{2}$ 1

(b) Given that $\cos\alpha = \frac{5}{13}$ find the value of $\cos 2\alpha$ 2

(c) Consider the cubic equation $x^3 - 7x - 6 = 0$. If two roots of this equation are -1 and 3, find the third root. 1

(d) Find $\frac{d}{dx}(x^2 e^{-x^2})$

(e) The acute angle between the lines $y = (m+2)x$ and $y = mx$ is $45°$

(i) Show that $\left|\frac{2}{m^2+2m+1}\right| = 1$ 1

(ii) Hence find the possible values for m 2

(f) Use the substitution $u = 1 + \ln x$ to evaluate 3

$$\int_1^e \frac{1}{x}(1+\ln x)^3\,dx$$

Question 2 (12 marks) **(Use Writing Booklet 1)** **Marks**

(a) The variable point $P(t+1,\ 2t^2+1)$ lies on a parabola. 2
Find the Cartesian equation of the parabola.

(b) Solve the equation $\frac{2x+3}{x-4} \le 1$ 2

(c) Evaluate $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}$ 2

(d) Evaluate $\int_0^{\pi} 2\cos^2\frac{x}{4}\,dx$ 2

(e) Find the coefficient of x^5 in the binomial expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$ 2

(f) $A(-3,4)$ and $B(1,2)$ are two points. Find the coordinates of the point (x,y) which divides the interval AB *externally* in the ratio **3 : 1** 2

Question 3 (12 marks) **(Use Writing Booklet 2)** **Marks**

(a) Consider the function $f(x)=\dfrac{x-2}{x-1}$

(i) Show that the function is increasing for all values of x in the function's domain. **2**

(ii) Sketch the graph of the function showing clearly any intercepts on the coordinate axes and the equations of any asymptotes. **2**

(iii) Find the equation of the inverse function $f^{-1}(x)$ **1**

(iv) Deduce, from your result in (iii), that the graph of the function $f(x)$ is symmetrical about the line $y=x$ **1**

(b) Consider the function $y=\dfrac{1}{2}\cos^{-1}(x-1)$

(i) Find the domain and range of the function. **2**

(ii) Sketch *neatly* the graph of the function, showing clearly the coordinates of the end points. **1**

(iii) The region in the first quadrant bounded by the curve $y=\dfrac{1}{2}\cos^{-1}(x-1)$ and the coordinate axes is rotated through $360°$ about the y axis.

Find the volume of the solid of revolution, giving your answer in simplest exact form **3**

Question 4 (12 marks) **(Use Writing Booklet 2)** **Marks**

(a) Consider the equation $4e^{-x}-\tan x+1=0$ which has a root $x=\alpha$

(i) Show that $1<\alpha<1.5$ **1**

(ii) Using $x=1$ as a first approximation of the root use one application of Newton's method to find a better approximation of this root. **3**

Write your answer correct to 4 significant figures

(b) The rate of growth of a bacteria colony is proportional to the excess of the colony's population over 5000 and is given by

$\dfrac{dN}{dt}=k(N-5000)$ where k is a positive constant and t is the time in days

(i) Show that $N=5000+Ae^{kt}$ is a solution of the above differential equation. **1**

(ii) If the initial population is 15000, and it reaches 20000 after 2 days, find the value of A and k **3**

(iii) Hence, calculate the expected population after a further 5 days. **1**

(c) A committee of 3 women and 7 men are to be seated randomly at a round table

(i) What is the probability that the three women are seated together. **1**

(ii) The committee elects a President and a Vice President. What is the probability that they are seated opposite one another. **2**

Question 5 (12 marks) **(Use Writing Booklet 3)** **Marks**

(a) Use mathematical induction to prove to prove that for all $n \geq 2$ **4**

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+.....+\frac{(n-1)}{n!}=1-\frac{1}{n!}$$

(b) Show that $\frac{d^2x}{dt^2}=\frac{d}{dx}(\frac{1}{2}v^2)$ **2**

(c) A particle moves in a straight line so that, when x m from an origin, its acceleration is given by $-9e^{-2x}$ ms^{-2}. Initially, it is at the origin where the velocity is 3 ms^{-1}.

(i) Determine the velocity as a function of x in simplest form, justifying any choice you may have to make. **2**

(ii) Determine x as a function of t, where t is the number of seconds after it leaves the origin. **2**

(iii) Find the particle's velocity and acceleration 3 seconds after leaving the origin. **2**

Question 6 (12 marks) **(Use Writing Booklet 3)** **Marks**

(a) A particle's motion is defined by the equation; $v^2=12+4x-x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}. Initially the particle 6 metres to the right of the origin.

(i) Show that the particle is moving in Simple Harmonic Motion **1**

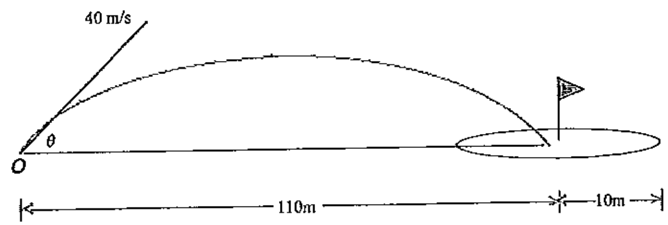
(ii) Find the centre, period and amplitude of the motion **3**

(iii) The displacement of the particle at any time t is given by the equation. **2**

$$x=a\sin(nt+\alpha)+b$$

Find the values of α and b, given $0 \leq \theta \leq 2\pi$

(b) A golfer hits a golf ball from a point O with velocity 40 m/s at an angle θ to the horizontal. The ball travels in a vertical plane where the acceleration due to gravity is 10 ms^{-2}.



(i) Write down expressions for the horizontal displacement x metres, and the vertical displacement y metres, of the golf ball from O after time t seconds. **1**

(ii) Hence show that the horizontal range, R metres, of the golf ball until it returns to ground level is given by $R=160\sin 2\theta$ **2**

(iii) The golfer is aiming over horizontal ground at a circular green of radius 10 metres, with the centre of the green 110 metres from O. Find the possible set of values of θ for the ball to land on the green, giving your answers correct to the nearest degree. **3**

Question 7 (12 marks) **(Use Writing Booklet 4)** — **Marks**

(a) Four dice are rolled simultaneously. Any die showing a 6 on the uppermost face is set aside, and the remaining dice are rolled again.
(Note: a die has six faces numbered 1 to 6 with each face equally likely to fall uppermost)

(i) Find the probability (correct to 2 decimal places) that after the first roll of the dice, exactly one of the four dice is showing a 6 on the uppermost face. **1**

(ii) Find the probability (correct to 2 decimal places) that after the second roll of the dice exactly two of the four dice are showing a 6 on the uppermost face. **3**

(b) (i) Show that $\dfrac{x^n + x^{n+2}}{1+x^2} = x^n$ **1**

(ii) An Integral is defined by $I_n = \displaystyle\int_0^1 \frac{x^n}{1+x^2}\,dx$, for $n \geq 0$

1. Evaluate I_0 **1**

2. Use part (i) to show that $I_n + I_{n+2} = \dfrac{1}{n+1}$ **2**

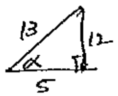
3. Evaluate I_2 **1**

(c) (i) Show that $\dfrac{\binom{3n}{k}}{\binom{3n}{k-1}} = \dfrac{3n-k+1}{k}$ **1**

(ii) Find the greatest coefficient in the expansion of $\left(1+\dfrac{x}{2}\right)^{3n}$, **2**

(n a positive integer).

END of PAPER

Q	Solution	Marks/Comments
1a)	$y = \tan^{-1}\left(\frac{x}{2}\right)$	
	$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2}$	
	$= \frac{4}{4+x^2} \cdot \frac{1}{2}$	
	$= \frac{2}{4+x^2}$	1
b)	$\cos\alpha = \frac{5}{13}$ 	
	$\cos 2\alpha = 2\cos^2\alpha - 1$ or use other definitions of $\cos 2\alpha$	1
	$= 2\left(\frac{5}{13}\right)^2 - 1$	
	$= -\frac{119}{169}$	1
c)	$x^3 - 7x - 6 = 0$	
	$x^3 + 0x^2 - 7x - 6 = 0$	
	(−) (+) (−)	
	let roots be $-1, 3, \alpha$	
	$\therefore -1 + 3 + \alpha = 0$ (or $-3\alpha = 6$	
	$\therefore \alpha = -2$ $\alpha = -2$)	1
d)	$\frac{d}{dx}(x^2 e^{-x^2})$ using product rule $u = x^2$ $v = e^{-x^2}$	
	$u' = 2x$ $v' = -2xe^{-x^2}$	1
	$\therefore \frac{d}{dx}(x^2 e^{-x^2}) = (2xe^{-x^2}) + (-2x^3 e^{-x^2})$ [use $vu' + uv'$ or $uv' + vu'$]	
	$= 2xe^{-x^2}(1-x^2)$	1
	OR $= \frac{2x(1-x^2)}{e^{x^2}}$	

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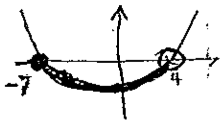
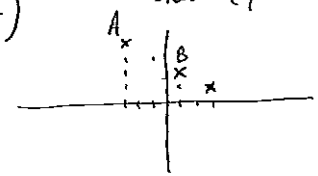
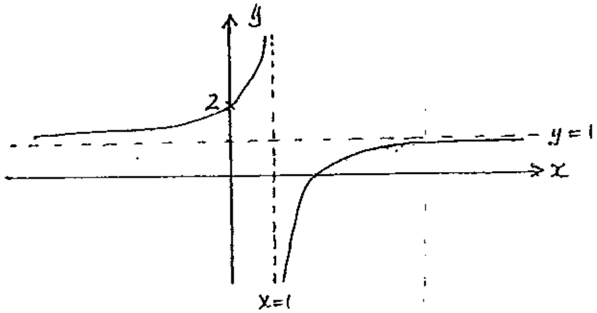
Q	Solution	Marks/Comments
1e)	(i) $\tan 45^\circ = \left\lvert \frac{m+2-m}{1+m(m+2)} \right\rvert$	
	$\therefore 1 = \left\lvert \frac{2}{m^2+2m+1} \right\rvert$	1
	(ii) $\frac{2}{m^2+2m+1} = \pm 1$	
	$m^2+2m+1 = 2$ or $-m^2-2m-1 = 2$	1
	$m^2+2m-1 = 0$ $\therefore m^2+2m+1 = -2$	
	$m = \frac{-2 \pm \sqrt{8}}{2}$ $m^2+2m+3 = 0$	$-\frac{1}{2}$ for not showing (case 2)
	$b^2 - 4ac < 0 \therefore$ no sols	
	$\therefore m = -1 \pm \sqrt{2}$	1
	or $m = 0.41$ or -2.41 (to 2dp)	
f)	$I = \int_1^e \frac{1}{x}(1+\ln x)^3 dx$ $u = 1 + \ln x$	
	$du = \frac{1}{x} dx$	1
	for $x = e$ $u = 2$	
	$x = 1$ $u = 1$	1
	$\therefore I = \int_1^2 u^3 du$	
	$= \left[\frac{u^4}{4}\right]_1^2$	
	$= \frac{16}{4} - \frac{1}{4}$	
	$= 3\frac{3}{4}$	1

Q	Solution	Marks/Comments
2a)	$P(t+1, 2t^2+1)$ $\therefore x = t+1$ — ① $\quad y = 2t^2+1$ — ② from ① $t = x-1$ sub in ② $y = 2(x-1)^2+1$ $= 2(x^2-2x+1)+1$ $= 2x^2-4x+3$	1 1
b)	$\dfrac{2x+3}{x-4} \le 1 \quad x \neq 4$ $\times$ by $(x-4)^2$ $(2x+3)(x-4) \le (x-4)^2$ $2x^2-5x-12 \le x^2-8x+16$ $x^2+3x-28 \le 0$ $(x-4)(x+7) \le 0$  $-7 \le x < 4$	1 $\frac{1}{2}$ off for $x \le 4$ 1
c)	$\lim\limits_{x \to 0} \dfrac{\sin 3x}{5x} = \lim\limits_{x \to 0} \left(\dfrac{3}{5} \cdot \dfrac{\sin 3x}{3x}\right)$ $= \dfrac{3}{5} \lim\limits_{x \to 0} \dfrac{\sin 3x}{3x}$ $= \dfrac{3}{5}$	1 1
d)	$I = \int_0^{\pi} 2\cos^2 \frac{x}{4}\, dx$ $\quad$ Note $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\therefore \cos^2 \frac{x}{4} = \frac{1}{2}\left(1+\cos\frac{x}{2}\right)$ $\therefore I = \int_0^{\pi}\left(1+\cos\frac{x}{2}\right)dx$ $\quad$ or $2\cos^2\theta = \cos 2\theta + 1$ $= \left[x + 2\sin\frac{x}{2}\right]_0^{\pi}$ $\quad$ $\therefore 2\cos^2\frac{x}{4} = \cos 2\left(\frac{x}{4}\right)+1$ $= \cos\frac{x}{2}+1$ $= \pi + 2$	1 1
2e)	Note $T_{k+1} = {}^nC_k a^{n-k} b^k$ for $\left(x^2+\frac{2}{x}\right)^{10}$ $\therefore T_{k+1} = {}^{10}C_k (x^2)^{10-k}\left(\frac{2}{x}\right)^k$ $= {}^{10}C_k\, x^{20-2k} \cdot \frac{2^k}{x^k}$ $= {}^{10}C_k\, x^{20-3k} \cdot 2^k$ $\therefore 20-3k = 5$ $15 = 3k$ $5 = k$ $\therefore T_{k+1} = {}^{10}C_5 (x^2)^5\left(\frac{2}{x}\right)^5$ $\therefore$ Coefficient is ${}^{10}C_5 \cdot 2^5$ $(= 8064)$	1 $\frac{1}{2}$ $\frac{1}{2}$
f)	$A(-3,4)$ $\quad$ $B(1,2)$ $\quad$ (x,y) $\left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$ $\quad$ 3 : -1 $\therefore P\left(\dfrac{3+3}{2}, \dfrac{-4+6}{2}\right)$ $\quad$ check (optional)  $\therefore P(3,1)$	1 1

Q	Solution	Marks/Comments
3 a)	$f(x) = \dfrac{x-2}{x-1} \quad (x \neq 1)$	
(i)	$f'(x) = \dfrac{(x-1)-(x-2)}{(x-1)^2}$	1
	$= \dfrac{1}{(x-1)^2} > 0$ for all $x \neq 1$ since $(x-1)^2 > 0$	1
(ii)		2
(iii)	let $y = \dfrac{x-2}{x-1} \; (= f(x))$	
	inverse is $x = \dfrac{y-2}{y-1}$	
	$xy - x = y - 2$	
	$xy - y = x - 2$	1
	$y(x-1) = x-2$	
	$y = \dfrac{x-2}{x-1}$	
	$\therefore f^{-1}(x) = \dfrac{x-2}{x-1}$	
(iv)	the function is the inverse of itself	
	$\therefore$ the function is symmetrical about $y=x$	1

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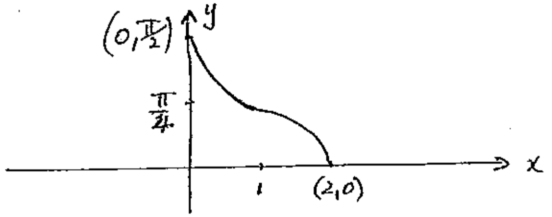
Q	Solution	Marks/Comments
3 b)	$y = \frac{1}{2}\cos^{-1}(x-1)$	
(i)	D_f: $-1 \le x-1 \le 1$	
	$0 \le x \le 2$	1
	R_f: $0 \le y \le \frac{\pi}{2}$	
	Note: $0 \le \cos^{-1}(x-1) \le \pi$	1
	$\therefore 0 \le \frac{1}{2}\cos^{-1}(x-1) \le \frac{\pi}{2}$	
(ii)		1
(iii)	$y = \frac{1}{2}\cos^{-1}(x-1)$	
	$\therefore \cos^{-1}(x-1) = 2y$	
	$x - 1 = \cos 2y$	1
	$x = 1 + \cos 2y$	
	$V = \pi \int_0^{\frac{\pi}{2}} (1+\cos 2y)^2 \, dy$	
	$= \pi \int_0^{\frac{\pi}{2}} (1 + 2\cos 2y + \cos^2 2y) \, dy$	
	$= \pi \int_0^{\frac{\pi}{2}} \left[1 + 2\cos 2y + \frac{1}{2}(1+\cos 4y)\right] dy$	1
	$= \pi \int_0^{\frac{\pi}{2}} \left(\frac{3}{2} + 2\cos 2y + \frac{1}{2}\cos 4y\right) dy$	
	$= \pi \left[\frac{3y}{2} + \sin 2y + \frac{1}{8}\sin 4y\right]_0^{\frac{\pi}{2}}$	
	$= \pi\left[\left(\frac{3\pi}{4} + 0 + 0\right) - (0 + 0 + 0)\right]$	
	$= \frac{3}{4}\pi^2$ units^3	1

Q	Solution	Marks/Comments
4a) (i)	$f(x) = 4e^{-x} - \tan x + 1$ $f(1) = 0.91411004 > 0$ $f(1.5) = -12.20889 < 0$ $\therefore 1 < \alpha < 1.5$	1
(ii)	now $f'(x) = -4e^{-x} - \sec^2 x$	1
	$\therefore f'(1) = -4.897036585$ $x_2 = 1 - \frac{f(1)}{f'(1)}$	1
	$x_2 = 1 + \frac{0.91411004}{4.897036585}$ $\therefore x_2 = 1.187$ (4 sig. figs)	1
	Note: $f(1.187) = -0.255801167$ which is closer to zero than $f(1)$ $\therefore$ is a better approximation	
b) (i)	$N = 5000 + Ae^{kt}$ — ① from ① $Ae^{kt} = N - 5000$ Now $\frac{dN}{dt} = Ae^{kt}.k$ $= k(N-5000)$	1
(ii)	when $t=0$ $N = 15000$ $\therefore 15000 = 5000 + Ae^0$ $\therefore A = 10000$	1
	$\therefore N = 5000 + 10000e^{kt}$ when $t = 2$ $N = 20000$ $\therefore 20000 = 5000 + 10000e^{2k}$	1
	$\therefore e^{2k} = \frac{15000}{10000} = 1.5$ $\therefore 2k = \ln 1.5$ $\therefore k = \frac{\ln 1.5}{2} \doteqdot 0.20273$	1

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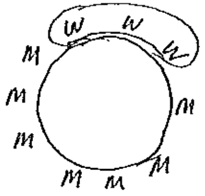
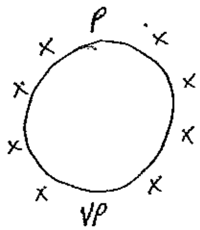
Q	Solution	Marks/Comments
4b) (iii)	when $t = 7$ $N = 5000 + 10000e^{7 \times \frac{\ln 1.5}{2}}$ $= 46335$ (nearest whole number)	1
c) (i)	 If the three women are to sit together there are essentially 8 items to arrange around the table $\therefore P(\text{3 women sit together}) = \frac{7!\,3!}{9!}$ $= \frac{1}{12}$	1
(ii)	 $P(\text{P \& VP sit opp}) = \frac{8!}{9!}$ $= \frac{1}{9}$	1
	Note: Once P & VP have been seated opposite one another there are 8 seats left to fill $\therefore 8!$	1

Q	Solution	Marks/Comments
5a)	Prove. $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\dots+\frac{(n-1)}{n!}=1-\frac{1}{n!}$ $\quad n\geq 2$	
	Step 1 Prove true for $n=2$	
	$LHS=\frac{1}{2!}=\frac{1}{2}$ $\quad RHS=1-\frac{1}{2!}=\frac{1}{2}$	$\frac{1}{2}$
	$\therefore$ true for $n=2$	
	Step 2 Assume true for $n=k$ $(2\leq k<n)$	$\frac{1}{2}$
	i.e. $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\dots+\frac{k-1}{k!}=1-\frac{1}{k!}$	
	Step 3 Aim to prove true for $n=k+1$	
	i.e. A.T.P. $\underbrace{\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\dots+\frac{k-1}{k!}}+\frac{k}{(k+1)!}=1-\frac{1}{(k+1)!}$	$\frac{1}{2}$
	$L.H.S.=1-\frac{1}{k!}+\frac{k}{(k+1)!}$ $\quad$ [L.C.D. $=(k+1)!$]	1
	$=\frac{(k+1)!-(k+1)+k}{(k+1)!}$	
	$=\frac{(k+1)!-1}{(k+1)!}$	
	$=1-\frac{1}{(k+1)!}$	1
	$=R.H.S.$	
	Step 4 $\therefore$ true for $n=k+1$ if true for $n=k$	$\frac{1}{2}$
	Since true for $n=2$ then true for $n=3,4,\dots$	
	$\therefore$ by principal of mathematical induction	
	true for $n\geq 2$ (n integer)	

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Q	Solution	Marks/Comments
5.b)	$\frac{d^2x}{dt^2}=\frac{d}{dt}\cdot\frac{dx}{dt}$	
	$=\frac{dv}{dt}$	
	$=\frac{dv}{dx}\times\frac{dx}{dt}$	1
	$=v\cdot\frac{dv}{dx}$	
	$=\frac{d}{\cancel{dv}}\left(\frac{1}{2}v^2\right)\times\frac{\cancel{dv}}{dx}$	
	$=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	1
c) (i)	Using part (b) $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-9e^{-2x}$	
	$\frac{1}{2}v^2=-9\int e^{-2x}\,dx$	
	$\frac{1}{2}v^2=\frac{9}{2}e^{-2x}+c$	1
	when $x=0$ $v=3$	
	$\therefore \frac{9}{2}=\frac{9}{2}+c$	
	$\therefore c=0$	
	$\therefore \frac{1}{2}v^2=\frac{9}{2}e^{-2x}$	
	$v^2=9e^{-2x}$	
	$v=\pm\sqrt{9e^{-2x}}$	$\frac{1}{2}$
	when $x=0$ $v=+3$ $\therefore$ $v=\sqrt{9e^{-2x}}$	
	$\therefore$ $v=3e^{-x}$	
	Could also justify the positive choice because initially velocity is positive and v never equals zero $\therefore$ must continue positive direction	$\frac{1}{2}$

Q	Solution	Marks/Comments
Q5c (ii)	$\frac{dx}{dt} = 3e^{-x}$ from (i) $\frac{dt}{dx} = \frac{1}{3e^{-x}} = \frac{1}{3}e^{x}$ $t = \frac{1}{3}\int e^{x}\,dx$ $= \frac{1}{3}e^{x} + c$	1
	when $t=0$ $x=0$ $\therefore 0 = \frac{1}{3} + c$ $\therefore c = -\frac{1}{3}$ $\therefore t = \frac{1}{3}e^{x} - \frac{1}{3}$ $3t = e^{x} - 1$ $e^{x} = 3t + 1$	1
	$\therefore x = \ln(3t+1)$	
(iii)	$v = \frac{dx}{dt}$ $\therefore v = \frac{3}{3t+1}$ $= \frac{3}{9+1}$ $= 0.3\, ms^{-1}$ or when $t=3$ $x = \ln 10$ $\therefore v = 3e^{-\ln 10}$ $= 3e^{\ln\frac{1}{10}}$ $= 3 \times \frac{1}{10}$ $= 0.3\, m/s$	1
	$a = \frac{dv}{dt}$ $= -3(3t+1)^{-2}.3$ $= -\frac{9}{(3t+1)^2}$ $= \frac{-9}{(9+1)^2}$ $= -0.09\, ms^{-2}$ $a = -9e^{-2\ln 10}$ $= -9e^{\ln\frac{1}{100}}$ $= -9.\frac{1}{100}$ $= -0.09\, m/s^2$	1

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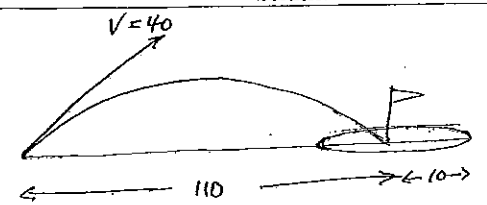
Q	Solution	Marks/Comments
Q6 a) (i)	$v^2 = 12 + 4x - x^2$ now $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $\ddot{x} = \frac{d}{dx}\left(6 + 2x - \frac{x^2}{2}\right)$ $= 2 - x$ $= -1(x-2)$ $\therefore$ SHM. as $a = -n^2(x-b)$	1
(ii)	Centre of motion is $x=2$ where $\ddot{x} = 0$	1
	$n=1$ $\therefore$ Period $T = \frac{2\pi}{n} = 2\pi$	1
	extremities of motion where $v=0$ $\therefore 12 + 4x - x^2 = 0$ $(2+x)(6-x) = 0$ $\therefore x = -2, 6$ $\therefore$ amplitude $= 4$	1
(iii)	$a=4$ $n=1$ and centre of motion is $b=2$ $\therefore x = 4\sin(t+\theta) + 2$	1
	when $t=0$ $x=6$ (given) $\therefore 6 = 4\sin\theta + 2$ $\therefore \sin\theta = 1$ $\theta = \frac{\pi}{2}$	1
	$\therefore x = 4\sin\left(t + \frac{\pi}{2}\right) + 2$	

Q	Solution	Marks/Comments
6 d)		
(i)	$x = 40t\cos\theta \qquad y = 40t\sin\theta - 5t^2$	1
(ii)	When ball returns to horizontal $y = 0$ $\therefore 40t\sin\theta - 5t^2 = 0$ $5t(8\sin\theta - t) = 0$ $t = 0, 8\sin\theta$	1
	when $t = 8\sin\theta \quad x = 320\sin\theta\cos\theta$ $= 160\sin 2\theta$ (Note: $\sin 2\theta = 2\sin\theta\cos\theta$)	1
(iii)	$x = 100$ $16\sin 2\theta = 100$ $\sin 2\theta = \frac{5}{8}$ $2\theta = 39^\circ, 141^\circ$ (nearest degree) $\theta = 19^\circ, 71^\circ$ (nearest degree)	1
	$x = 120$ $160\sin 2\theta = 120$ $\sin 2\theta = \frac{3}{4}$ $2\theta = 49^\circ \; 131^\circ$ (nearest deg) $\theta = 24^\circ \; 66^\circ$ (nearest deg)	1
	$\therefore 19^\circ < \theta < 24^\circ$ OR $66^\circ < \theta < 71^\circ$	1

Q	Solution	Marks/Comments
Q7a) (i)	$P(\text{one six on first roll}) = {}^4C_1\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^1$ $= 0.39$ (2dp)	1
(ii)	$P(\text{2 sixes on first roll } \underline{\text{and}} \text{ no 6's on second roll})$ $= {}^4C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 \times {}^2C_0\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^0 \doteq 0.0804$	1
	$P(\text{1 six on first roll } \underline{\text{and}} \text{ one 6 on second roll})$ $= {}^4C_1\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^1 \times {}^3C_1\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^1 \doteq 0.1340$	1
	$P(\text{no 6's on first roll } \underline{\text{and}} \text{ two 6's on second roll})$ $= {}^4C_0\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^0 \times {}^4C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 = 0.0558$	1
	$\therefore P(\text{two sixes overall}) = 0.0804 + 0.1340 + 0.0558$ $= 0.27$ (2dp)	
b i)	$\dfrac{x^n + x^{n+2}}{1 + x^2} = \dfrac{x^n(1+x^2)}{1+x^2}$ $= x^n$	1
ii)	$I_n = \int_0^1 \frac{x^n}{1+x^2}\,dx$ $I_0 = \int_0^1 \frac{1}{1+x^2}\,dx$ $= \left[\tan^{-1}x\right]_0^1$ $= \frac{\pi}{4}$	1

Q	Solution	Marks/Comments
Q7b (ii) 2.	$I_n = \int_0^1 \frac{x^n}{1+x^2}\,dx \qquad I_{n+2} = \int_0^1 \frac{x^{n+2}}{1+x^2}\,dx$	
	$\therefore I_n + I_{n+2} = \int_0^1 \frac{x^n}{1+x^2}\,dx + \int_0^1 \frac{x^{n+2}}{1+x^2}\,dx$	1
	$= \int_0^1 \frac{x^n + x^{n+2}}{1+x^2}\,dx$	
	$= \int_0^1 x^n\,dx$ from part (i)	
	$= \left[\frac{x^{n+1}}{n+1}\right]_0^1$	
	$= \frac{1}{n+1}$	1
3.	from 2. put $n=0$	
	$\therefore I_0 + I_2 = \frac{1}{0+1}$	
	$\therefore I_2 = 1 - I_0$	
	$= 1 - \frac{\pi}{4}$	1
	OR $I_2 = \int_0^1 \frac{x^2}{1+x^2}\,dx$	
	$= \int_0^1 \frac{1+x^2-1}{1+x^2}\,dx$	
	$= \int_0^1 1\,dx - \int_0^1 \frac{1}{1+x^2}\,dx$	
	$= [x]_0^1 - [\tan^{-1}x]_0^1$	
	$= 1 - \frac{\pi}{4}$	OR 1

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Q	Solution	Marks/Comments
Q7c) (i)	$\frac{\binom{3n}{k}}{\binom{3n}{k-1}} = \frac{\cancel{3n!}}{(3n-k)!\,k!} \times \frac{(3n-k+1)\cancel{!}\,(k-1)!}{\cancel{3n!}}$	1
	$= \frac{3n-k+1}{k}$	
(ii)	$\frac{T_{k+1}}{T_k} = \frac{\binom{3n}{k} a^{3n-k} b^k}{\binom{3n}{k-1} a^{3n-k+1} \cdot b^{k-1}}$	
	$= \frac{\binom{3n}{k}}{\binom{3n}{k-1}} \cdot \frac{b}{a}$	
	$= \frac{3n-k+1}{k} \cdot \frac{\frac{1}{2}}{1}$ from (i)	
	$= \frac{3n-k+1}{2k}$	1
	Now $\frac{3n-k+1}{2k} > 1$ for increasing coefficients	
	$\therefore 3n-k+1 > 2k \quad (k>0)$	
	$\therefore 3k < 3n+1$	
	$k < n + \frac{1}{3}$	
	$\therefore k = n$ for greatest coefficient	
	$\therefore$ greatest coefficient $= {}^{3n}C_n \left(\frac{1}{2}\right)^n$	
	[Note: $T_{k+1} = {}^{3n}C_k\, a^{3n-k} b^k$] $a=1$, $b=\frac{1}{2}$	1