CHRISTIAN BROTHERS' HIGH SCHOOL, LEWISHAM

MATHEMATICS
H.S.C.ASSESSMENT TASK
April 2010

Topics covered:	Sequences and Series- Exponentials and Indices involving arithmetic / algebra Logarithms involving arithmetic / algebra Integrations including primitives, methods of approximation, Definite and Indefinite integrals, Areas under curves and Volumes of rotation around axes

Assessable Outcomes

H2
H3
H4
H5
H6
H8

Time allowed —90 minutes

All necessary working must be shown
Board approved calculators may be used
Begin each new question in a new booklet

Question 1 (*New Booklet*)

a) Find $\int (2x+3)^{10}dx$. (2)

b) Find a primitive function of $6-x^{-3}$. (2)

c) Evaluate $\int_1^2 \frac{1}{x^2}dx$. (2)

d) Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of

i) $\log_a\left(\frac{b}{c}\right)$ (1)

ii) $\log_a(bc)^2$. (2)

e) (i) For what values of r does the geometric series $a+ar+ar^2+\ldots$ have a limiting sum? (1)

For these values of r write down the limiting sum (1)

(ii) Find a geometric series with a common ratio $\frac{1}{w}$ that has a limiting sum $\frac{1}{1-w}$. (1)

Question 2 (*New Booklet*)

a) Find an expression for the nth term of the series $2+7+12+\ldots$ (2)

b) Evaluate $\sum_{r=0}^{5} 2r-1$ (2)

A layer of plastic cuts out 15% of the light and lets through the remaining 85% .

(i) Show that two layers of the plastic let through 72.25% of the light.

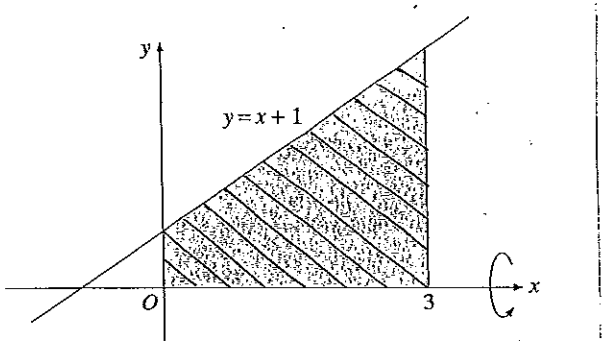
(ii) How many layers of the plastic are required to cut out at least 90% of the light?

d) Evaluate $\int_1^9 (1+\sqrt{x})dx$. (3)

e) Solve the equation $2\ln x = \ln(5+4x)$ (2)

Question 3 (*New Booklet*)

a)



(i) Calculate the area of the shaded region (3)

(ii) This region is rotated about the x axis to form a solid. Find the volume of the solid. (3)

b) The third term of an arithmetic series is 32 and the sixth term is 17.

i) Find the common difference (1)

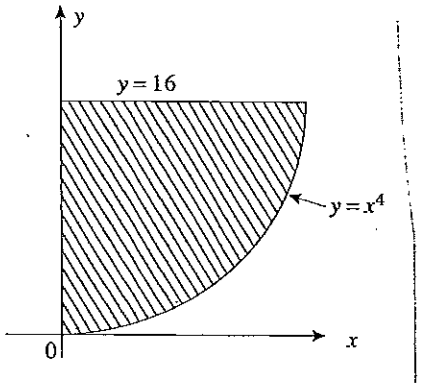
ii) Find the sum of the first ten terms. (2)

c) The positive multiples of 7 are 7, 14, 21....

i) What is the largest multiple of 7 less than 1000? (1)

What is the sum of the positive multiples of 7 which are less than 1000? (2)

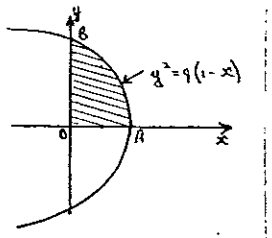
Question 4 (*New Booklet*)



The shaded region in the diagram is bounded by the curve $y = x^4$, the y axis, and the line $y = 16$.

Calculate the volume of the solid of revolution when this region is rotated about the **y axis** (3)

b)



The diagram shows the graph of $y^2 = 9(1-x)$.

The graph intersects the axes at the points A and B as shown

i) Find the coordinates of A and B. (2)

ii) Find the area of the shaded region. (3)

c) A ball is dropped from a height of 2 metres onto a hard floor and bounces. After each bounce, the maximum height reached by the ball is 75% of the previous maximum height. Thus, after it first hits the floor it reaches a height of 1.5 metres before falling again, and after the second bounce, it reaches a height of 1.125 metres before falling again.

i) What is the maximum height reached after the third bounce? (1)

ii) What kind of sequence is formed by the successive maximum heights? (1)

iii) What is the total distance travelled by the ball from the time it was first dropped until it eventually comes to rest on the floor? (2)

Question 5. (*New Booklet*)

a) The table shows the values of a function $f(x)$ for five values of x.

x	1	1.5	2	2.5	3
$f(x)$	5	1	-2	3	7

Use Simpson's rule with these five values to estimate

$\int_1^3 f(x)dx$. (3)

b) Let $f(x)=\sqrt{25-x^2}$.

i) Copy the following table of values into your writing booklet and supply the missing values (2)

x	0	1	2	3	4	5
$f(x)$	5.000		4.583			0.000

ii) Use these six values of the function and the trapezoidal rule to find the approximate value of $\int_0^5 \sqrt{25-x^2}\,dx$ (2)

iii) Draw the graph of $x^2+y^2=25$ and shade the region whose area is represented by the integral

$$\int_0^5 \sqrt{25-x^2}\,dx \qquad (2)$$

iv) Use your answer to part iii) to explain why the exact value of the integral is $\frac{25\pi}{4}$ (2)

v) Use your answers to part ii) and part iv) to find an approximate value for π. (1)

☺End of Test☺

Start here for Question Number 1.

a). $\int (2x+3)^{11} dx$

$= \frac{(2x+3)^{12}}{22} + C.$ ✓ 2

b). $\int (6 - x^{-3}) dx$

$= 6x + \frac{x^{-2}}{2} + C.$ ✓ 2

c). $\int_1^2 x^{-2} dx$

$= \left[\frac{x^{-1}}{1} \right]_1^2$

$= \left[-\frac{1}{x} \right]_1^2$ 2

$= \frac{1}{2}.$ ✓

d). i. $\log_a b - \log_a c$

$= 2.75 - 0.25$

$= 2.5.$ ✓

ii. $2(\log_a b + \log_a c)$

$= 2(2.75 + 0.25)$

$= 6.$ ✓ 3

e). i. $|r| < 1$ { $-1 < r < 1$ } ✓ limiting sum $\frac{a}{1-r}$.

ii. $\frac{a}{1-\frac{1}{w}} = \frac{1}{1-w}$

$\frac{aw}{w-1} = \frac{1}{1-w}$

$\frac{aw}{w-1} = -\frac{1}{w-1}$

$aw = -1$

$a = -\frac{1}{w}.$

$\sum_{r=1}^{n} \frac{1}{w}\left(\frac{1}{w}\right)^{r-1} = -\frac{1}{w} - \frac{1}{w^2} - \frac{1}{w^3} - \ldots$ ✓ 3

Start here for Question Number: 2

a). $T_n = a + (n-1)d$.

$= 2 + 5n - 5$

$= 5n - 3$ //

b). $\sum_{r=0}^{5} = -1 + 1 + 3 + 5 + 7 + 9$

$= 24$. //

c). i. $100(0.85)^n$ %

$= 72.25$ % $\times$?

ii. $100(0.85)^n < 10$.

$0.85^n < \frac{1}{10}$

$\log_{0.85} \frac{1}{10} \leq n$

$n \geq 14.168\ldots$

$n = 15$ layers. //

d) $\int_1^9 (1 + x^{\frac{1}{2}})\,dx$.

$= \left[x + \frac{2x^{\frac{3}{2}}}{3}\right]_1^9$

$= 9 + \frac{2(27)}{3} - 1 - \frac{2}{3}$

$= 25\frac{1}{3}$ ///

e). $\ln x^2 = \ln(5+4x)$

$x^2 = 5 + 4x$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5, -1$

but $2\ln(-1)$ is undefined, i.e. $x = -1$ is outside domain of $\ln x$.

$\therefore x = 5$ only.

//

Start here for Question Number: 3.

a) i. $\int_0^3 (x+1)\,dx$

$= \left[\frac{x^2}{2} + x\right]_0^3$

$= \frac{9}{2} + 3 - 0 - 0.$

$= 7\frac{1}{2}\ u^2.$

ii. $\pi \int_0^3 (x+1)^2\,dx$

$= \pi \int_0^3 x^2 + 2x + 1\,dx$

$= \pi \left[\frac{x^3}{3} + x^2 + x\right]_0^3$

$= \pi \left[9 + 9 + 3 - 0 - 0 - 0\right]$

$= 21\pi\ u^3$

b) i. $a + 2d = 32$ ①

$a + 5d = 17$ ②

① - ②

$-3d = 15$

$d = -5.$

Sub into ①

$a = 42$

ii. $\frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{10}{2}\left(84 - 9 \times 5\right)$

$= 195$

c) i. $T_n = 7n$

$7n < 1000$

$n < 142.85\ldots$

$n = 142.$

$2 \times 142 = 994.$

ii. $\frac{n}{2}\left(2a + (n-1)d\right).$

$\frac{142}{2}\left(14 + 141 \times 7\right).$

$= 71071$

Start here for Question Number: 4

a),

$y = 16, \quad y = x^4$

$16 = x^4$

$x = \pm 2.$

$y = x^4$

y limits $0, 16.$

$x^2 = \pm\sqrt{y}.$

$$\pi \int_0^{16} (16)^2 \, dy - \pi \int_0^{16} (\sqrt{y})^2 \, dy.$$

$$= \pi \int_0^{16} 256 - y \, dy,$$

$$= \pi \left[256y - \frac{y^2}{2} \right]_0^{16}$$

$$= 3968\pi \ u^3$$

$$\pi \int_0^{16} (y^{\frac{1}{2}}) \, dy.$$

$$= \pi \left[\frac{2y^{\frac{3}{2}}}{3} \right]_0^{16}$$

$$= \frac{128\pi}{3} \ u^3$$

OR.

$$= 42\tfrac{2}{3}\pi \ u^3$$

b). i. At A, when $y = 0$,

$0 = 9(1 - x).$

$x = 1$

$A(1, 0);$

B, when $x = 0.$

$y^2 = 9.$

$y = \pm 3.$

But B's ordinate $> 0.$

$\therefore B(0, 3).$

ii). $y = \pm 3\sqrt{1 - x}$

$= \pm 3(1 - x)^{\frac{1}{2}}.$

But region OAB is above x axis.

$\therefore y = 3(1 - x)^{\frac{1}{2}}.$

$$\int_0^1 3(1 - x)^{\frac{1}{2}} \, dx.$$

$$= 3 \left[\frac{2(1 - x)^{\frac{3}{2}}}{3x - 1} \right]_0^1$$

$$= 3 \left[\frac{2 \cdot (1 - x)^{\frac{3}{2}}}{3} \right]_0^1.$$

$= 2 \ u^2$

(ii). $x = 1 - \frac{y^2}{9}$

$$\int_0^3 \left(1 - \frac{y^2}{9}\right) dy$$

$$= \left[y - \frac{y^3}{27} \right]_0^3$$

$= 2 \ u^2.$

c) i. $2(0.75)^n$

$2(0.75)^3 = 0.84375$ m ✓

ii. Geometric sequence, common ratio 0.75, first term 1.5. ✓

iii. $2 + 2\sum_{r=1}^{\infty} 2(0.75)^r$

$= 2 + 2\left(\frac{1.5}{1-0.75}\right)$

$= 14$ m. ✓✓

11001

Start here for Question Number: 5

a) ~~$\frac{h}{3}(y_0 + \ldots y_n)$~~

$\frac{0.5}{3}\left(5 + 7 + 4(1+3) + 2 \times 2\right)$

$= 4$ ✓ 3

b) i.

x	0	1	2	3	4	5
$f(x)$	5.000	4.899	4.583	4.000	3.000	0.000

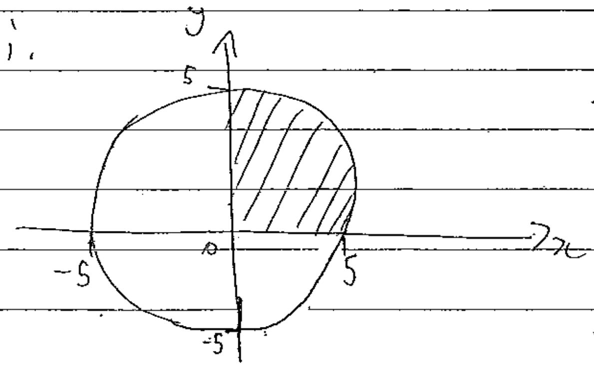
✓ 2

ii. $\frac{1}{2}\left[5.000 + 0.000 + 2(4.899 + 4.583 + 4.000 + 3.000)\right]$

$= 18.982$ ✓ 2

$= 19.0$ (3sf).

iii.



✓ 2

11001

the area of

IV, Region is a quarter of a circle with radius 5.

$\therefore = \frac{1}{4} \times \pi (5)^2$

$= \frac{25}{4} \pi$ ✓2

v). ~~78.982~~

$\pi = \frac{25}{4}$

$\pi \doteqdot 3.03712$ ✓1

$\doteqdot 3.04$ (3sf).