2011
HIGHER SCHOOL CERTIFICATE
ASSESSMENT 3

Mathematics

General Instructions

- Working Time - 45 mins.
- Write using a blue or black pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (30)

- Attempt Questions 1-3.
- All questions are of equal value.

Remember to use the standard integrals sheet

Question 1 (10 Marks) Start a new page

Marks

(a) Differentiate with respect to x

(i) xe^x — 2

(ii) $\dfrac{x+2}{\tan x}$ — 2

(iii) $(\log_e x+1)^3$ — 2

(b) Evaluate

(i) $\displaystyle\int \frac{10x}{x^2+1}\,dx$ — 2

(ii) $\displaystyle\int \sec^2 3x\,dx$ — 2

Question 2 (10 Marks) Start a new page

Marks

(a) The curve $y=e^{2x}$ is rotated about the x-axis.

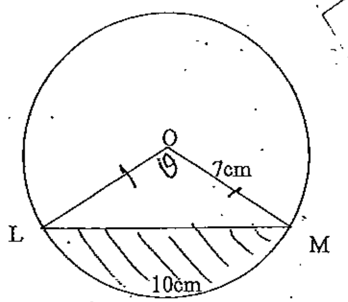
Show that the volume of the solid of revolution formed between

x = 0 and x = 3 is given by $\frac{\pi}{4}(e^{12}-1)u^3$ — 3

Question 2 continued

Marks

(b) The diagram shows a circle with centre O and radius 7cm.
The length of the arc LM is 10cm.



(i) Find $\angle LOM$ **1**

(ii) Hence find the area of the minor segment bounded by LM correct to 1 decimal place **2**

(c) The rate at which people flow into a train station during afternoon peak hour is given by:

$$\frac{dP}{dt} = 60 - 16t \quad \text{for } t \geq 0$$

where P is the number of people and t is the time in measured in hours.

(i) Find the initial rate at which people flow into the station **1**

(ii) Find the number of people in the station as a function of t. **1**

(iii) Initially, there are 240 people in the station.
Find out how many people remain in the station after 5 hours **2**

Question 3 (10 Marks) Start a new page

Marks

(a) A particle moves in a straight line and at any time t seconds $t \geq 0$, its displacement $x(t)$ from 0 is measured in metres.

Its velocity in metres per second is given by:

$$\dot{x}(t) = 1 - \sin(2t) \quad \text{for } 0 \leq t \leq 2\pi$$

(i) Find when the particle is first at rest **2**

(ii) Find an expression for the acceleration **1**

(iii) The particle is initially 1.5m from O.

Find an expression for the displacement $x(t)$. **2**

(b) Evaluate $\frac{d}{dx}\sin(x^{-3})$ **2**

(c) Current i is measured in Amps and time is measured in seconds.
A current i_0 is established in an electrical circuit.
After the source of the current has been removed, the current in the circuit decays according to the equation $\frac{di}{dt} = -ki$

(i) Show that $i = i_0 e^{-kt}$ satisfies the differential equation $\frac{di}{dt} = -ki$. **1**

(ii) The current in the circuit decays to 36.8% of the original current in a 0.1 of a second. Find k to 1 decimal place. **2**

Q1

Q1. a) i. let $y = xe^x$

$u = x \quad v = e^x$

$u' = 1 \quad v' = e^x$

10

$y' = vu' + uv'$ ✓

$y' = e^x + xe^x$ ✓

2

ii. let $y = \frac{x+2}{\tan x}$ let $u = x+2 \quad v = \tan x$

$u' = 1 \quad v' = \sec^2 x$

$y' = \frac{vu' - uv'}{v^2}$

$\therefore y' = \frac{\tan x - (x+2)(\sec^2 x)}{(\tan x)^2}$ ✓

2

iii) let $y = (\log_e x + 1)^3$

$\therefore y' = 3(\log_e x + 1)^2 \cdot \left(\frac{1}{x}\right)$ ✓ ✓

2

b) i) $\int \frac{10x}{x^2+1} \, dx \qquad u = x^2+1, \quad u' = 2x$

$= 5 \int \frac{2x}{x^2+1} \, dx$

$= 5 \ln(x^2+1) + c.$ ✓

2

ii) $\int \sec^2 3x \, dx \qquad a = 3.$

$= \frac{1}{3} \tan 3x + c.$ ✓ ✓

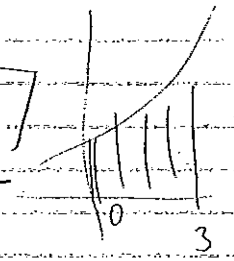
2

Q2

a) $y = e^{2x}$

10

$\boxed{V = \pi \int_0^3 y^2 \, dx}$



$y = e^{2x}$

$\therefore y^2 = e^{4x}$

$\therefore V = \pi \int_0^3 e^{4x} \, dx$ ✓

$= \pi \left[\frac{1}{4} e^{4x}\right]_0^3$

$= \frac{\pi}{4} \left[e^{4x}\right]_0^3$ ✓

3

$= \frac{\pi}{4} \left[e^{12} - e^0\right]$

$= \frac{\pi}{4} (e^{12} - 1) \ u^3$ ✓ // as req.

b) i) $l = r\theta$

$10 = 7\theta$

$\therefore \theta = \frac{10}{7}$ radians. (1)

minor segment:

ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ 1dp:

$A = \frac{1}{2}r^2\left(\frac{10}{7} - \sin\frac{10}{7}\right)$

$= \frac{1}{2}(7)^2\left(\frac{10}{7} - \sin\left(\frac{10}{7}\right)\right)$ (radians mode)

$\therefore A = 10.747$

$= 10.7\ cm^2$ (1dp). (2)

c) $\frac{dP}{dt} = 60 - 16t$ $t \geq 0$

i) initial @ $t = 0$

$\frac{dP}{dt} = 60 - 16(0)$

$= 60$ people /hr. (1)

ii) $\int \frac{dP}{dt} = 60 - 16t \; dt$

$\therefore P = 60t - \frac{16t^2}{2} + c$

$\therefore$ ie $P = 60t - 8t^2 + c$ (1)

iii) $t = 0$, $P = 240$.

~~$P = 60t - 80$~~

$P = 60t - 8t^2 + c$

$240 = 60(0) - 8(0) + c$

$\therefore c = 240$

$\therefore P = 60t - 8t^2 + 240$.

($P = ?$ $t = 5$)

$P = 60(5) - 8(5)^2 + 240$

$\therefore P = \boxed{340 \text{ ppl}}$ remain at station after 5 hrs, (2)

~~remaining:~~

BRNO

3

10

a) $t \geq 0$ meters

$\rightarrow$ m/sec.

$x = 1 - \sin(2t)$ $\quad 0 \leq t \leq 2\pi$

i) rest when $v = 0$

$v = 1 - \sin(2t)$

$1 - \sin 2t = 0$

$\sin 2t = 1$

$2t = \frac{\pi}{2}$

$\therefore t = \frac{\pi}{4}$ sec. first @ revt.

2

ii) find $\ddot{x}$

$sc - s - c$

differentiate

$\dot{x} = 1 - \sin(2t)$

$\ddot{x} = \cancel{}\ -2\cos(2t)$

1

iii) $t=0$, $x = 1.5$ cm

$sc - s - c$

$\int \dot{x} = 1 - \sin(2t)\ dt$

$x = t + \frac{1}{2}\cos(2t) + c$ $\rightarrow$ 1.5 cm = 0.015m

(@t=0, x=0.015)

$0.015 = 0 + \frac{1}{2}\cos(0) + c$

$\therefore\ 0.015 = \frac{1}{2}(1) + c$

$c = -0.485$

$\therefore x(t) = t + \frac{1}{2}\cos(2t) - 0.485$

m/s.

PTO

$s\ c - s - c$

a) iii) $\int \dot{x} = 1 - \sin 2t\ dt$

$x = t + \frac{1}{2}\cos 2t + c$

$x = 1.5\,m$, $t = 0$

$1.5 = 0 + \underbrace{\frac{1}{2}\cos 2(0)}_{0.5} + c$

$c = 1.5 - 0.5$

$c = 1$

$\therefore x(t) = t + \frac{1}{2}\cos 2t + 1$ for m/s.

2

b) $y = \sin(x^{-3})$

$y' = -3x^{-4}\cos(x^{-3})$ let $u = x^{-3}$

$u' = -3x^{-4}$

$\therefore \frac{dy}{dx} = \frac{-3}{x^4}\cos\left(\frac{1}{x^3}\right)$

$\cancel{u = x^{-3}}$

$\cancel{u' = 3x^{-4}}$

2

e) i) $i = i_0 e^{-kt}$

$\frac{di}{dt} = -k\cdot(i_0 e^{-kt})$

since $i = i_0 e^{-kt}$

$\therefore \frac{di}{dt} = -k(i)$

$\frac{di}{dt} = -ki$ as req.

1

ii) decays to 36.8%

~~hence $i =$~~

$i = i_0 e^{-kt}$ i = current (amps)

because it decays to 36.8% of original.

~~$i =$~~ 36.8% = 0.368

$\therefore i = 0.368\, i_0$

$i = i_0 e^{-kt}$ subing in

$0.368\, \cancel{i_0} = \cancel{i_0} e^{-kt}$

$e^{-kt} = 0.368$ $t = 0.1$ sec.

$\cancel{e^{(-0.1)k}}$ $e^{-k(0.1)} = 0.368$

$\therefore k = \frac{1}{-0.1}\ln_e(0.368)$

2

hence $k = 9.9967$

$k = 10.0$ (1 dp).