NAME :

CLASS:

Randwick Girls High School

MATHEMATICS
Extension II

Assessment Task No.2
March 2011

General Instructions

- Reading time – 5 minutes
- Working Time – 45 minutes
- Write using a black or blue pen
- Approved calculators may be used
- All necessary working should be shown for every question.
- Work down the page, not across!

Total Marks

- Attempt All Questions
- Narks for each question are indicated on the paper

Question 1.

a) Use implicit differentiation to show that $\frac{dy}{dx}$ of $5x^2 - y^2 + 4xy = 18$ is equal to $\frac{5x+2y}{y-2x}$ **3**

b) Use calculus to find the turning points of $y = x^3 - 3x + 5$ hence sketch the curve. $x = -2.27$ **3**

c) Use your answer in part (b) to assist you in drawing neat sketches of the following where $f(x) = x^3 - 3x + 5$

i) $y = |f(x)|$ **1**

ii) $y = \sqrt{f(x)}$ **2**

iii) $y = \frac{1}{f(x)}$ **3**

iv) $y = \frac{x}{f(x)}$ **3**

v) $y^2 = f(x)$ **1**

Question 2.

a) The equation $x^3 - 4x^2 + 2 = 0$ has roots α, β, γ.
Find an equation with roots

i) $-\alpha, -\beta, -\gamma$. **2**

ii) $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$ **2**

b)

Find real numbers a, b such that

$$\frac{x}{(x+2)(x-3)} = \frac{a}{x+2} + \frac{b}{x-3}$$ **3**

c)

Find integers p and q such that $(x+1)^2$ is a factor of
$x^3 + 2x^2 + px + q$. **3**

d)

The complex polynomial with real coefficients **4**
$z^3 + 3z + 2i = (z-\alpha)^2(z-\beta)$ find the values
of α and β.

xt 11 Solutions Task 2

a) (i) $\frac{d}{dx}\left(5x^2 - y^2 + 4xy = 18\right)$

$$10x - 2y\frac{dy}{dx} + 4x\frac{dy}{dx} + 4y = 0$$

$$\frac{dy}{dx}\left(4x - 2y\right) + 10x + 4y = 0$$

$$\frac{dy}{dx} = \frac{-10x - 4y}{4x - 2y}$$

$$= \frac{2y + 5x}{y - 2x}$$

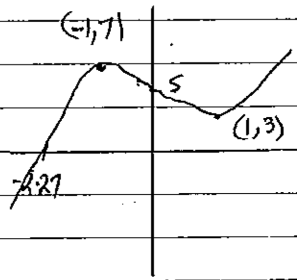
b) $y = x^3 - 3x + 5$

$y' = 3x^2 - 3$ Stat pts when $y' = 0$ $3x^2 - 3 = 0$

$x = \pm 1$

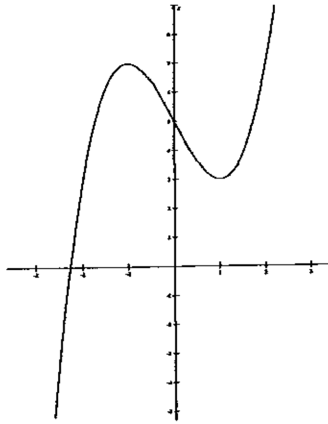
$y(1) = 1 - 3 + 5 = 3$

$y(-1) = -1 + 3 + 5 = 7$

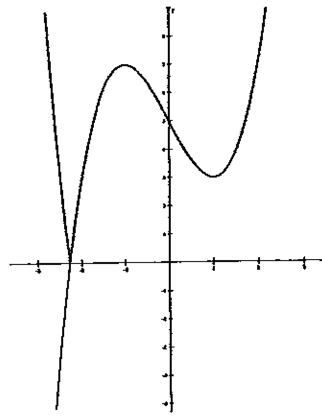


$x \doteqdot -2.27$ $y = 0$

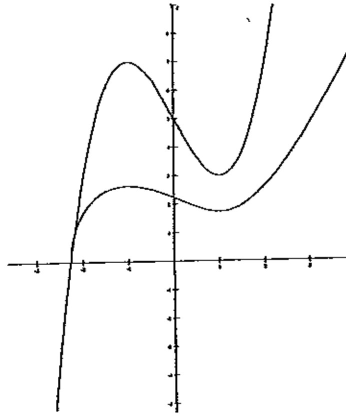
Graphs see attached



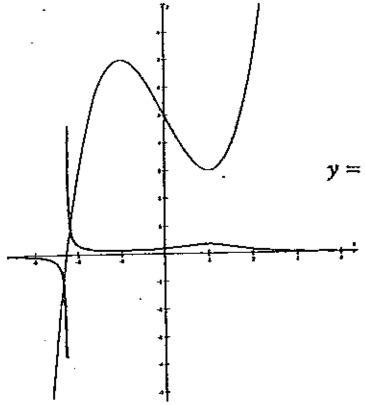
$y = f(x)$



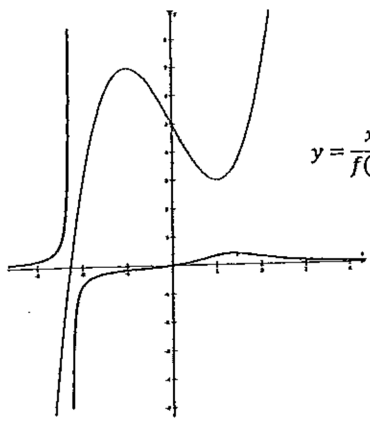
$y = |f(x)|$



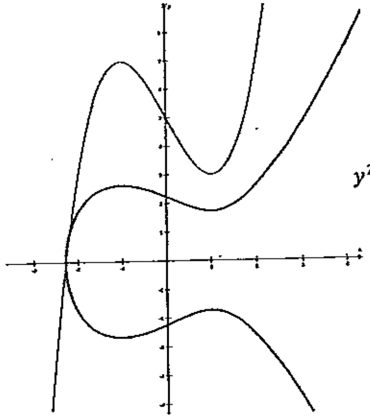
$y = \sqrt{f(x)}$



$y = \frac{1}{f(x)}$



$y = \frac{x}{f(x)}$



$y^2 = f(x)$

?a) $x^3-4x^2+2=0$

i) $-\alpha\ -\beta\ -\gamma$

$y=-x$

$x=-y$

$(-y)^3-4(-y)+2=0$

$-y^3-4y+2=0$

$\therefore x^3+4x-2=0$

ii) $\alpha^{-1}\ \beta^{-1}\ \gamma^{-1}$

$y=1/x$

$x=1/y$

$(1/y)^3-4(1/y)^2+2=0$

$1-4y+2y^3=0$

$\therefore 2x^3-4x+1=0$

b) $a(x-3)+b(x+2)=x$

let $x=3$

$5b=3$

$b=3/5$

let $x=-2$

$-5a=-2$

$a=2/5$

c) $(x+1)^2$ is a double root $\therefore P(-1)=P'(-1)=0$

$P(-1)=(-1)^3+2(-1)^2+p(-1)+q$

$=-1+2-p+q=0$

$\therefore -p+q=-1$

$P'(x)=3x^2-4x+p$

$P'(-1)=3(-1)^2-4(-1)+p$

$=3-4+p=0$

$p=1$

$\therefore q=0$

?) $P(z)=z^3+3z+2i=(z-\alpha)^2(z-\beta)$

$P'(z)=3z^2+3$

$P'(\alpha)=3\alpha^2+3=0$

$3(\alpha^2+1)=0$

$\alpha^2=-1$

$\alpha=\pm\sqrt{-1}$

$=\pm i$

$P(-i)=(-i)^3+3(-i)+2i$

$=i-3i+2i$

$=0$

$\therefore -i$ is the double root

Prod of roots

$\alpha\beta\gamma=-2i$

$\alpha^2\beta=-2i$

$-\beta=-2i$

$\beta=2i$

N.B. There are other methods of solution.