SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2016
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes.
- Working time – 90 Minutes.
- Write using black or blue pen. .
- Board approved calculators maybe used.
- The Board Approved Reference Sheet is Provided.
- ALL necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for untidy or badly arranged work.
- Answer in simplest EXACT form unless otherwise instructed.

- Attempt Questions 1–9
- The mark value of each question is shown on the right hand side.
- Attempt Questions 1–6 on the Multiple Choice answer sheet provided.
- Each section is to be answered in a NEW writing booklet, clearly labelled **Question 7, Question 8** and **Question 9**.
- Total Marks – 63

Examiner A. Wang

Section I – Multiple Choice

6 Marks
Attempt Questions 1–6

Use the multiple-choice answer sheet for Questions 1–6

---

1 What is the derivative of $y = 3\sin x - 4\cos x$?

(A) $\frac{dy}{dx} = 3\cos x - 4\sin x$

(B) $\frac{dy}{dx} = 3\cos x + 4\sin x$

(C) $\frac{dy}{dx} = -3\cos x + 4\sin x$

(D) $\frac{dy}{dx} = -3\cos x - 4\sin x$

2 In how many ways can 12 students of unique height be arranged in a line, so that the tallest and the shortest student never come together?

(A) $11! \times 10$

(B) $10! \times 10$

(C) $11! \times 9$

(D) $10! \times 11$

3 Consider the function $f(x) = x^4 + 3x^2 - x - 5$. It has one root at $x = -1$. Take $x = 2$ as a first approximation for this root.

Using two applications of Newton's method, which of the following is a better approximation for the root?

(A) $x = -1.257$

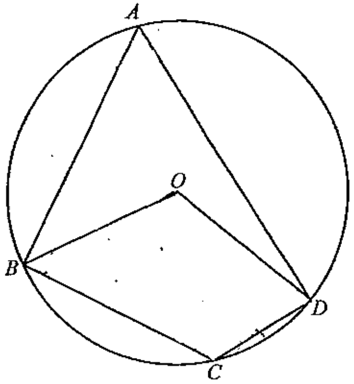
(B) $x = 1.257$

(C) $x = -1.512$

(D) $x = 1.512$

4 In the diagram A, B, C and D are points on a circle with centre O.

$\angle BAD = x°$ and $\angle BOD = \angle BCD$.



NOT TO SCALE

What is the value of x?

(A) 75°

(B) 120°

(C) 90°

(D) 60°

5 What is the solution to $\dfrac{{}^{n}C_4}{{}^{n-2}C_2} = 1$?

(A) 5

(B) 4

(C) 6

(D) 2

6 What is the length of the chord of the parabola $x^2 = 4ay$ passing through the vertex and having slope $\tan \alpha$?

(A) $4a \operatorname{cosec} \alpha \cot \alpha$

(B) $4a \sec \alpha \tan \alpha$

(C) $4a \cos \alpha \cot \alpha$

(D) $4a \sin \alpha \tan \alpha$

Section II

57 marks
Attempt Questions 7–9

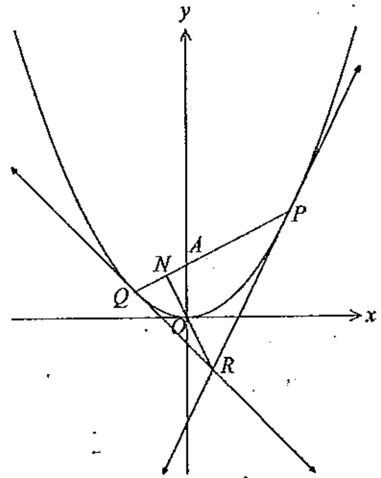
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 7–9, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 (19 marks) **Start a NEW Writing Booklet**

(a) The diagram below shows a parabola defined by the parametric equations

$$x = 2t \text{ and } y = t^2$$



i) Write down the equations of the tangents at the points $P(2p, p^2)$ and $Q(2q, q^2)$. **1**

ii) Show that the point of intersection of the two tangents is $R(p+q, pq)$. **2**

iii) Show that the equation of the chord PQ is $(p+q)x - 2y - 2pq = 0$. **2**

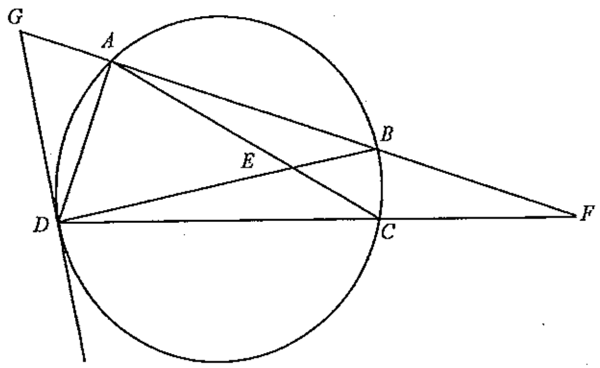
iv) Points P and Q move on the parabola in such a way that $pq = -2$. Prove that the chord PQ always passes through the point $A(0, 2)$. **1**

v) N is the intersection of the chord PQ and the line through R and O. Show that RN is perpendicular to PQ. **2**

Question 7 continues on the next page

Question 7 (continued)

b) In the diagram below, DG is a tangent to the circle at D. $GABF$ and DCF are straight lines.



i) Copy the diagram into your writing booklet.

ii) Prove $2 \times \angle ADG = \angle BEC + \angle BFC$. **3**

c) Evaluate $\lim_{x \to 0} \frac{1}{x} \sin \frac{x}{2\pi}$. **1**

d) Find:

i) $\frac{d}{dx} \cos(\sqrt{x})$ **2**

ii) $\frac{d}{dx} \tan(\sin 3x)$ **2**

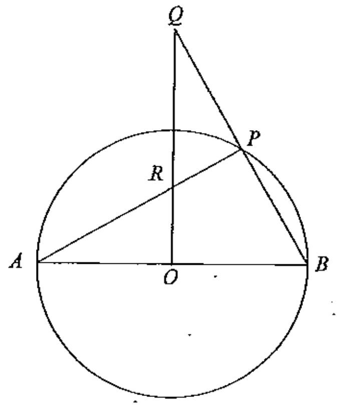
e) Show that $\frac{d}{dx}(\sec x) = \sec x \tan x$ **1**

f) Find the area bounded by the curve $y = \sin\left(\frac{x}{2}\right)$, the lines $x = -\pi$, $x = \pi$, and the x-axis. **2**

End of Question 7

Question 8 (19 marks) **Start a NEW Writing Booklet**

a) O is the centre of the circle APB.
BPQ, ORQ, ARP and AOB are straight lines. $\angle QOB = 90°$.



i) Copy the diagram into your writing booklet.

ii) Prove that A, O, P and Q are concyclic points. **3**

b) In how many rearrangements of the letters of the word SCINTILLATING will no two T's appear together? **2**

c) The function $f(x) = x^2 - \ln(x + 1)$ has one root between 0.5 and 1.

i) Show that the root lies between 0.7 and 0.8. **2**

ii) Hence use the halving interval method to find the value of the root correct to 1 decimal place. **2**

Question 8 continues on the next page.

Question 8 (continued)

d) Let A be a point on the parabola $x^2 = 4by$ whose coordinates are $(2bt, bt^2)$.
Tangents are drawn from A to another parabola $x^2 = 4ay$, where $b > a$.
These tangents touch the parabola $x^2 = 4ay$ at $P(2ap, ap^2)$ and $Q(2aq, aq^2)$.

From P and Q, normals are drawn to intersect at N.

i) Sketch a diagram to represent the above information. **1**

ii) Show that $bt^2 - 2bpt + ap^2 = 0$ and $bt^2 - 2bqt + aq^2 = 0$ **2**

iii) Show that the coordinates of N are given by **2**
$$x = -apq(p+q),\ y = a[\,(p+q)^2 - pq + 2\,].$$

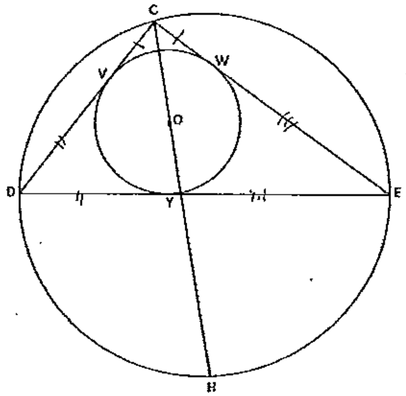
iv) From part ii) above, show that $a(p+q) = 2bt$ and $apq = bt^2$. **2**

v) Hence show that the locus of N, the intersection of the normals to $x^2 = 4ay$, is the curve **3**
$$x^2(4b-a)^3 = 4ab(y-2a)^3.$$

End of Question 8

Question 9 (19 marks) Start a NEW Writing Booklet

a) The incircle of triangle CDE has centre O and touches the sides of $\Delta\, CDE$ at V, W and Y.
The circumcircle of triangle CDE meets CO produced to H.



i) Copy the diagram to your writing booklet.

ii) Prove that H is the midpoint of arc DE. **2**

iii) Prove that $\Delta\, ODV \equiv \Delta\, ODY$. **2**

iv) Prove that $\angle\, ODH = \angle\, DOH$. **2**

v) Prove $HD = HO$. **1**

b) i) Simplify $\cos(A-B) - \cos(A+B)$ **1**

ii) Prove by the method of mathematical induction that **3**
$$\sin w + \sin 3w + \sin 5w + \ldots + \sin(2n-1)w = \frac{1-\cos 2nw}{2\sin w}$$
for a constant w and for integers $n \geq 1$.

c) The straight line $y = mx + b$ meets the parabola $x^2 = 4y$ at two points L and N.
M is the midpoint of LN.

i) Find the coordinates of M in terms of m and b. **3**

ii) Find the locus of M if $b = -2$. **3**

iii) What are the restrictions on the domain of M? Justify your answer. **2**

End of paper

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2016

HSC Task #2

Mathematics Extension 1

Suggested Solutions & Markers' Comments

QUESTION	Marker
1 – 6	–
7	EC
8	BK
9	AF

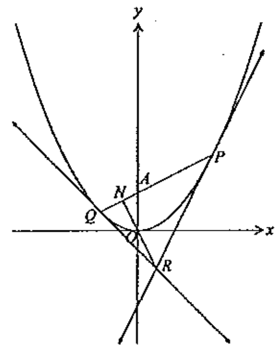
Multiple Choice Answers

1. B
2. A
3. B
4. D
5. B
6. B

Question 7 — Solutions

(a) The diagram below shows a parabola defined by the parametric equations

$$x = 2t \text{ and } y = t^2$$



i) Write down the equations of the tangents at the points $P(2p, p^2)$ and $Q(2q, q^2)$. **1**

Tangent at P: $y = px - p^2$
Tangent at Q: $y = qx - q^2$

ii) Show that the point of intersection of the two tangents is $R(p + q, pq)$. **2**

$y = px - p^2 \quad -(1)$
$y = qx - q^2 \quad -(2)$
Equating (1) and (2): $(p - q)x = p^2 - q^2$
$\therefore (p - q)x = (p - q)(p + q)$
$\therefore x = p + q$
Substitute into (1): $y = p(p + q) - p^2$
$\therefore y = pq$
$\therefore$ the point of intersection is $(p + q, pq)$.

iii) Show that the equation of the chord PQ is $(p + q)x - 2y - 2pq = 0$. **2**

$$m_{PQ} = \frac{q^2 - p^2}{2q - 2p} = \frac{(q-p)(q+p)}{2(q-p)} = \tfrac{1}{2}(p+q)$$

$\therefore y - q^2 = \frac{1}{2}(p+q)(x - 2q)$
$\therefore 2y - 2q^2 = (p+q)x - 2q(p+q)$
$\therefore (p+q)x - 2y + 2q^2 - 2q(p+q) = 0$
$\therefore (p+q)x - 2y - 2pq = 0$

(a) iv) Points P and Q move on the parabola in such a way that $pq = -2$. Prove that the chord PQ always passes through the point $A(0, 2)$. **1**

Substitute $pq = -2$ into $(p+q)x - 2y - 2pq = 0$

$\therefore (p+q)x - 2y + 4 = 0$

y-intercept when $x = 0$

$\therefore -2y + 4 = 0$

$\therefore y = 2$

i.e. (0, 2) always lies on the chord PQ.

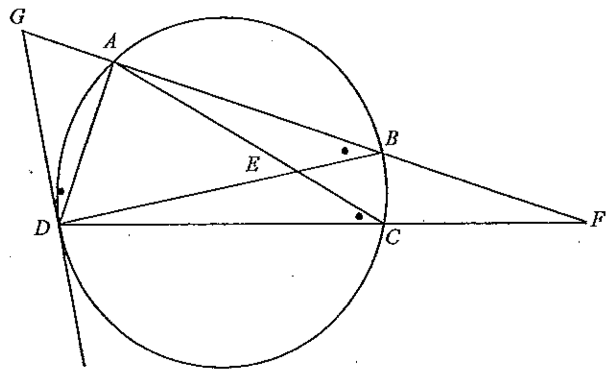
v) N is the intersection of the chord PQ and the line through R and O. Show that RN is perpendicular to PQ. **2**

$$m_{RN} = \frac{pq}{p+q}$$
$$= \frac{-2}{p+q}$$

$$m_{PQ} \times m_{RN} = \tfrac{1}{2}(p+q) \times \frac{-2}{p+q}$$
$$= -1$$

$\therefore PQ \perp RN$

b) In the diagram below, DG is a tangent to the circle at D. $GABF$ and DCF are straight lines.



i) Copy the diagram into your writing booklet.

ii) Prove $2 \times \angle ADG = \angle BEC + \angle BFC$. **3**

$\angle ADG = \angle ABE$ (angle between tangent and chord)

Similarly, $\angle ADG = \angle ACD$

$\angle BEC + \angle BFC = 2\pi - (\angle EBF + \angle ECF)$ (angle sum of quad. $BECF$)

$\angle EBF = \pi - \angle ABE$ (straight $\angle$)

Similarly, $\angle ECF = \pi - \angle ACD$

$\therefore \angle BEC + \angle BFC = \angle ACD + \angle ABE$

$\therefore \angle BEC + \angle BFC = 2 \times \angle ADG$

c) Evaluate $\lim_{x \to 0} \frac{1}{x} \sin \frac{x}{2\pi}$. **1**

$$\lim_{x \to 0} \frac{1}{x} \sin \frac{x}{2\pi} = \lim_{x \to 0} \frac{\sin \frac{x}{2\pi}}{x}$$
$$= \lim_{x \to 0} \left(\frac{\sin \frac{x}{2\pi}}{\frac{x}{2\pi}} \times \frac{1}{2\pi} \right)$$
$$= \frac{1}{2\pi} \times \lim_{x \to 0} \left(\frac{\sin \frac{x}{2\pi}}{\frac{x}{2\pi}} \right)$$
$$= \frac{1}{2\pi}$$

d) Find:

i) $\frac{d}{dx}\cos(\sqrt{x})$ **2**

$$\frac{d}{dx}\cos(\sqrt{x}) = \frac{d}{dx}\cos(x^{\frac{1}{2}})$$
$$= -\sin(x^{\frac{1}{2}}) \times \tfrac{1}{2}x^{-\frac{1}{2}}$$
$$= -\frac{\sin(x^{\frac{1}{2}})}{2x^{\frac{1}{2}}}$$

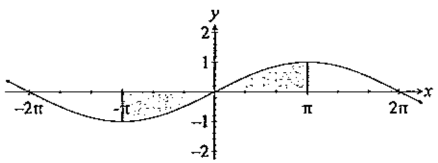
ii) $\frac{d}{dx}\tan(\sin 3x)$ **2**

$$\frac{d}{dx}\tan(\sin 3x) = \sec^2(\sin 3x) \times \frac{d}{dx}(\sin 3x)$$
$$= 3\cos 3x \sec^2(\sin 3x)$$

e) Show that $\frac{d}{dx}(\sec x) = \sec x \tan x$ **1**

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}(\cos x)^{-1}$$
$$= -(\cos x)^{-2} \times (-\sin x)$$
$$= \frac{\sin x}{\cos^2 x}$$
$$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$$
$$= \tan x \sec x$$

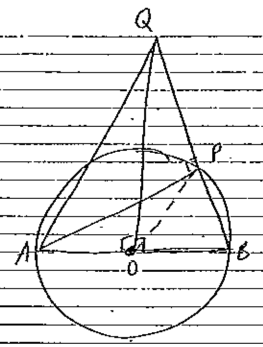
f) Find the area bounded by the curve $y = \sin\left(\frac{x}{2}\right)$, the lines $x = -\pi$, $x = \pi$, and the x-axis. **2**



$$\text{Area} = 2 \times \int_0^{\pi} \sin\left(\frac{x}{2}\right) dx$$
$$= 2 \times \left[-2\cos\left(\frac{x}{2}\right)\right]_0^{\pi}$$
$$= -4\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right]$$
$$= 4$$

Q8.

(a) (i)



$\angle AOB = 90°$ (given)

(ii) Prove A, O, P, Q are concyclic.

$\angle APB = 90°$ (Angle in a semi-circle) ✓

$\Rightarrow \angle APQ = 90°$ (Supplementary ∠s on straight line)
and $\angle AOQ = 90°$ (Same reason)

Then $\angle APQ = \angle AOQ = 90°$ ✓

Then P and O are angles at the circumference of a circle with QA as the diameter, i.e. they are angles in a semi-circle ✓

∴ AOPQ is a circle and A, O, P, Q are concyclic.

Mostly done well. Some did a longer solution using congruent triangles.

(3)

8(b) SCINTILLATING

1S, 1C, 3I, 2N, 2T, 2L, 1A, 1G — 13 letters

Method 1 Put all 10 letters, apart from the I's, down first

$= \frac{10!}{2!2!2!}$ ways. ✓

_/_/_/_/_/_/_/_/_/_/_

Then there are 11 possible spaces in which to place the 3 I's ⇒ choose 3 spaces from 11

$= {}^{11}C_3$ ways $= \frac{11!}{3!8!}$ ways ✓

Then total ways $= \frac{10!}{2!2!2!} \times \frac{11!}{3!8!}$

$= 74\,844\,000$ ways. (2)

Method 2

No. of ways without restriction $= \frac{13!}{3!2!2!2!}$ ✓

Case with 3 I's together $= \frac{11!}{2!2!2!}$ ✓

Cases with 2 I's together $= \frac{12!}{2!2!2!}$

∴ Total ways $= \frac{13!}{3!2!2!2!} - \frac{12!}{2!2!2!} - \frac{11!}{2!2!2!}$

$= 74\,844\,000$ ways (2)

About two-thirds of the students used the spaces method and got the correct answer. Of the remaining students most forgot to subtract the case of the 3 Is together.

8(c)(i) $y = x^2 - \ln(x+1)$ — Most did this question well.

$f(0.7) = -0.0406 < 0$ ✓

$f(0.8) = 0.0522 > 0$ ✓ (2)

∴ there is a value between 0.7 and 0.8 for which $f(x) = 0$ since $f(x)$ is a continuous function

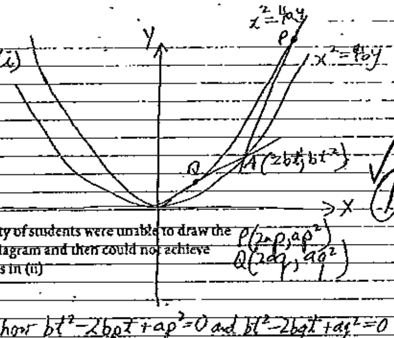
(ii) $f\left(\frac{0.7+0.8}{2}\right) = f(0.75) = 0.00288 > 0$ ✓

0.7 — 0.75 — 0.8

∴ root is closer to $x = 0.7$ (to 1dp). ✓ (2)

Many students spent time doing multiple applications of the halving the interval method before realising that they only needed the x-value to 1 dp. A few used the wrong method.(ie Newton's Method)

8(d)(i)



✓ (1)

A majority of students were unable to draw the correct diagram and then could not achieve full marks in (ii)

$P(2ap, ap^2)$

$Q(2aq, aq^2)$

(ii) Show $bt^2 - 2bpt + ap^2 = 0$ and $bt^2 - 2bqt + aq^2 = 0$

$y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

At P, $y' = \frac{2ap}{2a} = p$.

∴ eqn of tangent at P is $y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$ ✓

But A lies on tangent

ie $(2bt, bt^2)$ satisfies eqn

$\Rightarrow bt^2 = 2btp - ap^2$

$\Rightarrow bt^2 - 2btp + ap^2 = 0$. ✓ (2)

Similarly, tangent at Q is $y = qx - aq^2$ and A lies on tangent

$\Rightarrow bt^2 - 2btq + aq^2 = 0$.

All those who did not have correct diagram in (i) received 1 mark for the correct process of finding a tangent at A and the equations through the other points.

8(d)

(iii) Equation of Normal at P:

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$yp - ap^3 = -x + 2ap$

$yp + x = ap^3 + 2ap$ (1)

Similarly the Normal at Q has equation

$yq + x = aq^3 + 2aq$ (2)

(1) − (2) $\Rightarrow y(p-q) = a(p^3 - q^3) + 2a(p-q)$

$y(p-q) = a(p-q)(p^2+pq+q^2) + 2a(p-q)$

$\Rightarrow y = a(p^2+pq+q^2) + 2a$ since $p \neq q$

$\Rightarrow y = a\left[(p+q)^2 - pq + 2\right]$ # ✓

Also $q \times$ (1) $\Rightarrow ypq + qx = aqp^3 + 2apq$ (3)

and $p \times$ (2) $\Rightarrow ypq + px = apq^3 + 2apq$ (4)

(4) − (3) $\Rightarrow (p-q)x = apq(q^2 - p^2)$

$(p-q)x = -apq(p^2 - q^2)$ ✓

$x = -apq(p+q)$ #

Done well overall. (2)

8(d)

(iv) $bt^2 - 2bpt + ap^2 = 0$ (1)

$bt^2 - 2bqt + aq^2 = 0$ (2)

$p \times$ (2) $\Rightarrow bpt^2 - 2bpqt + apq^2 = 0$ (3)

$q \times$ (1) $\Rightarrow bqt^2 - 2bpqt + aqp^2 = 0$ (4)

(3) − (4) $\Rightarrow bt^2(p-q) + apq(q-p) = 0$

$bt^2(p-q) = apq(p-q)$

$\Rightarrow bt^2 = apq$ ($p \neq q$)

$pq = \frac{bt^2}{a}$ (5) ✓ (2)

Also (1) − (2) $\Rightarrow -2bpt + 2bqt + a(p^2 - q^2) = 0$

$-2bt(p-q) = -a(p^2 - q^2)$

$2bt = a(p+q)$ ✓

$\frac{2bt}{a} = p+q$ (6) #

Most students did this question successfully.

(v) $N = \left(-apq(p+q),\ a\left[(p+q)^2 - pq + 2\right]\right)$ (8)

$\Rightarrow x = -apq(p+q)$ (7) and $y = a\left[(p+q)^2 - pq + 2\right]$

sub (5), (6) into (7) from part (iv)

$\Rightarrow x = -a\left(\frac{bt^2}{a}\right)\left(\frac{2bt}{a}\right)$

$\Rightarrow x = \frac{-2b^2t^3}{a}$

$\Rightarrow t^3 = \frac{-ax}{2b^2}$

$\Rightarrow t = \frac{-a^{1/3}x^{1/3}}{2^{1/3}b^{2/3}}$ (9) ✓

8 (d)(v) (continued)

Sub ⑤, ⑥ in ⑧

$$y = a\left[\frac{4b^2t^2}{a^2} - \frac{bt^2}{a} + 2\right]$$

$$y = \frac{4b^2t^2}{a} - bt^2 + 2a$$

$$y - 2a = t^2\left(\frac{4b^2}{a} - b\right)$$

Sub in $t = \frac{a^{\frac{1}{3}}x^{\frac{1}{3}}}{2^{\frac{1}{3}}b^{\frac{2}{3}}}$

$$\Rightarrow y - 2a = \frac{a^{\frac{2}{3}}x^{\frac{2}{3}}}{2^{\frac{2}{3}}b^{\frac{4}{3}}}\left(\frac{4b^2}{a} - b\right) \checkmark$$

Cube both sides:

$$(y-2a)^3 = \frac{a^2x^2}{4b^4}\left(\frac{4b^2}{a} - b\right)^3$$

$$(y-2a)^3 = \frac{a^2x^2}{4b^4}\left[\frac{b}{a}(4b-a)\right]^3$$

$$(y-2a)^3 = \frac{a^2x^2}{4b^4}\frac{b^3}{a^3}(4b-a)^3$$

$$(y-2a)^3 = \frac{x^2}{4ab}(4b-a)^3$$

$$4ab(y-2a)^3 = x^2(4b-a)^3 \quad \# \quad \checkmark \ (3)$$

8 (d)(v) (continued)

Alternate Method

From $t^3 = \frac{-ax}{2b^2} \Rightarrow t^6 = \frac{x^2a^2}{4b^4}$ ①

Also from $y - 2a = t^2\left(\frac{4b^2}{a} - b\right)$

$$y - 2a = t^2\left(\frac{4b^2 - ab}{a}\right)$$

$$\Rightarrow t^2 = \frac{(y-2a)a}{4b^2 - ab}$$

$$t^2 = \frac{(y-2a)a}{b(4b-a)}$$

$$\Rightarrow t^6 = \frac{(y-2a)^3a^3}{b^3(4b-a)^3} \quad ②$$

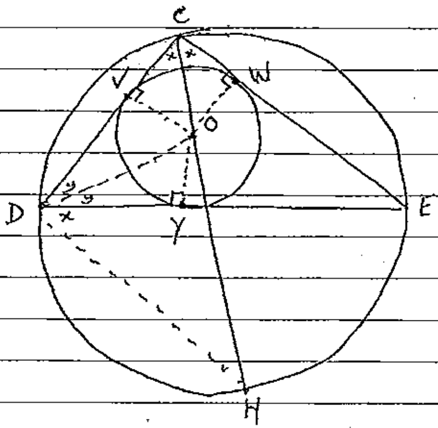
Then ① = ② $\Rightarrow \frac{x^2a^2}{4b^4} = \frac{(y-2a)^3a^3}{b^3(4b-a)^3}$

$$\Rightarrow x^2(4b-a)^3 = 4ab(y-2a)^3 \ \#$$

Only a few students got this question out correctly. 1 mark weas assigned if students correctly expressed x and y in terms of a,b and t

Question 9

a) i)



ii) In Δ's COV & COW

CO is common

OV = OW (equal radii)

$C\hat{V}O = C\hat{W}O = 90°$ (radius ⊥ tangent)

ΔCOV ≡ ΔCOW (RHS)

$V\hat{C}O = W\hat{C}O$ (corresponding angles, ΔCOV ≡ ΔCOW)

arc DH = arc HE (arcs that subtend equal angles are equal)

∴ H is the midpoint of arc DE.

iii) In Δ's DVO & DYO

DO is common

OV = OY (equal radii)

$D\hat{V}O = D\hat{Y}O = 90°$ (radius ⊥ tangent)

ΔDVO ≡ ΔDYO (RHS)

iv) let $V\hat{C}O = W\hat{C}O = x$ (proven in (ii))

let $V\hat{D}O = Y\hat{D}O = y$ (corresponding angles, ΔDVO ≡ ΔDYO)

$H\hat{D}E = x = H\hat{C}E$ (angles in the same segment)

$D\hat{O}H = x + y$ (exterior angle of ΔDOC)

$H\hat{D}O = x + y$

$\therefore\ O\hat{D}H = D\hat{O}H$

v) $HD = HO$ (sides opposite equal angles, ΔODH)

COMMENTS:

- Part (v) should have been an easy mark for all students regardless of their success with the other questions.
- Part (iii) could be proven using the congruency tests SSS, SAS, RHS, and was generally well attempted.
- Very few students made any progress with part (iv).
- There were students who oversimplified the diagram by making Y the point where CO meets DE.

b) i) $\cos(A-B) - \cos(A+B)$

$= \cos A\cos B + \sin A\sin B - (\cos A\cos B - \sin A\sin B)$

$= 2\sin A\sin B$

ii) Prove true for $n=1$

$LHS = \sin\omega$

$RHS = \dfrac{1-\cos 2\omega}{2\sin\omega}$

$= \dfrac{1-(1-2\sin^2\omega)}{2\sin\omega}$

$= \sin\omega$

$LHS = RHS$

$\therefore$ true for $n=1$

Assume true for $n=k$ where $k\in N$

$\sin\omega + \sin 3\omega + \dots + \sin(2k-1)\omega = \dfrac{1-\cos 2k\omega}{2\sin\omega}$

Prove true for $n=k+1$

ie $\sin\omega + \sin 3\omega + \dots + \sin(2k-1)\omega + \sin(2k+1)\omega = \dfrac{1-\cos 2(k+1)\omega}{2\sin\omega}$

$LHS = \sin\omega + \sin 3\omega + \dots + \sin(2k-1)\omega + \sin(2k+1)\omega$

$= \dfrac{1-\cos 2k\omega}{2\sin\omega} + \sin(2k+1)\omega$

$= \dfrac{1-\cos 2k\omega + 2\sin(2k+1)\omega\sin\omega}{2\sin\omega}$

$= \dfrac{1-\cos 2k\omega + \cos 2k\omega - \cos(2k+2)\omega}{2\sin\omega}$

$= \dfrac{1-\cos 2(k+1)\omega}{2\sin\omega}$

$= RHS$

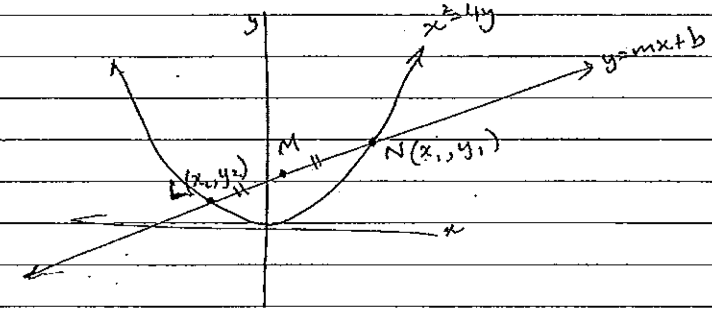
$\therefore$ true for $n=k+1$

$\therefore$ true by induction for integers $n\geq 1$.

COMMENT:

- Students should look to use part (i)
- Students should not assume what they are required to prove.

c)



i) $y = mx+b$ —— ①

$y = \frac{x^2}{4}$ —— ②

sub ① into ②

$\frac{x^2}{4} = mx+b$

$x^2 - 4mx - 4b = 0$ has roots x_1 & x_2

$x_1 + x_2 = \frac{-B}{A}$

$= \frac{-(-4m)}{1}$

$= 4m$

x coordinate of M is $\frac{x_1+x_2}{2} = \frac{4m}{2}$

$= 2m$

M lies on $y=mx+b$

y coordinate of M is $y = m(2m)+b$

$y = 2m^2+b$

∴ M has coordinates $(2m, 2m^2+b)$

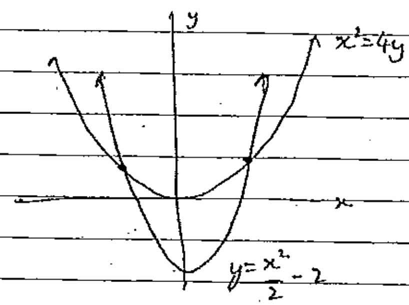
ii) $x = 2m$

$y = 2m^2 + (-2)$ when $b=-2$

$y = 2\left(\frac{x}{2}\right)^2 - 2$

$y = \frac{x^2}{2} - 2$ which is a parabola.

iii)



M is the midpoint of a chord & so must lie within the parabola $x^2=4y$.

$y = \frac{x^2}{4}$ —— ①

$y = \frac{x^2}{2} - 2$ —— ②

sub ① into ②

$\frac{x^2}{4} = \frac{x^2}{2} - 2$

$\frac{-x^2}{4} = -2$

$x^2 = 8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

∴ $x > 2\sqrt{2}$, $x < -2\sqrt{2}$

COMMENTS:

- Students who first found the coordinates of L & N tended to get bogged down in the algebra.
- The discriminant could have been used for part (iii). However, it would take a little longer, since we were asked for the restriction on the domain of M not m.