2011 Assessment Task 1

(November 2010)

MATHEMATICS

Extension 2

Year 12

Time allowed - 90 minutes (plus 5 minutes reading time)

Topics: Curve Sketching and Circular Motion

Instructions

Name and Class ______________________

- Attempt all questions.
- Questions are NOT of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only.
- Questions do not necessarily appear in order of difficulty.
- Diagrams are not to scale
- Use $g = 10ms^{-2}$ where needed

Question One (8 Marks)

Given $f(x) = 1 + x$ draw sketch graphs of the following:

a) $f(x)$ [1]

b) $f^{-1}(x)$ [1]

c) $[f(x)]^2$ [2]

d) $\dfrac{1}{f(x)}$ [2]

e) $-\sqrt{f(x)}$ [2]

Question Two (6 marks)

Sketch the following showing any important features.

a) $y = x^3(x+3)^2(x-2)$ [2]

b) $y = -\sqrt{9-x^2}$ [2]

c) $x^2 + 4y^2 = 1$ [2]

Question Three (3 marks)

Given $f(x) = (x-2)^2; x \geq 2$

a) Find $f^{-1}(x)$ [1]

b) Find the co ordinates of the point of intersection of $f(x)$ and $f^{-1}(x)$ [2]

Question Four (12 marks)

Sketch the following showing any intercepts or asymptotes

a) $y = \dfrac{x^2}{x^2 - 4}$ [3]

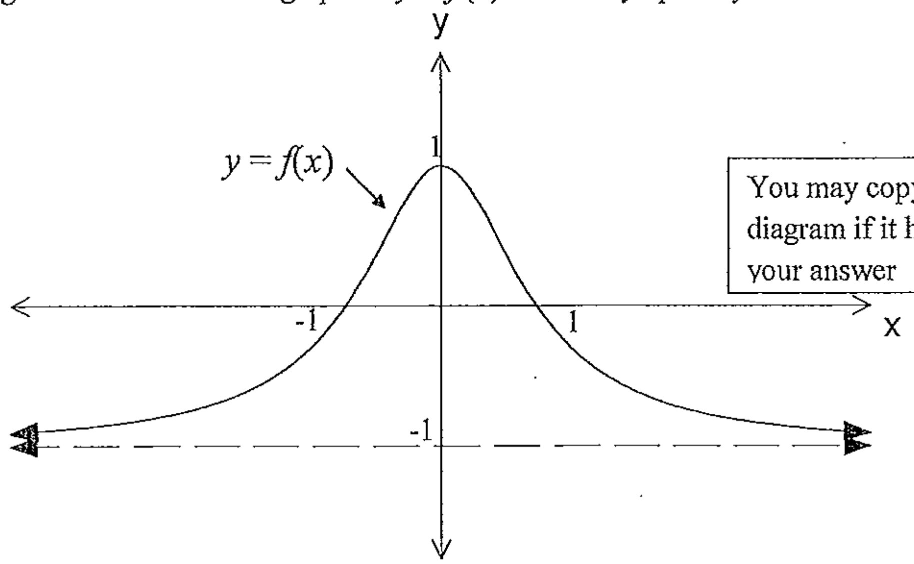
b) $(x-1)(y+2) = -1$ [3]

c) $y^2 - x^2 = 4$ [3]

d) $y = \dfrac{x^2 + 1}{x}$ [3]

Question Five (8 marks)

The diagram below shows the graph of $y = f(x)$ and its asymptote $y = -1$



You may copy or trace the diagram if it helps with your answer

Draw separate 1/3 page sketches of the graphs of the following:

a) $y = |f(x)|$ [2]

b) $y = \dfrac{1}{\sqrt{f(x)}}$ [2]

c) $y = 2^{f(x)}$ [2]

d) $y = xf(x)$ [2]

Question Six (4 marks)

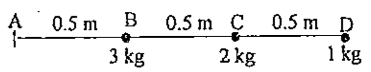
The angular speed of a grindstone is 2400 revolutions per minute. The radius of the grindstone is 20 cm.

a) Calculate the angular velocity in $rad.s^{-1}$ [2]

b) Find the linear velocity of a point on the circumference [2]

Question Seven (7 marks)

A 1.5 metre piece of string AD has masses of 3kg, 2kg and 1kg placed at B, C and D respectively.

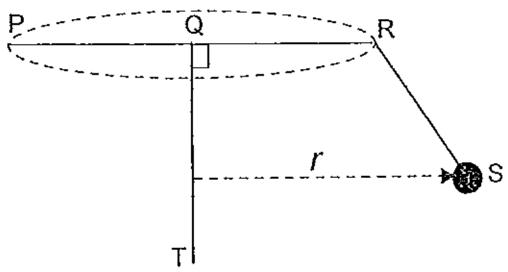


The string rotates in a horizontal circle on a smooth table about A and will break if the angular speed exceeds 8 $rad.s^{-1}$.

a) Find the breaking strain (maximum tension of the string) [4]

b) Find the maximum angular velocity of the string if the positions of the 3kg and 1kg masses are reversed. [3]

Question Eight (6 marks)
A schematic diagram of the 'Wave Swinger' at Luna Park is shown below.

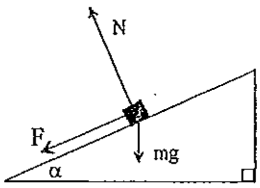


The system is designed so that it rotates about QT with the arm RS swinging out. The radius of motion r is shown. A mass m is attached at S.

If $QR = 4$m, $RS = 2$m, find the speed (in ms^{-1}) that S moves at if $\angle QRS = 135^0$.

Question Nine (6 marks)

An object of mass m kg is travelling around a banked circular track of radius r metres. The track is banked at α to the horizontal and the mass is moving at $v\ ms^{-1}$. The forces acting on the object are the gravitation force mg, a sideways frictional force F and the normal reaction force N.

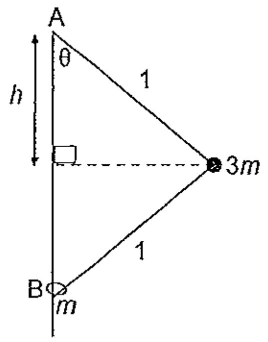


a) By resolving forces both vertically and horizontally, find an expression for F independent of N [4]

b) Given that the radius of the curve is 500m and that $\tan\alpha = \frac{1}{100}$, find [2]

the ideal velocity so that there is no tendency to slip sideways(use $g = 10ms^{-2}$).

Question Ten (4 marks)

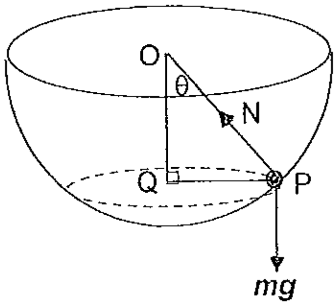


In the diagram above, a light string of length two metres is attached to a smooth metal rod at A and B. A mass of $3m$ kilograms is attached to the middle of the string. A second mass of m kilograms in the form of a ring is attached to the end of the string at B. The $3m$ kg mass is rotating in circular motion at $w\ rad.s^{-1}$ and the m kg mass is free to move up and down the smooth vertical rod. The string makes an angle of θ with the vertical. Acceleration due to gravity is $g\ ms^{-2}$.

Given that h is the distance between A and the centre of the circular motion, find an expression for h in terms of g and w

Question Eleven (8 marks)

A smooth hemispherical bowl of internal radius R metres and centre O is positioned so that its rim is horizontal. A particle P of mass m rotates at velocity $v\ ms^{-1}$ on the inner surface in horizontal circle of radius r metres. OP makes an angle θ with OQ as shown below.

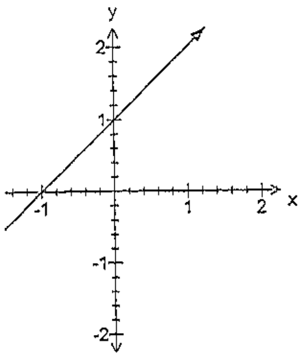


Note: OP = R metres
QP = r metres

a) Find an expression for v^2 in terms of g, R and θ [4]

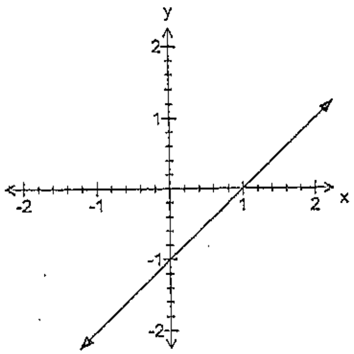
b) Hence show that the normal force is given by $N = \dfrac{mv^2 \pm \sqrt{m^2v^4 + 4R^2m^2g^2}}{2R}$ [4]

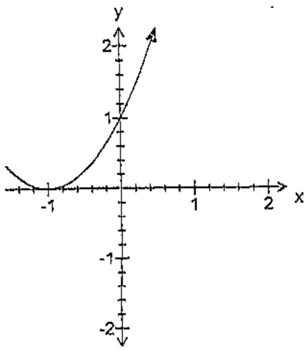
Extn 2 Task 1 2010 (for HSC 2011)



b.

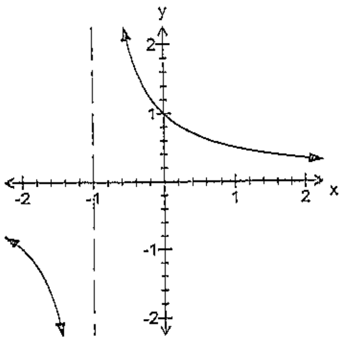
$f^{-1}(x)$

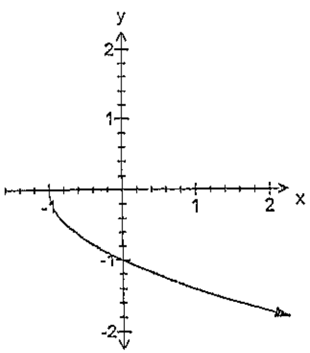




d.

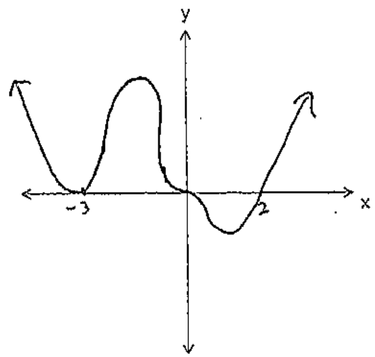
$\frac{1}{f(x)}$



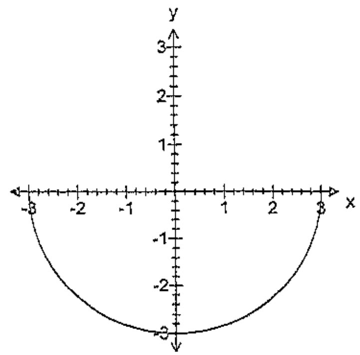


2.

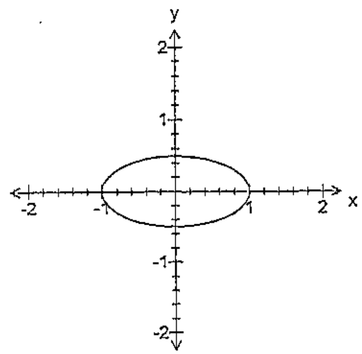
a.



b.



c.



3.

a.

$x = (y-2)^2$

$\sqrt{x} = y-2$

$y = \sqrt{x}+2$

b.

$x = (x-2)^2$

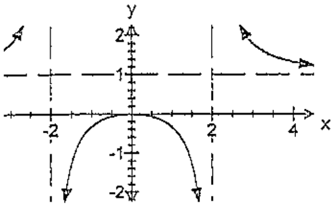
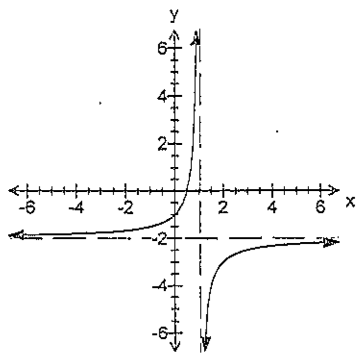
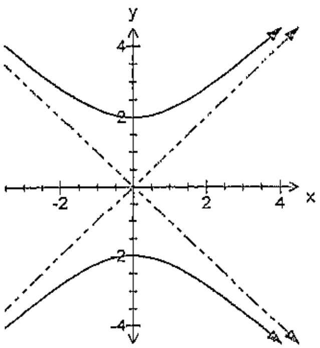
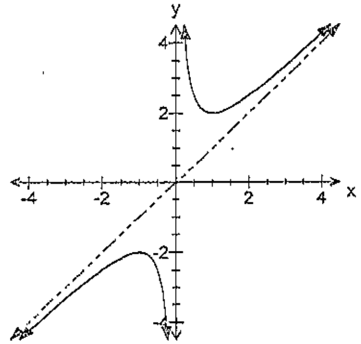
$x = x^2 - 4x + 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

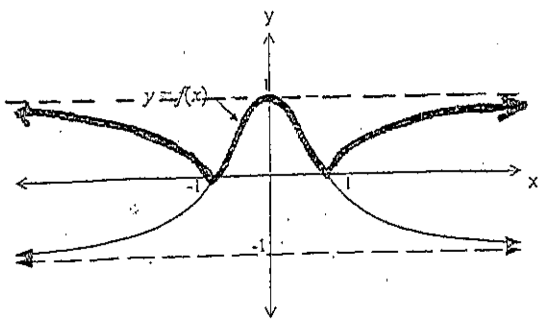
When $x = 4, y = 4$

$\therefore$ the curves intersect at (4, 4)

$\frac{x^2}{\ldots -4}$ 	b. $(x-1)(y+2)=-1$ 
$=4$ 	d. $y=\frac{x^2+1}{x}$ 

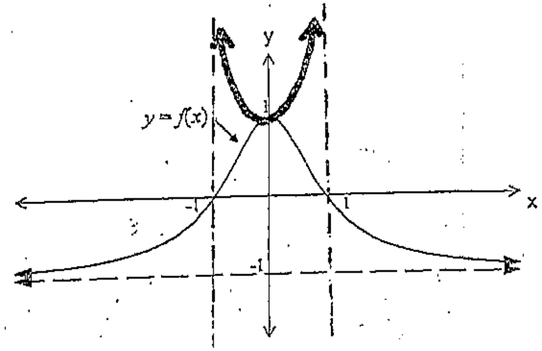
Question Five [2+2+2+2=8 marks]

(a)



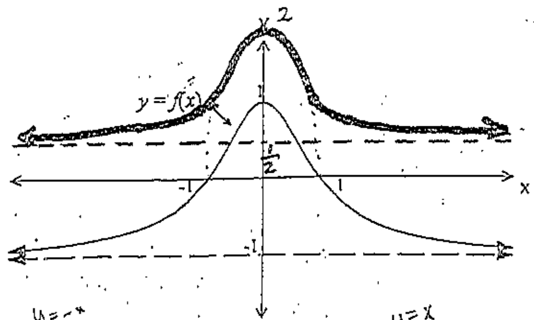
$y = |f(x)|$

(b)



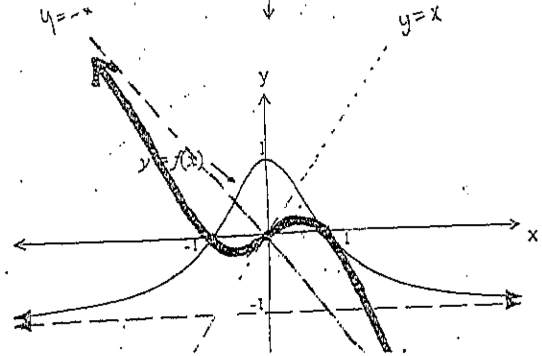
$y = \frac{1}{\sqrt{f(x)}}$

(c)



$y = 2^{f(x)}$

(d)



$y = x\,f(x)$

Question Six [2+2=4 marks]

(a)

$$\begin{aligned}\omega &= 2400 \text{ revs/min}\\ &= \frac{2400\times 2\pi}{60} \text{ rads/s}\\ &= 80\pi \text{ rads/s}\\ \omega &\doteq 251.33 \text{ rads/s}\end{aligned}$$

(b)

$$\begin{aligned}v &= r\omega\\ &= 0.2\times 80\pi \text{ m/s}\\ &= 16\pi \text{ m/s}\\ v &\doteq 50.265 \text{ m/s}\end{aligned}$$

Question Seven [4+3=7 marks]

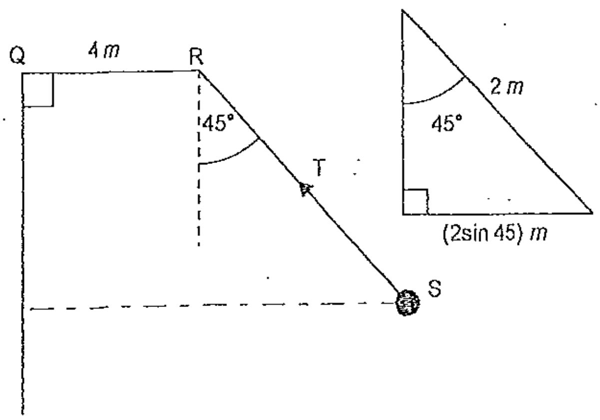
(a)

$$\begin{aligned}T &= \left(1\times 1.5\times 8^2\right)+\left(2\times 1\times 8^2\right)+\left(3\times 0.5\times 8^2\right)\\ &= 96+128+96\\ T &= 320 \text{ N}\end{aligned}$$

(b)

$$\begin{aligned}512 &= \left(3\times 1.5\times \omega^2\right)+\left(2\times 1\times \omega^2\right)+\left(1\times 0.5\times \omega^2\right)\\ 7\omega^2 &= 320\\ \omega &= \sqrt{\frac{320}{7}} \doteq 6.76 \text{ rad/s}\end{aligned}$$

Question Eight [6 marks]



$$\left.\begin{aligned}T\sin 45 &= \frac{mv^2}{r}\\ T\cos 45 &= mg\end{aligned}\right\}$$

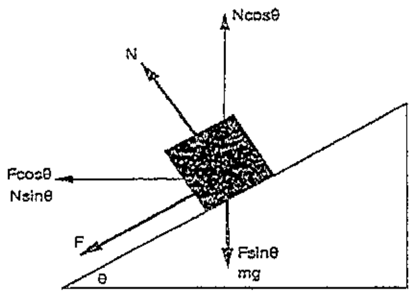
$$\tan 45 = \frac{v^2}{gr}$$

$$v = \sqrt{gr} \Rightarrow r = 4+2\sin 45 = 4+\sqrt{2}$$

$$v = \sqrt{10\left(4+\sqrt{2}\right)} \doteq 7.36 \text{ m/s}$$

Question 9:

a)



$$N\cos\theta = F\sin\theta + mg$$

$$N\cos\theta - F\sin\theta = mg \qquad (1)$$

$$N\sin\theta + F\cos\theta = \frac{mv^2}{r} \qquad (2)$$

$$(1)\times\sin\theta: \quad N\cos\theta\sin\theta - F\sin^2\theta = mg\sin\theta \qquad (3)$$

$$(2)\times\cos\theta: \quad N\cos\theta\sin\theta + F\cos^2\theta = \frac{mv^2}{r}\cos\theta \qquad (4)$$

$$(4)-(3): \quad F\cos^2\theta + F\sin^2\theta = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

b) If there is no tendency to slip, $F = 0$:

$$\frac{mv^2}{r}\cos\theta - mg\sin\theta = 0$$

$$\frac{v^2}{r}\cos\theta = g\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg}$$

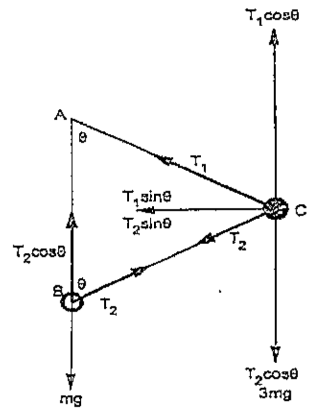
$$\tan\theta = \frac{v^2}{rg}$$

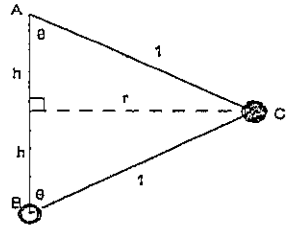
$$\frac{1}{100} = \frac{v^2}{500\times10}$$

$$v^2 = 50$$

$$v = 5\sqrt{2}\ ms^{-1}$$

Question 10:





At B:

$$T_2\cos\theta = mg$$

$$T_2 = \frac{mg}{\cos\theta}$$

At C (vertically):

$$T_1\cos\theta = T_2\cos\theta + 3mg$$

$$= mg + 3mg$$

$$= 4mg$$

$$T_1 = \frac{4mg}{\cos\theta}$$

At C (horizontally):

$$T_1\sin\theta + T_2\sin\theta = 3mr\omega^2$$

$$\frac{mg}{\cos\theta}\cdot\sin\theta + \frac{4mg}{\cos\theta}\cdot\sin\theta = 3mr\omega^2$$

$$mg\tan\theta + 4mg\tan\theta = 3mr\omega^2$$

$$5g\tan\theta = 3r\omega^2$$

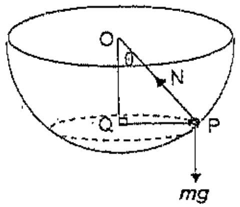
$$\tan\theta = \frac{3r\omega^2}{5g}$$

$$\frac{r}{h} = \frac{3r\omega^2}{5g}$$

$$h = \frac{5g}{3\omega^2}$$

Question 11:

a)



Note: OP = R metres
QP = r metres

$$N\cos\theta = mg$$
$$N\sin\theta = \frac{mv^2}{r}$$
$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$

But $\sin\theta = \frac{r}{R}$ and so $r = R\sin\theta$

$$v^2 = Rg\sin\theta\tan\theta$$

b)

$$N\cos\theta = mg \quad (1)$$
$$N\sin\theta = \frac{mv^2}{r} \quad (2)$$
$$(1)\times\cos\theta: \ N\cos^2\theta = mg\cos\theta \quad (3)$$
$$(2)\times\sin\theta: \ N\sin^2\theta = \frac{mv^2}{r}\sin\theta \quad (4)$$
$$(3)+(4): N = mg\cos\theta + \frac{mv^2}{r}\sin\theta$$
$$N = mg\cdot\frac{mg}{N} + \frac{mv^2}{R\sin\theta}\cdot\sin\theta \quad \left[\text{using } N\cos\theta = mg \text{ and } \sin\theta = \frac{r}{R}\right]$$
$$= \frac{m^2g^2}{N} + \frac{mv^2}{R}$$
$$RN^2 = m^2g^2R + mv^2N$$
$$RN^2 - mv^2N - m^2g^2R = 0$$
$$N = \frac{mv^2 \pm \sqrt{(-mv^2)^2 + 4R(m^2g^2R)}}{2R}$$
$$= \frac{mv^2 \pm \sqrt{m^2v^4 + 4R^2m^2g^2}}{2R}$$