

PROBABILITY: COMBINATIONS

The order in which elements of the set are selected is often not important. For instance if you are playing cards, the cards in your hand are important while the order in which you were dealt those cards is not.

Example: In how many ways can a committee of three be chosen from a group of 10?
Answer:

If order is important then the number of ways is ${}^{10}P_3 = 720$ ways.

Suppose Tom, Dick and Harry are the three representatives. The chance of them being

chosen in that order is $\frac{1}{720}$. But there are 3! i.e.6 ways in which Tom, Dick & Harry can be chosen. (TDH, THD, DTH, DHT, HTD, HDT) so the probability of Tom Dick

and Harry being on the committee becomes $\frac{6}{720} = \frac{1}{120}$

And the number of possible arrangements is 120.

This is known as the number of combinations and is written nC_r and may be calculated by :

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

This can be determined directly by most calculators.

Exercise 1:

1. What is the number of possible different hands (combinations of cards) if a hand consists of 5 cards?
2. In how many ways can a group of 10 students be chosen from a class of 25?
3. What is the probability of someone being dealt a royal flush (10, J, Q, K, A of the same suite) in a hand of 5 cards?
4. Keno consists of 80 numbers of which 20 are drawn in each game. How many combinations are possible in a game of keno.

Sometimes we combine the product rule with combinations.

Example: In how many ways can a committee of 3 boys and 3 girls be selected from a class of 16 boys and 14 girls?

Answer: The boys may be chosen ${}^{16}C_3$ ways and the girls ${}^{14}C_3$ ways.

$$\text{Boys } {}^{16}C_3 = 560 \text{ ways} \quad \text{Girls } {}^{14}C_3 = 364 \text{ ways}$$

$$\text{Total} = 560 \times 364 = 203840 \text{ ways.}$$

Exercise 2:

1. How many ways may a Parliamentary Committee be formed if it is to consist of 4 government members and 3 opposition members if 8 government and 10 opposition members apply?
2. A bookshelf contains 20 mathematics books and 14 science books. In how many ways can a person choose 2 mathematics books and two science books?
3. A class consists of 17 boys and 13 girls. In how many ways is it possible to choose a group of 3 boys and 2 girls?

Answers:

Ex.1.	1. 2598960	2. 3268760	3. $\frac{4}{2598960} = \frac{1}{649740}$	4. 3.54×10^{18}
Ex.2.	1. 8400	2. 17290	3. 53040	