

PROBABILITY: PERMUTATIONS

Permutations refer to the number of ways a given set can be arranged. For instance the letters abc can be arranged 6 ways: abc, acb, bac, bca, cab, cba.

Example: In how many ways can 5 people queue in single file for movie tickets?

Answer: The first position in the queue may be filled in one of 5 ways. There are now 4 people left so the second position may be filled in one of 4 ways. Similarly the third position can be filled in 3 ways, the fourth position in 2 ways and the fifth position in 1 way.

Hence the number of arrangements for 5 people in a queue is $5 \times 4 \times 3 \times 2 \times 1 = 120$.

The number of arrangements for n items is: $n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$

The product of an integer n and all the positive integers below it is called “factorial n” and is written n!

Exercise 1:

1. In how many ways may the letters of the word “MONDAY” be arranged?
2. Cathy received 5 DVDs for her birthday. In how many ways can she arrange the order of viewing her 5 DVDs?
3. Thirty students enter a classroom in single file. In how many ways can the order of them entering the classroom be arranged?

Sometimes not all of the positions have to be allocated.

Consider a raffle where there are 3 prizes and 100 tickets.

The first prize may be allocated in one of 100 ways. Once the first prize has been drawn there are 99 tickets left so the second prize may be allocated in one of 99 ways. There are 98 tickets left for the third prize so the third prize may be allocated 98 ways. Thus the number of ways that the first three prizes may be distributed is:

$$100 \times 99 \times 98 = 970200$$

Another way of writing this is $\frac{100!}{97!}$ i.e. $\frac{100!}{(100-3)!}$

The number of ways r objects can be arranged from n different objects is: $\frac{n!}{(n-r)!}$
This is written ${}^n P_r$ and can be determined directly by most calculators.

Exercise 2:

1. A club of 40 members is to elect a president, vice-president, secretary and treasurer. In how many ways may these positions be filled?
2. (i) What is the probability of picking the trifecta (first 3 places in correct order) in a horse race of 18 runners of equal ability?
(ii) What is the probability of picking the first 5 places in correct order from a field of 18 horses of equal ability?
3. A librarian has 80 equal size books but only has room for 10 of them on the shelf. In how many ways can she arrange the ten books on the shelf?

Answers : Ex.1. 1. $6! = 720$ 2. $5! = 120$ 3. $30! = 2.65 \times 10^{32}$
Ex.2. 1. 2193360 2. (i) $\frac{1}{4896}$ (ii) $\frac{1}{1028160}$ 3. 5.97×10^{18}