HSC Mathematics 2010
HSC Task #1

Name: _____

Time: 50 minutes
Value: 40 marks
Teacher: RDS HRK GHW BG1 CRA RABS

☺ *Show all working to gain maximum marks*

☹ *Marks will be deducted for poor or illegible work*

Part A (10 marks) **Start a new booklet** **Marked by RABS**

1. Evaluate $\sum_{p=2}^{14}(2p-4)$ **2**

2. $(x-8)$, x and 36 are three consecutive terms in a geometric series.
Find possible value(s) of x. **3**

3. The 8^{th} term in an arithmetic series is 45 and the 13^{th} term is 80.
Find the value of the first term and common difference. **2**

4. Consider the following series:

$$1+\left(\sqrt{6}-2\right)+\left(\sqrt{6}-2\right)^2+..$$

a. Explain why this series has a limiting sum. **1**

b. Find the exact value of the limiting sum. Write your answer with a rational denominator. **2**

Part B (10 marks)	Start a new booklet	Marked by RDS

1. Janet is learning to make pretzels. On the first day she made 12 pretzels but each day she found she could make 4 more than the day before.

a. How many pretzels would she be able to make on the 5^{th} day? **1**

b. How many pretzels will she have made after 5 days? **1**

c. When will she have made a total of over 200 pretzels? **2**

2. Kate pays \$1900 into her superannuation fund at the start of each year. The fund will pay interest at 10% per annum for the next 25 years.

a. Find the value of the first \$1900 investment after 25 years **1**

b. Show that the value of the fund after 25 years will be $20900\left[(1.1)^{25}-1\right]$ **3**

c. Calculate the total interest gained **2**

Part C (7 marks) **Start a new booklet** **Marked by CRA**

1. Anthony borrowed \$50000 with interest charged at the rate of 15% per annum compounded monthly. The loan is to be paid off in equal monthly instalments of *\$M* over a period of 15 years.

Let A_n be the amount owing after n months

a. Show that the amount owing after 3 months is
$A_3 = 50000 \times 1.0125^3 - M(1+1.0125+1.0125^2)$ **3**

b. Find the value of the monthly instalments, \$M (to the nearest dollar). **3**

c. Hence, calculate the total amount paid for the loan **1**

Part D (13 marks) **Start a new booklet** **Marked by HRK**

1. A function $y = f(x)$ is continuous for all values.
After finding the first and second derivatives a student discovers the following, for all values of x: **2**

When $x < 2$, $f'(x) < 0$ and $f''(x) > 0$
When $x = 2$, $f'(x) = 0$ and $f''(x) = 0$
When $x > 2$, $f'(x) < 0$ and $f''(x) < 0$.

Draw a neat sketch of $y = f(x)$, showing all the important characteristics of the function given that $f(2) = 0$.

2. For the function $y = x^3 - 6x^2 + 9x + 1$ find the:

a. Values of x for which the curve is increasing. **2**

b. Stationary points and determine their nature. **4**

c. Co-ordinates of any points of inflection. **2**

d. Sketch curve $y = x^3 - 6x^2 + 9x + 1$ showing all important features **3**

HSC MATHS SERIES / GEOM APPS
ASSESS. TASK 2010

1. $T_1 = 0$ $\quad a = 0$
$T_2 = 2$ $\quad d = 2$
$T_3 = 4$ $\quad n = 13$ ✓

$$S_{13} = \frac{13}{2}\left[0 + (12 \times 2)\right]$$
$$= 156 \checkmark$$

- GREAT. MOST ANSWERED CORRECTLY!

2. $\frac{x}{x-8} = \frac{36}{x}$ ✓ COMMON RATIO!

$$x^2 = 36(x-8)$$
$$x^2 = 36x - 288$$
$$0 = x^2 - 36x + 288 \checkmark$$
$$(x-12)(x-24)$$
$$x = 12 \text{ or } 24 \checkmark$$

- MOSTLY CORRECT. IF YOU'RE GETTING RECURRING DECIMALS, GO BACK & FIND YOUR ERROR!
- REMEMBER IT'S A GP WITH A COMMON RATIO.

3. $T_8 = 45 = a + 7d$ — ①

$T_{13} = 80 = a + 12d$ — ②

② - ①: $35 = 5d$
$d = 7$ ✓
$a = -4$ ✓ (SUB. INTO ANY)

- AGAIN, MOSTLY CORRECT.

4. $\sqrt{6} - 2 = 0.44\ldots$

(a) AS $|0.44\ldots| < 1$, A LIMITING SUM EXISTS. ✓

- NOT ANSWERED TO WELL. MANY DID NOT PROVE THAT $\sqrt{6}-2$ HAD A VALUE BETWEEN -1 & 1 OR JUST SIMPLY STATED THE CRITERIA.

(b) $S_\infty = ?$
$a = 1$
$r = (\sqrt{6} - 2)$

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{1}{1-(\sqrt{6}-2)} \checkmark$$
$$= \frac{1}{3-\sqrt{6}} \times \frac{3+\sqrt{6}}{3+\sqrt{6}}$$
$$= \frac{3+\sqrt{6}}{3} \checkmark$$

- NOT DONE TOO WELL. MANY HAD TROUBLE RATIONALISING THE DENOM.
- DON'T FORGET FORMULAE!

PART B SOLUTIONS

① $12 + 16 + 20 + 24 + \cdots$ (+4, +4, +4)

AP, $a = 12$, $d = 4$.

(a) $T_5 = 12 + (5-1)4$
$= 12 + 16$
$= 28$ pretzels ✓ → Answered well!

(b) $S_5 = \frac{5}{2}[12 + 28]$
$= 100$ pretzels ✓

(c) $S_n > 200$ ← remember your formulas
$\frac{n}{2}[2\times12 + (n-1)4] > 200$
$n(24 + 4n - 4) > 400$
$n(20 + 4n) > 400$
$20n + 4n^2 > 400$
$4n^2 + 20n - 400 > 0$ ✓
$n^2 + 5n - 100 > 0$
$n = \frac{-5 \pm \sqrt{5^2 - 4(1)(-100)}}{2}$
$n = \frac{-5 + \sqrt{425}}{2}$ or $n = \frac{-5 - \sqrt{425}}{2}$
$n = 7.807\ldots$ or $n = -12.807\ldots$ ✗
↓
∴ since $n > 0$,
After 8 days there will be over 200 pretzels ✓

↳ many students stopped at the quadratic formula! Work both out!

② (a) $A_1 = 1900(1.1)^{25}$ ✓
$= \$20,585.94$

(b) $A_1 = 1900(1.1)^{25}$
$A_2 = 1900(1.1)^{24}$ → Most students were ok creating the first few terms.
$\vdots$
$A_{25} = 1900(1.1)$

Total $= A_1 + A_2 + A_3 + \cdots$
$= 1900(1.1)^{25} + 1900(1.1)^{24} + \cdots + 1900(1.1)$ ✓
$= 1900(1.1 + 1.1^2 + \cdots + 1.1^{25})$
GP, $a = 1.1$, $n = 25$, $r = 1.1$
$= 1900\left[\frac{1.1(1.1^{25} - 1)}{1.1 - 1}\right]$ ✓ → Many students tried to use a prepared formula. Do not do this!!! Write out every step.
$= 1900\left[\frac{1.1(1.1^{25} - 1)}{0.1}\right]$
$= 20900\left[(1.1)^{25} - 1\right]$ ✓

(c) Total $= \$205545.35$ ✓
Total investment $= 1900 \times 25$
$= 47500$
∴ Total interest $= 205545.35 - 47500$
$= \$158,045.35$ ✓

PART C SOLUTIONS

① \$50000 borrowed 15% per annum (comp. monthly).
↳ paid back over 15 years.

(a) $r = \frac{15}{12/100} = 0.0125$ ✓ $n = 15$ yrs $= 180$ months.
→ One mark awarded for correct ratio.
→ remember to change ratio of n, to monthly, not years.

$A_1 = 50000(1.0125) - M$ ✓
$A_2 = A_1(1.0125) - M$
$= 50000(1.0125)^2 - M(1.0125) - M$
$A_3 = A_2(1.0125) - M$
$= 50000(1.0125)^3 - M(1.0125)^2 - M(1.0125) - M$.
$= 50000(1.0125)^3 - M(1 + 1.0125 + 1.0125^2)$ ✓

(b) $A_{180} = 50000(1.0125)^{180} - M(1 + 1.0125 + \cdots + 1.0125^{179})$ ✓

$0 = 50000(1.0125)^{180} - M\left[\frac{1(1.0125^{180} - 1)}{1.0125 - 1}\right]$

$\therefore M = \frac{50000(1.0125)^{180}}{\left[\frac{1.0125^{180} - 1}{0.0125}\right]}$ ✓ → AGAIN! Do not use a pre-prepared formula. Write out every step!

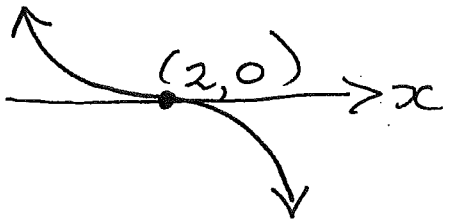
$\therefore M = \$699.79$
$= \$700$ ✓ → Many, many, many students lost a mark for not rounding to the nearest \$. Read the question carefully!

(c) Total $= \$700 \times 180$ payments
$= \$126,000$ ✓

Error carried forward (E.C.F) was applied with whatever M was calculated in part (b)

SECTION D

1/ LEARN THE RULES and connections !!
TRANSLATE $f(2)=0$ is the POINT $(2,0)$
$f'(x)<0$ decreasing $f''(x)>0$ concave up etc



2/ $y = x^3 - 6x^2 + 9x + 1$

$y' = 3x^2 - 12x + 9$

$y'' = 6x - 12$

(a) increasing means $y' > 0$

$3x^2 - 12x + 9 > 0$

FACTORISE

$\frac{3}{3}(x^2 - 4x + 3) > \frac{0}{3}$

$(x-3)(x-1) > 0$

WHEN $x<1$, $x>3$ curve increases.

Too many students had problems with factorising +/or quadratic inequalities. YOU MUST work on these if you are one of these students They are skills which are needed in a number of topics and will not go away!!

(b) FOR ST PTS $y' = 0$

$3x^2 - 12x + 9 = 0$ FROM (a)

$(x-3)(x-1) = 0$

$x = 1, 3$

then $y = 5, 1$

$f''(1) = -6 < 0$ ∩ concave DOWN $\therefore$ MAX at $(1,5)$

$f''(3) = 6 > 0$ ∪ concave UP $\therefore$ MIN at $(3,1)$

c) $f''(x) = 0$ for POSSIBLE inflexions

$6x - 12 = 0$

$x = 2$, then $y = 3$

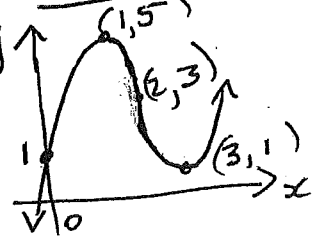
$\therefore (2,3)$ is a possible inflexion

MUST CHECK HERE OR Refer back to b)

x	1	2	3
y''	−	0	+

$\therefore$ inflexion at $(2,3)$

d) 1ST draw  THEN LABEL



MANY NEED TO PRACTISE Sketching