SYDNEY GIRLS HIGH SCHOOL

LABOR OMNIA VINCIT

# YEAR 12 MATHEMATICS

## ASSESSMENT TASK 2

March 2011

Time allowed: 75 minutes +5 min reading

Logs and exponentials, Trig functions I and Polynomials

Instructions:

- There are Four (4) questions. Questions **are not** of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Name: ............ ..............................

## STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\ a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x>0$

6300 - 3

## QUESTION ONE (19 marks)

a) Change $80^\circ$ into radians. (1)

b) Change $\frac{2\pi}{9}$ into degrees. (1)

c) Find the domain and the range of $y = \ln(x+3)$. (2)

d) Differentiate the following

i) $y = 3\cos x$ (1)

ii) $y = e^{4x+3}$ (1)

e) Find

i) $\int 3e^{5x+6}\,dx$ (1)

ii) $\int \sin 5x\,dx$ (1)

f) Differentiate the following

i) $y = \sin x \cos x$ (2)

ii) $y = \frac{2x}{e^x + 1}$ (2)

iii) $y = (e^{3x} + 2)^4$ (2)

g) Find the equation of the tangent to the curve $y = e^{4x}$ which passes through the origin. (3)

h) Use Newton's method once to find an approximation for the root of $x^3 + x^2 - 1$, given there is a root near $x = 1$. (2)

## QUESTION TWO (16 marks)

a) Find

i) $\int \frac{\sin x}{1+\cos x}\,dx$ (1)

ii) $\int \frac{e^x + e^{3x}}{e^{2x}}\,dx$ (2)

b) Find the area between the curve $y = -e^{2x}$, the $x$-axis and the lines $x = 0$ and $x = 1$. (2)

c) O is the centre of a circle with radius 10 cm. Angle $AOB = 72^\circ$. $A$ and $B$ are points on the circumference of the circle.

i) Find the length of the arc $AB$ (2)

ii) Find the area of the sector $AOB$ (1)

iii) Find the area of the minor segment bounded by the chord $AB$ (2)

d) i) State the amplitude and the period of $y = 2\sin 4x$ (2)

ii) Sketch the curve in the domain $0 \le x \le 2\pi$. (2)

e) The remainder when the cubic polynomial $p(x) = kx^3 + 4x + 9$ is divided by $x - 3$ is 75. Find the value of $k$. (2)

## *QUESTION THREE (20 marks)*

a) Differentiate

i) $y = \ln\left(\dfrac{2x+1}{1-3x}\right)$ (3)

ii) $y = -4\sin^2 x$ (2)

b) The roots of the $x^3 + 6x^2 + 8x + 3 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find the value of

i) $\alpha + \beta + \gamma$ (1)

ii) $\alpha\beta\gamma$ (1)

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ (2)

iv) $\alpha^2 + \beta^2 + \gamma^2$ (2)

v) $4\alpha^2\beta\gamma + 4\alpha\beta^2\gamma + 4\alpha\beta\gamma^2$ (2)

c) Solve $\log_e(2x+2) + \log_e x - \log_e 12 = 0$ (3)

d) Find the volume generated when the curve $y = \tan x$ is rotated about the $x$-axis between $x = 0$ and $x = \dfrac{\pi}{4}$. (4)

## *QUESTION FOUR ( 20 marks)*

a) i) Factorise the polynomial $y = 2x^3 - x^2 - 2x + 1$ (3)

ii) Without using calculus sketch the curve $y = 2x^3 - x^2 - 2x + 1$ (2)

iii) Hence or otherwise, find the solution to $2x^3 - x^2 - 2x + 1 \leq 0$ (1)

b) i) Show that $\dfrac{x+3}{x+5} = 1 - \dfrac{2}{x+5}$ (1)

ii) Hence find $\displaystyle\int \frac{x+3}{x+5}\,dx$ (2)

c) A vertical line $x = \dfrac{5\pi}{6}$ meets the curve $y = 4\sin\left(x - \dfrac{\pi}{6}\right)$ at $Q$ and $y = -3\cos\left(x + \dfrac{\pi}{3}\right)$ at $P$. Find the length of $PQ$ in exact form. (3)

d) The roots of $x^3 - 15x^2 - 6x + k = 0$ are in arithmetic progression. Find the value of $k$. (4)

e) When $P(x) = ax^3 + bx + c$ is divided by $x - 1$, the remainder is $-4$. When $P(x)$ is divided by $x^2 - 4$, the remainder is $-4x + 3$. Find the values of $a$, $b$ and $c$. (4)

Ext 1 Assessment Task 2 2011 March

Q1

a) $\pi = 180^\circ$

$80^\circ = \frac{\pi}{180} \times 80$

$= \frac{4\pi}{9}$

b) $\frac{2 \times 180^\circ}{9}$

$= 40$

c) Domain: $x > -3$

range: all real $y$

d) i) $y' = -3\sin x$

ii) $y' = 4e^{4x+3}$

e) i) $y = \frac{3e^{5x+6}}{5} + c$

iii) $y' = 4(e^{3x}+2)^3 \times 3e^{3x}$

$= 12e^{3x}(e^{3x}+2)^3$

g) $y' = 4e^{4x}$

$y - 0 = 4e^{4x}(x-0)$

$y = 4e^{4x} x$ } equate to find p. of Intersec

$y = e^{4x}$

$e^{4x} = 4e^{4x} x$

$\therefore x = \frac{1}{4}$

$\therefore y = 4e^{\frac{1}{4}\times 4} x$ $\quad \therefore \boxed{y = 4ex}$

h) $a_1 = 1 - \frac{(1+1-1)}{3+2}$

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

ii) $y = \frac{-\cos 5x}{5} + c$

f) i) $y' = \cos x \cos x + \sin x(-\sin x)$

$= \cos^2 x - \sin^2 x$

ii) $y' = \frac{2(e^x+1) - e^x(2x)}{(e^x+1)^2}$

Q2)

a) i) $y = \ln(1+\cos x) + C$

ii) $\int e^{-x} + e^{x}\, dx$

$= -e^{-x} + e^{x} + C$

b) $A = \left| \int_0^1 -e^{2x}\, dx \right|$

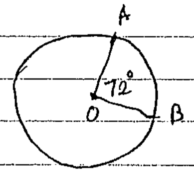
$= \left| \left[ -\frac{e^{2x}}{2} \right]_0^1 \right|$

$= \frac{-e^1}{2} + \frac{1}{2}$

$= \frac{1}{2} - \frac{e}{2}$

$= \left| \frac{1-e}{2} \right|$

c)




i) $\ell = \theta r$

$= \frac{72\pi}{180} \times 10$

$= 4\pi$

ii) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(10^2) \frac{2\pi}{5}$

$= \frac{200\pi}{10}$

$= 20\pi$

iii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} 100 \left(\frac{2\pi}{5} - \sin\frac{2\pi}{5}\right)$

$= 50(0.3056)$

$\div 15.28$

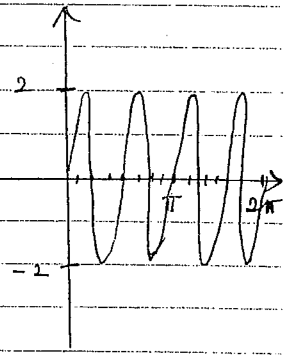
d) i) amp $= 2$

$P = \frac{2\pi}{b}$

$= \frac{2\pi}{4}$

$= \frac{\pi}{2}$

ii)



e)

$P(3) = 27k + 12 + 9$

$75 = 27k + 21$

$54 = 27k$

$\boxed{k = 2}$

Q3) a)

i) $y = \ln(2x+1) - \ln(1-3x)$

$y' = \frac{2}{2x+1} - \frac{-3}{1-3x}$

$= \frac{2(1-3x) + 3(2x+1)}{(2x+1)(1-3x)}$

$= \frac{5}{(2x+1)(1-3x)} = \frac{5}{1-6x^2+2x}$

ii) $y = -4(\sin x)^2$

$y' = -8\sin x \cos x$

b) i) $\alpha + \beta + \gamma = -6$

ii) $\alpha\beta\gamma = -3$ $\quad \alpha\beta + \beta\gamma + \gamma\alpha = 8$

iii) $\frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{8}{-3} = -2\frac{2}{3}$

iv) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$
$= (-6)^2 - 2(8)$
$= 36 - 16$
$= 20$

v) $4\alpha\beta\gamma(\alpha + \beta + \gamma)$
$= 4(-3)(-6)$
$= 72$

c) $\log_e (2x+2)x = \log_e 12$

$2x^2 + 2x = 12$

$2x^2 + 2x - 12 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

$x \neq -3$

$\therefore \underline{\underline{x = 2}}$

d) $V = \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

$= \pi \int \sec^2 x - 1 \, dx$

$= \pi \left[ \tan x - x \right]_0^{\frac{\pi}{4}}$

$= \pi \left[ \tan\frac{\pi}{4} - \frac{\pi}{4} - \tan 0 - 0 \right]$

$= \pi \left[ 1 - \frac{\pi}{4} \right]$

$= \frac{4\pi - \pi^2}{4} \ u^3$

Q4) a)

$P(1) = 2 - 1 - 2 + 1$
$= 0$

$\therefore (x-1)$ is a factor

$$
\begin{array}{r}
2x^2 + x - 1 \\
(x-1)\,\overline{)\,2x^3 - x^2 - 2x + 1} \\
\underline{2x^3 - 2x^2 \qquad\qquad} \\
x^2 - 2x \qquad \\
\underline{x^2 - x \qquad} \\
-x + 1 \\
\underline{-x + 1} \\
0
\end{array}
$$

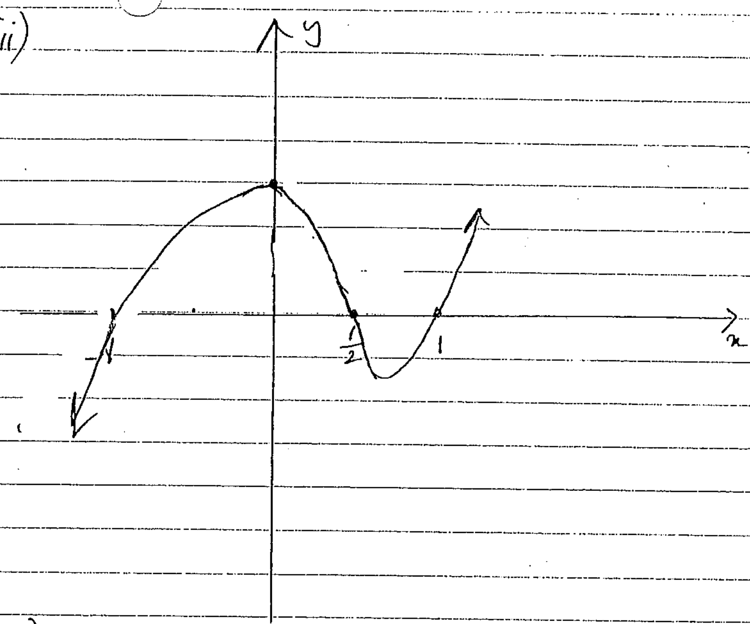
$2x^3 - x^2 - 2x + 1 = (x-1)(2x^2 + x - 1)$

$= (x-1)(2x^2 + 2x - x - 1)$

$= (x-1)(2x(x+1) - (x+1))$

$= (x-1)(2x-1)(x+1)$

ii)




iii) $x \leq -1$, $\frac{1}{2} \leq x \leq 1$

b) i) $1 - \frac{2}{x+5}$

$= \frac{x+5-2}{x+5}$

$= \frac{x+3}{x+5}$

ii) $\int \frac{x+3}{x+5} \, dx = \int 1 - \frac{2}{x+5} \, dx$

$= x - 2 \log_e (x+5) + C$

c) $x = \frac{5\pi}{6}$

$y = 4\sin\left(\frac{4\pi}{6}\right)$

$= 4\sin\left(\frac{2\pi}{3}\right)$

$= \frac{4\sqrt{3}}{2}$

$x = \frac{5\pi}{6}$

$y = -3\cos\left(\frac{5\pi}{6} + \frac{\pi}{3}\right)$

$= -3\cos\left(\frac{7\pi}{6}\right)$

$= -3\left(\frac{-\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2}$

$PQ = \frac{\sqrt{3}}{2}$

d) $a-d,\ a,\ a+d$

$3a = 15$

$a = 5$

$(a-d)a + a(a+d) + (a+d)(a-d) = -6$

$a^2 - ad + a^2 + ad + a^2 - d^2 = -6$

$3a^2 - d^2 = -6$

$75 - d^2 = -6$

$d^2 = 81 \qquad d = \pm 9$

$d = 9 \quad a = 5$

$(a-d)(a)(a+d) = -k$

$(5-9)(5)14 = -k$

$\boxed{k = 280}$

e) $P(1) = a+b+c$

$\boxed{a+b+c = -4}$ — (1)

$P(2) = 8a+2b+c$

$-4(2)+3 = 8a+2b+c$

$-8+3 = 8a+2b+c$

$\boxed{-5 = 8a+2b+c}$ — (2)

$P(-2) = -8a-2b+c$

$8+3 = -8a-2b+c$

$\boxed{11 = -8a-2b+c}$ — (3)

$c = -4-a-b$ (4)

$-5 = 8a+2b-4-a-b$ (5)

$11 = -8a-2b-4-a-b$ (6)

$-5 = 7a+b-4$

$11 = -9a-3b-4$

$7a+b = -1$

$-9a-3b = 15$

$21a + \cancel{3b} = -3$

$-9a-3b = 15$

$12a = 12$

$\boxed{a = 1}$

$7+b = -1$

$\boxed{b = -8}$

$c = -4-1+8$

$\boxed{c = 3}$